Microwave Theory and Techniques Prof. Girish Kumar Department of Electrical Engineering Indian Institute of Technology, Bombay

Module - 05 Lecture - 24 Microwave Filters - IV: Band Pass Filters

Hello, in the last few lectures we have been talking about low pass filters. In the low pass filter we discuss about different types of filter realizations, such as Butterworth, Chebyshev, Bessel filter, and elliptic filters.

We saw how to find g parameters for these different Butterworth and Chebyshev filter. For Butterworth the expression for g parameters was very very simple. However, for chebyshev we have to first define how much ripple we want in the pass band? And, as I mention please do not designed for ripples more than 0.5 db, in the pass band because more than 0.5 dB ripples in the pass band will lead to very poor VSWR in the pass band region.

And, we had also seen that for a faster transition from the pass band to stop band, which we had taken a design example as omega 1 divided by omega c equal to 1.2. Where we had taken that design example of omega 1 divided by omega c equal to 1.2, where we wanted attenuation of 30 dB. We saw that a Butterworth filter would require and 19th order filter. Whereas, a chebyshev with the ripple in the pass band of 1 dB would require only 8th order filter.

So, you can see that if you use chebyshev filter, you can reduce the number of components. So, we noted that for the chebyshev filter the order required was only 8th order whereas, for Butterworth the order required was 19th. So, that means, you will require 8 lc components for the chebyshev filter and 19 lc components for Butterworth filter.

So, depending upon the requirement one should do that design accordingly. So, we had also seen that instead of realizing these filters using lumped elements like, inductors and capacitors, we can also use microstrip lines, we had earlier seen a small transmission line of length less than lambda by 4, if it is shorted it realizes inductor and if it is open circuited, it realizes capacitor.

So, we had used that concept to a design low pass filter. Now, let us look into how we can do the transformation from low pass filter to high pass filter or bandpass filter or band stop filter?.

(Refer Slide Time: 02:56)

Transformation from LPF to HPF, BPF, and BSFLow Pass Filter:
$$H(s) = \frac{1}{s+1}$$
High Pass Filter: $H(s) |_{s \to \frac{1}{s}} = \frac{1}{\frac{1}{s+1}} = \frac{s}{s+1}$ Band Pass Filter: $H(s) |_{s \to \frac{s^2 + \omega_0^2}{Bs}} = \frac{Bs}{s^2 + Bs + \omega_0^2}$ Band Stop Filter: $H(s) |_{s \to \frac{Bs}{s^2 + \omega_0^2}} = \frac{s^2 + \omega_0^2}{s^2 + Bs + \omega_0^2}$

In fact, we had seen in the previous lecture that a transfer function of a low pass filter is given by this particular expression. And since we know that high pass filter is nothing, but inverse of low pass filter all we have to do what is in this S over here, which we have designated as capital S over here. If, we change this S to 1 by S, then we can realize a high pass filter and we had seen very quick example let us just go through it quickly. So, if you put a S equal to 1 by S 1 by S over here.

So, that is the high pass filter expression for bandpass filters, what we have to do? We have to transfer low pass filter which was from 0 to normalize value of 1 to a bandpass filter, with the center frequency of omega 0 and bandwidth equal to b.

So, all we have to do it is shift this one to the central position, where center frequency is equal to omega 0. So, we use the transformation that is s we use this particular expression S square plus omega 0 squared divided by B S. So, that is the transfer function of bandpass filter. And, the transfer function of band stop filter can be obtained by knowing the fact, that band stop is nothing, but inverse of band pass filter. So, we use the inversion of this one here which is B S divided by S square plus omega 0 square. And, we get the transfer function of band stop filter.

Now, comes the next part how do we design these filter? Since, we have already found out the g parameters for low pass filter. We can use this concept of frequency transformation to design, high pass filter, band pass filter, and band stop filter. So, let us see how we can do that?

(Refer Slide Time: 04:47)



So, here let us see what we have, how to transform low pass filter to high pass filter to bandpass filter and band stop filter. Let us just go step by step. So, for low pass filter wherever we have a inductor, that inductor is to be replaced by capacitor. Why we had seen that for the impedance z is equal to SL.

And S has to be replaced by 1 by S. And, we know that for a capacitance z is equal to 1 over S C. So, that is how we can actually find out, the value of the capacitance. So, we can find the value of the capacitance by using this particular expression or I will show you in the next slide, how to use the g parameters itself to find the new values of the high pass filter component?

So, inductor becomes capacitor, capacitor becomes inductor. For bandpass filter, wherever there is an inductor that is to be replaced by series L N C and wherever there is a capacitor it has to be replaced by parallel L C combination. For band stop filter inductor has to be replaced by parallel combination of inductor and capacitor. And, the capacitor has to be replaced by series combination of inductor and capacitor. As, you can see again from band pass filter to band stop you have to do, you can say the cross of this.

So, this is because of the reason that band stop filter is nothing, but inverse of band pass filter.

Let us just see now one by one how to realize these components for different filters.



(Refer Slide Time: 06:32)

So, for low pass filter to high pass filter transformation, what we need to do? Let us first look at the low pass filter circuit configuration. So, what you do for the design? Let us say we want to design a Butterworth high pass filter. So, the first step would be actually speaking you design a Butterworth low pass filter.

Similarly, suppose you have to design a chebyshev filter of let us say ripple 0.5 dB in the pass band. So, what you do? First you design a low pass filter find all the g parameters; so, g 1, g 2, g 3, g 4 and so on. So, now, as I mentioned earlier so, inductor is to be replaced by capacitor, capacitor is to be replaced by inductor.

Now, you do not have to find g parameters again. These g parameters which are here g 1 g 2 g 3 and you can see that instead of writing here small gs we have written here capital C and capital L. So, here for these particular things all you need to do it is for low pass filter to high pass filter transformation. So, capital G k which is of course, again gks happens to be my initials also Girish Kumar. So, capital G k can be obtained by simply equal to 1 divided by g k. So, this small g ks are corresponding values of g parameters.

So, for example, C k dash is nothing, but equal to 1 by g 1, this will be 1 by g 2 1 by g 3 and so on. And, after that once you know what is C k, which is equal to 1 by g 1 simply use that impedance frequency scaling transformation to obtain the values of C.



(Refer Slide Time: 08:22)

Let us see, how now we can do the thing from low pass filter to bandpass filter.

Again what you do? You have to design let us say a Butterworth bandpass filter. So, you design a Butterworth low pass filter or you want to design a chebyshev bandpass filter, then you design first chebyshev low pass filter find all the g parameters. So, g 1 g 2 g 3 g 4, not this g 1 which I have actually represented here as capital L k, but this inductor has to be replaced by combination of inductor and capacitor.

So, do not get confused this L k dash does not mean inductor and capacitor, but actually speaking this is simply representation of this over here ok. So, inductor has be replaced by series, inductor and capacitor and the capacitor over here which has been not termed as C k, that capacitance corresponding to this number here is to be replaced by parallel combination.

So, how does this really happen? How this g 1 which is inductor gets transformed to series component. So, for that let us just see what we had done? For the transformation from low pass filter to band pass filter, we had used the expression that S was transformed to this particular expression.

Now, for inductor we know that what is Z Z is equal to S times g 1, because here g 1 represents inductor. So, now, substitute the value of s by this particular expression, which is the transformation from low pass filter to bandpass filter. Now, simplify this here we can see that there is a s square in the numerator and in the denominator we have S. So, separate this out this will become S g 1 divided by B, and if you look at the plus term after that the next term is omega 0 square divided by B as g 1.

Now, for impedance we can represent these terms as series combination of inductor and capacitor. Inductor is defined as s L, where as a capacitance is defined as 1 by s C. So, if you now compare this equation with this here we can say that one is nothing, but equal to g 1 by B and what is C equal to you can see that 1 by s C. So, s is here. So, C will be B divided by omega 0 square multiplied by g 1 ok. So, this is how you can find the values of L and C?

So, now let us see we have to transfer. The capacitance into this particular combination here for that I suggest, that you do these steps yourself, but to do that step let me just give you little bit of a hint, that for capacitance, you can write instead of Z write y. So, y is equal to S multiplied by g 2. So, over here then S will become this particular term and this will be g 2, remember now this is y.

And, in y if we have to add the 2 components, there will be in shunt. So, that is how these 2 comes in parallel or we can say these 2 are in shunt, than only they are added ok. So, this process is similar. So, you follow the same thing for inductors and capacitors for all these components and that way you can find out, the corresponding values of L C components for bandpass filter.

Now, for band reject filter I leave that exercise for you people to do it. So, we know that band reject filter is nothing, but inverse of bandpass filter. So, simply use the inverse of this particular transformation, simplify in this particular manner you can find out the components of band reject filter.

(Refer Slide Time: 12:24)



So, now let us just take an example. So, this is a third order Butterworth bandpass filter which we took as an objective to be designed at 100 megahertz, because that is easy then to transfer to any other frequency. So, the first step is that you actually design a third order low pass filter ok. At, 100 megahertz or you can actually speaking design it for the normalized frequency omega equal to 1.

So, the first step would be that you design a third order low pass filter at normalized frequency omega equal to 1. And, here just to show you what we have here? So, think about a low pass filter would be a capacitor, then inductor capacitor.

And, now you just look at the transformation inductor was replaced by series combination of inductor and capacitor, a capacitor what was supposed to be here is not replaced by parallel combination of inductor and capacitor. It is just shown slightly a different way you can see here this is ground this is ground. So, how do we drawn this particular thing on this side this capacitor will be over here and this capacitance will be over here.

So, capacitance in case of low pass filter is replaced by inductor, in parallel with capacitance, inductor in case of low pass filter is replaced by series inductor and capacitor, and the capacitor for low pass filter is replaced by inductor in parallel with capacitance.

And, now for the transformation we have used the bandwidth as 20 megahertz. Now, I just want to tell you that whatever those design value comes out to be they are not practically available in the market ok. So, what we have done? We have actually chosen L and C values which are practically available and these values are chosen close to what were the designed values? So, you can see here these are the values, which are relatively available in the market. So, you can see here this is 27 Nano Henry this is a 100 Pico Farad 8.2 Pico Farad. So, this is a 330 Nano Henry inductor and these values are same as this value over here.

Now, these are readily available component values. However, when you take these values as you will see in the next slide, the center frequency does not remain precisely 100 megahertz, but we will see what are the shift? Now, this particular thing has to be now fabricated. So, we have actually designed a general purpose pcb and I just want to mention to you that this general pcb we had designed for low pass filter, bandpass filter, band reject filter, attenuators also which we will discuss in the later lectures.

So, let us see what we have done here? So, here this is the input port this is the output port. And, the width of this particular line must be chosen corresponding to 50 ohm characteristic impedance. So, you can see here corresponding to this point here a sma connector has been connected here, the center of that pin is soldered over here for the output, you can see that at the output sma connector is connected.

So, now what you see over here these are the basically parts, where we can put the component. Now, what we have here you can see at the bottom side there are several plated through holes ok. So, basically these are done to provide proper grounding from the top to the bottom. Same thing has been done over here also.

Now, multiple plated through holes have been taken, the reason for that is if we just put only let us say 1 plated through hole over here. Then shorting happens at this particular point whereas, if we are going to put a component over here, it will see the path length something like this. And, that path length may provide additional inductor ok.

So, this is a general purpose thing and now you can see from here how we have done the realization? You can see here from here there is an inductor. So, this inductor is realized by this air core inductor over here. In fact, this kind of a thing can be very easily realized or you do it is take one of those transformer windings, and what we have done over here

is that just take the refill of the ball pen and wrap it around that. And, then of course, you have to do little bit of a calculation to find out the desired value of the inductor.

However, as I also mentioned that these inductor values are readily available, you can also purchase a lumped equivalent of this particular thing here. And, that can be soldered between this particular point over here, to this particular point over here. And, then the capacitor can be soldered between this and this and you can do a shorting over here. And, then you can keep following this particular thing. So, we have a capacitance over here you can see there is a capacitance.

And, since it is a large inductor, again this large inductor has been realized using a air core inductor, but again you can buy a chip inductor also. So, that is the air core inductor, then after that we need an inductor, which is going to ground, we need a capacitance which is going to ground. So, that is going to ground. And, whatever things you do not need to use simply put a copper strip and solder that particular portion.

(Refer Slide Time: 18:21)



So, let us see now what we got?. So, first let us see the simulated results. Now, these simulated results are not for the designed value, these simulated results are for the available component values. So, you can see here this is the plot for as you can see from here dot purple line here is for S 2 1. So, this is the S 2 1 simulated. I want to bring to your attention even though we had designed for 100 megahertz, but because of the

available component values the simulated values shifted slightly, but since this was just for testing the concept it is. So, it is fine, we can do that.

Let us see corresponding to this particular simulated S 1 1. So, S 1 1 is shown by blue color. So, this is the S 1 1 plot you can see that most of the time it is less than minus 15 dB in the desired a passband.

But, when we did the fabrication? So, you can see that there is a little bit of a shift in the resonance frequency. So, this is the S 2 1 measure. So, you can see that this is the measured plot, one can actually see that there is a small shift in the resonance frequency. And, also there is a small shift in the measured S 1 1 value also.

So, while there is a shift, because the reason for that is that, even though when we buy these readily available components. They do not have 0 percent tolerance majority of these components have 5 percent to 10 percent tolerance. So, because of those component tolerances resonance frequency can change.

Another thing I want to mention that we had taken those finite lengths in between which were acting as pad for this component. So, that also provides series inductor or maybe parasitic capacitance. So, because of that also generally speaking, resonance frequency decreases slightly.

So, when you have to design your next band pass filter or low pass filter remember that you have to take some of these precautions that all those line lengths do have parasitic inductance and capacitance and invariably, they try to reduce the designed resonance frequency slightly. So, please take care of these things when you actually do the design.

Now, you can see here that lot of inductors capacitors are required. And, then component tolerances and other things play very important role. So, now, we look into some of the designs, where you do not need inductors and capacitors, but use simply transmission line or modifications of transmission line to realize various filters.



So, let us look at these things now.

So, here 3 different configurations are shown over here. So, let me go through these things 1 by 1. So, this is the end coupled bandpass filter and this is generally designed, when the bandwidth required is less than 5 percent. I just want to mention here what is percentage bandwidth; percentage bandwidth is defined by bandwidth divided by center frequency multiplied by 100. So, end coupled filters are generally designed for narrow bandwidth, which is less than 5 percent.

So, what is this configuration here and why these are known as end coupled? So, I just want to tell you. So, each one of these things here are approximately lambda by 2 resonators. So, these resonators will act like a bandpass filter, we know that a lambda by 2 resonator will actually speaking have a bandpass filter response like this. And, when you use multiple of these bandpass filter section basically what happens, if you use only 1 section the transition from the pass band to the stop band will be relatively slow, but if you use larger number of these resonators then basically what happens, we can will have a sharper transition from the pass band to the stop band.

So, since these resonators are coupled at the end that is why these are known as end coupled bandpass filter?

Now, just I want to mention here that if you want to increase the bandwidth, while there is a problem the problem is that this gap here specially the gap between the first element and the last element becomes extremely small. In fact, this gap may become even of the order of 0.1 mm or sometimes even smaller, and you know that if you want to realize a 0.1 mm and do you go for a pcb fabrication there are chances that 0.11 mm gap may not be there and that 2 things will get shorted.

So, that is why generally speaking end couples are used only for smaller bandwidth. However, to increase the coupling the next step is we use coupled line band pass filter. And, generally these are designed for 5 percent to 20 percent bandwidth. So, here let us see what we have again these are lambda by 2 resonators, somewhat similar to this concept over here, but now instead of end coupled like this they are coupled like this, when we are talked about the coupled line micro strip and we had also talked about coupled line directional coupler I had mentioned to you, that the coupling is maximum between the resonators, when this length is equal to lambda by 4.

So, you can see that the total length is lambda by 2. So, this lambda by 4 is coupled with this lambda by 4. So, that becomes one coupled line configuration, this becomes another coupled line configuration and by using this particular configuration. What we are doing we are increasing the coupling between the input port and the next element. And, then similarly we keep on increasing the coupling and by increasing the coupling we are able to increase the bandwidth from 5 percent to 20 percent.

However, there are applications where we need bandwidth for more than 20 percent. So, in that particular case we can use the concept of direct couple. So, let us see what it is?. So, you can see here this one here shows only 1 section of course, you can see here there are 3 sections over here, there are multiple sections over here, right now let me explain you the concept with the single section. So, here this length is lambda by 4, which is shorted to ground.

Now, think about the transmission line theory. So, transmission line theory if you recall. So, if there is a lambda by 4 length, then the input impedance becomes Z 0 square divided by Z L. So, if Z L is equal to 0, which is shorted to ground what will happen over here? It will become open circuit. So, that means, corresponding to this particular length, where it is lambda by 4. So, that particular frequency when we give input here, this will act as a open circuit. So, everything go from here to there. At other frequencies loading of this short will come over here and that will get attenuated.



(Refer Slide Time: 26:00)

As I mentioned this is only a single section multiple sections can be realized using this particular configuration here. So, you can see here this is input, this is output, and we have several lambda by 4 sections over here. So, you can see here 1 2 3 4 and in between each lambda by 4 section, we have another lambda by 4 transformer in between.

So, these things generally are designed for sharper roll off and for wider bandwidth requirement. Of course, just to mention here instead of using a shorted lambda by 4 section, you can also use open ended lambda by 2 section also.

So, in case of open ended lambda by 2 section what will happen, this length will be let us say lambda by 2 and we know that for lambda by 2 section transmission line, whatever is the load impedance will become source impedance. So, in this case load impedance is open. So, source impedance will become open. So, if these things are open circuited whatever is the input that will go to the output. At other frequencies these things will provide attenuation to the signal.

(Refer Slide Time: 27:17)



So, let us see now how we can realize a band reject filter, we have to just remember the concept that band reject is nothing, but opposite of band pass filter. So, I have just shown the direct coupled concept here for other configuration, you can actually try to visualize yourself. So, here we have again a lambda by 4 section, but now this particular end is open circuited.

Since, this is lambda by 4 this open will act as a short over here. And if it is short so, whatever the input is given at that particular frequency it will get shorted. So, that will not go over here. So, basically it will act as a band reject filter at a frequency where this length is equal to lambda by 4.

Now, instead of having a length like this, you can always use a bent length also and the same thing can be done even for bandpass filter. Now, here depending upon how close is this bent? Ok. Suppose you make a bent very very close to it and the length is let us say lambda by 4. Then, there will be 2 modes of coupling 1 coupling will be of course, direct coupling like this here, other coupling can be because of the fringing fields, especially if this length is very very close to this here. So, you can basically increase the coupling and also you can make the filter instead of this width you can now make it relatively compact.

(Refer Slide Time: 28:55)



So, we will look at a one designed example and let us see then what kind of a results we get?. In fact, I have given their title slightly different here this is a bandpass filter and band reject filter depending on length. I have given here a little different title, it is a band pass filter and band reject filter depending on a length. The reason will be clear in a few minutes. So, let us see what we have done here? So, we have used the low cost FR 4 substrate, these are the substrate parameters and what we have chosen here line width, which is 1.5 mm here for the 50 ohm line. And, this is a lambda by 4 resonator width of that has been taken as 0.5 mm.

In the next lecture, I will show you, what is the effect of changing this particular width. But right now just to tell you we took this length of the line as 42 mm. Let us see the frequency response of this particular configuration. So, we are giving input here this is the output all we have here is a open circuited line, which is of length 42 mm. So, see here this is frequency and this is the S parameter response. So, let us just look at the green color which shows S 2 1. So, for S 2 1 I have written here bandpass filter, when I is equal to a lambda by 2, I have written here band reject filter, when I is equal to 3 lambda by 4.

So, let us see what are these things. So, let us just think about this is an open circuit. So, this open circuit will act as a short circuit at this particular point, when this length is

equal to lambda by 4. So, if these open here acts as a short circuit that will become like a band reject filter.

So, this is the response corresponding to when length is equal to a lambda by 4, you can quickly check now frequency is approximately equal to 1 gigahertz. So, at 1 gigahertz wave length is thirty centimeter, lambda by 4 will be thirty divided by 4 which is 7.5 centimeter or you can say 75 mm. Now, 75 mm is to be divided by square root of epsilon effective of this particular line, not this line because this is acting as a resonator.

Since the line is very thin, it is epsilon effective is also relatively small you can see that this is epsilon r 4.4 epsilon effective will be much smaller than 4.4. So, 75 mm is divided by this epsilon effective, which actually gives to the length of 42 mm.

So, when this length is lambda by 4, it acts like a band reject filter because this is open will act as a short. When this length is lambda by 2 and when this length will become lambda by 2, when the frequency is doubled which is approximately 2 gigahertz. So, at 2 gigahertz length becomes lambda by 2. So, open becomes open over here, when the length is 3 lambda by 4 what happens now? Open over here will act as a short circuit. So, you can see here this will again behave like a band reject filter and you can see approximately this frequency is around 3 gigahertz this is around 1 gigahertz.

So, you can actually see that this particular simple thing here can act as a band reject filter or band pass filter or band reject filter, depending upon the frequency of operation and you can see the corresponding S 1 1 plot. So, when bandpass filter characteristic is satisfied; that means, everything is going from here to here ; that means, nothing is reflected back. So, reflection coefficient is very small.

In case of band reject filter nothing is going over here. So, almost everything is reflected back. So, you can see that S 1 1 is very high at this particular point. So, we will continue from here in the next lecture, we will take more examples of how to realize these bandpass filters and band reject filter.

So, thank you very much we will see you next time, bye.