**Microwave Theory and Techniques**
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**Module – 05**
**Lecture – 22**
**Microwave Filters – II: Low Pass Chebyshev Filters**

Hello, everyone. In the last lecture, we had started discussion about filters. So, let us start with quickly what we had discussed in the last lecture.

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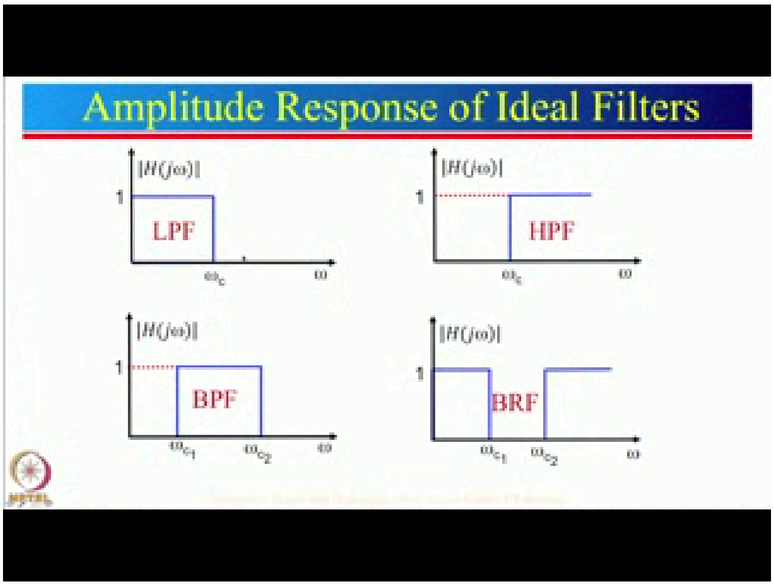


So, we had started with the ideal filter response. Here is a response of ideal low pass filter which passes all the frequencies up to omega c and blocks all the other frequency A high pass filter passes all the upper frequencies beyond omega c, a band pass filter passes all the frequencies between omega c 1 and omega c 2 and a band reject filter or band stop filter blocks all the frequencies between omega c 1 and omega c 2.

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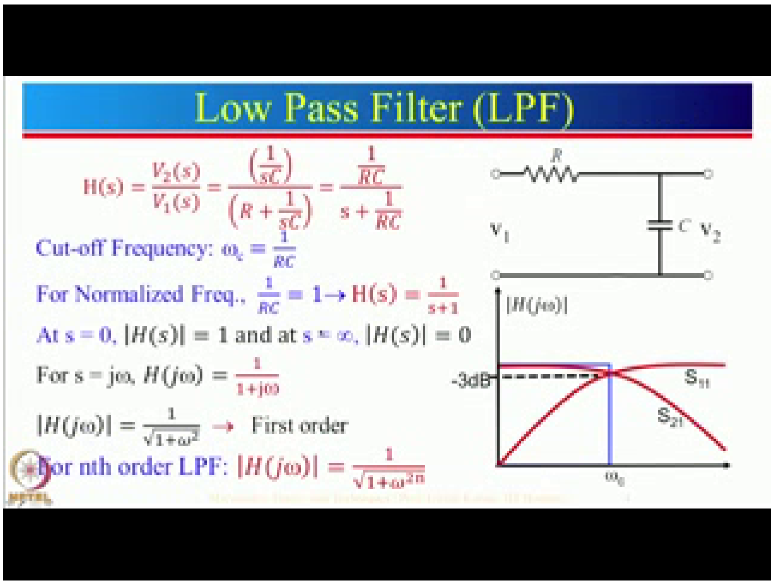


Then, we had seen a simple example of RC filter and I did mention that we do not use RC filter at microwave frequency because this R here gives additional loses.

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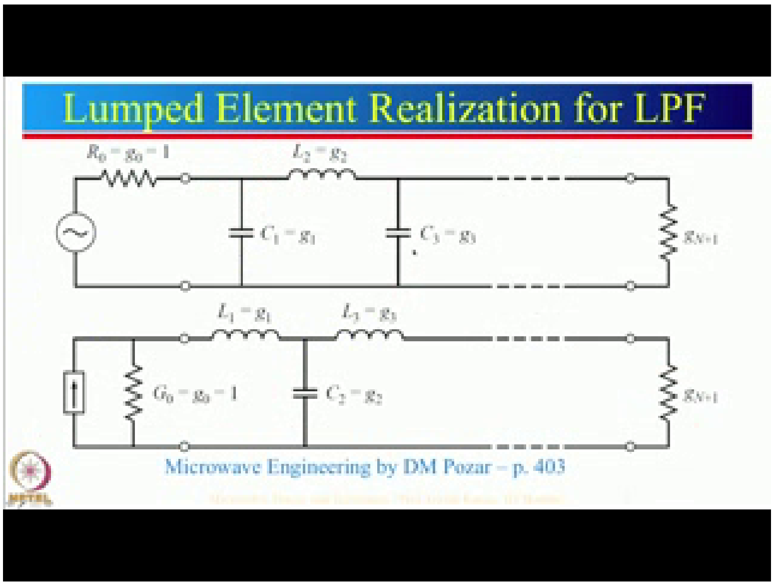


So, instead of using RC filter we in general use inductors and capacitor. So, one can start with the capacitor and then go to inductor or when can start with the inductor and then go to the capacitor.

So, the objective is now to find out these g parameters g 1, g 2, g 3 and so on.

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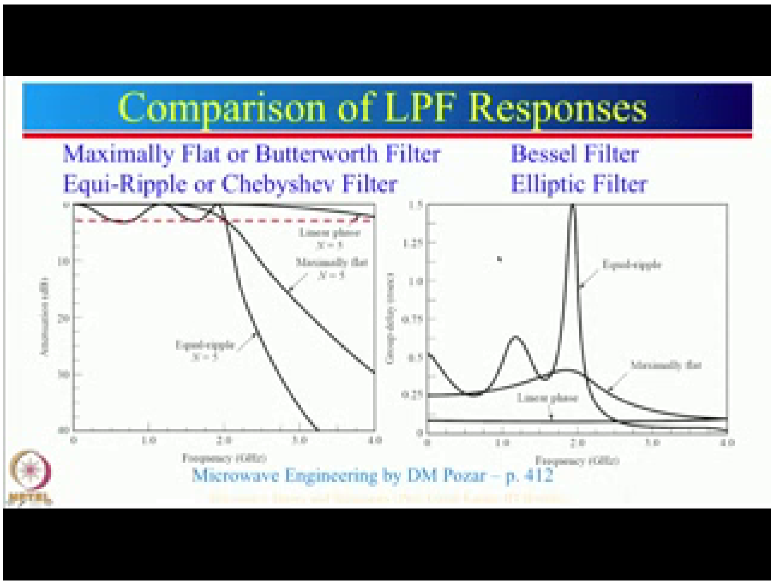


Then, we looked at four different types of realization. One was maximally flat or Butterworth filter, the response of that was given by this curve here; then we looked into equi ripple or Chebyshev filter, the response of that has equi ripple in the past band and then relatively sharper transition compared to the Butterworth filter.

Then, we had also looked at the Bessel filter, but we saw that the response is very very poor as for as the amplitude response is concern; however, phase responses nearly perfect. But, I just want to tell you very rarely people use Bessel filter because of this particular region that the responses very very slow.

Elliptic filters in general have equi-ripple in the pass band as well as they have equi-ripple in the stop band also, but the advantage of elliptic filter is that the response transition from pass band to top band is very very fast. So, suppose so, the same fifth order this is the response for maximally flat, this is the response for equi-ripple, but for elliptic filter the response will be even sharper and that will be something like this and then there will be some ripples in this particular region here.

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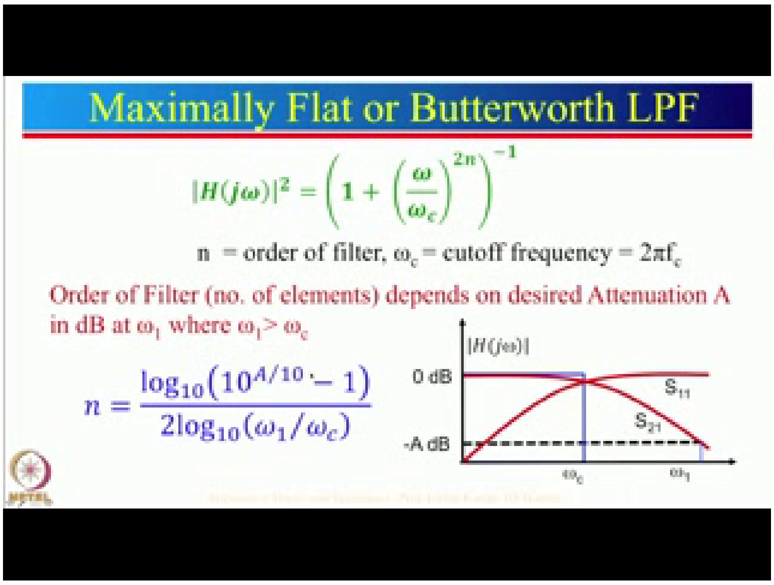


And, from here we actually defined another thing which is attenuation at frequency omega 1. So, if the cut of frequency is omega c so, it is generally desired that at a given frequency omega 1 how much is the attenuation, because there may be a some inter faring single over here which needs to be attenuated by may be 20 dB or 30 dB or more and the order of the filter can be obtained by using this particular equation which is obtained by taking log on both the sides and solving for n.

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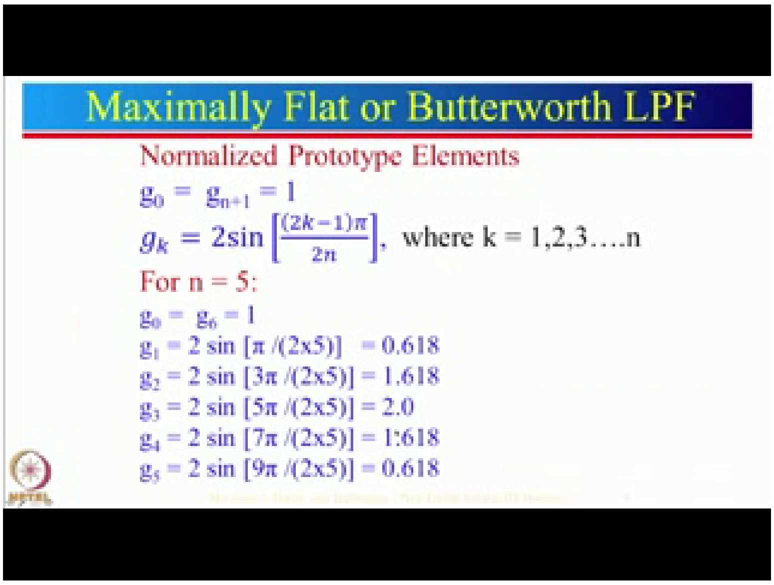


Then, we looked at the g parameters. So, how to find out the g parameters for Butterworth? So, we saw that g 0 is equal to g n plus 1 equal to 1. These are nothing, but corresponding to input and output, source as well as load impedances and g parameters can be found by using this simple expression and we had seen that these are the typical values for a fifth order filter.

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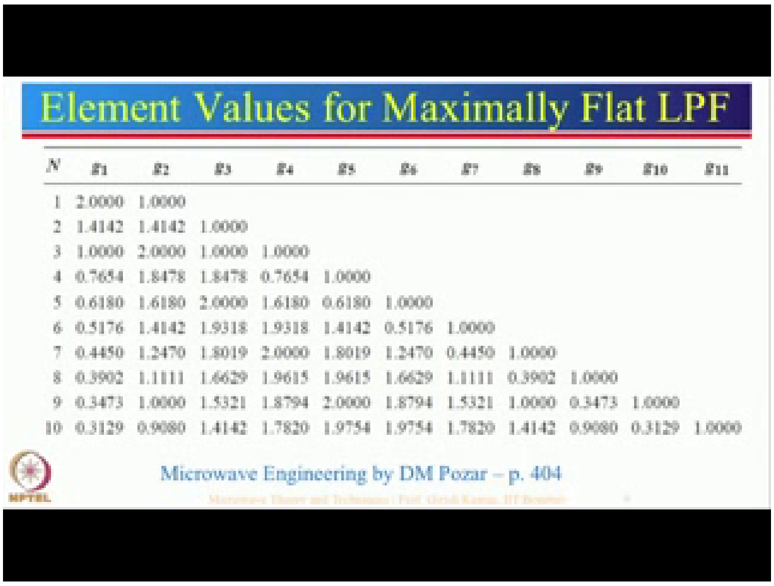


| N | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ | $g_9$ | $g_{10}$ | $g_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2.0000 | 1.0000 | | | | | | | | | |
| 2 | 1.4142 | 1.4142 | 1.0000 | | | | | | | | |
| 3 | 1.0000 | 2.0000 | 1.0000 | 1.0000 | | | | | | | |
| 4 | 0.7654 | 1.8478 | 1.8478 | 0.7654 | 1.0000 | | | | | | |
| 5 | 0.6180 | 1.6180 | 2.0000 | 1.6180 | 0.6180 | 1.0000 | | | | | |
| 6 | 0.5176 | 1.4142 | 1.9318 | 1.9318 | 1.4142 | 0.5176 | 1.0000 | | | | |
| 7 | 0.4450 | 1.2470 | 1.8019 | 2.0000 | 1.8019 | 1.2470 | 0.4450 | 1.0000 | | | |
| 8 | 0.3902 | 1.1111 | 1.6629 | 1.9615 | 1.9615 | 1.6629 | 1.1111 | 0.3902 | 1.0000 | | |
| 9 | 0.3473 | 1.0000 | 1.5321 | 1.8794 | 2.0000 | 1.8794 | 1.5321 | 1.0000 | 0.3473 | 1.0000 | |
| 10 | 0.3129 | 0.9080 | 1.4142 | 1.7820 | 1.9754 | 1.9754 | 1.7820 | 1.4142 | 0.9080 | 0.3129 | 1.0000 |

And instead of using this particular expression one can use the table to find out what are the different g parameters.

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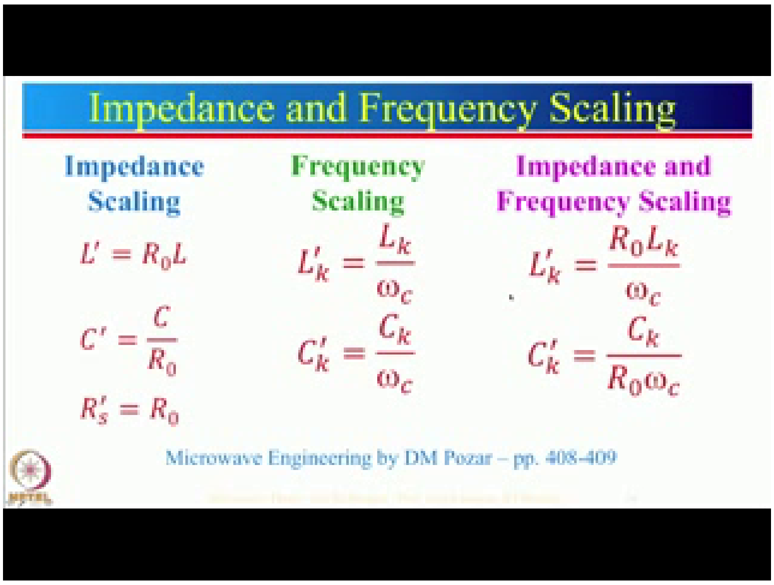


So, after that what we need to do we need to do impedance scaling as well as frequency scaling. Why we have to do that because we had taken input and output impedances to be equal to 1, but in the reality that will never with the case and we had taken an normalize frequency which is omega equal to 1, but we need to design filter at the desired frequency. So, ultimately we combine impedance as well as frequency scaling and what we had seen that the g parameters which are given by L k or C k we will see the example in a short while.

So, those g parameters have to be modified for inductor and capacitors. Slightly differently for case of g parameters which realize inductor we have to multiply by R 0 divide by omega c whereas, for the g parameters which will represent capacitance those things have to be divided by R 0 and omega c to realize the real capacitance.

So, let us take now an example.

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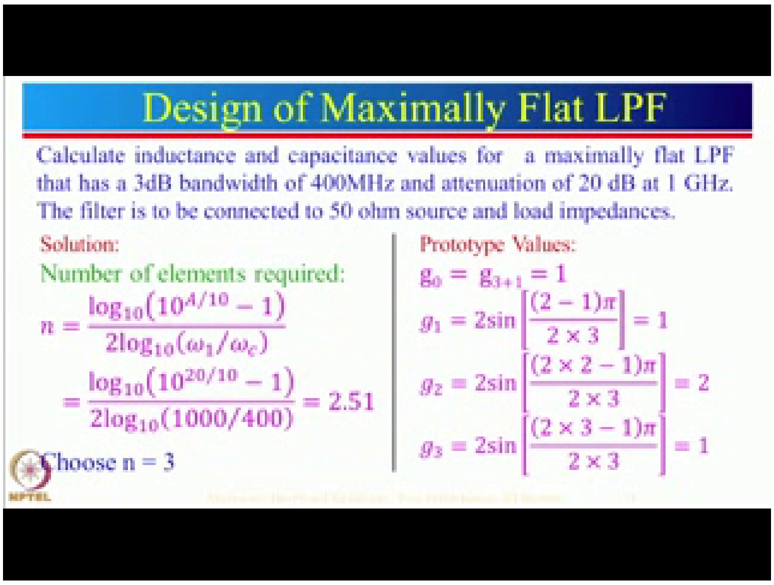


So, will a start with than example that we need to design of low pass filter with the cutoff frequency of 400 megahertz and we want attenuation of 20 dB at 1 gigahertz and it is given that the source and load impedances are 50 ohm.

So, the first thing what we need to do is we need to find out what is the order of the filter. So, we had seen the expression to find out the order of the filter n given by this particular expression. So, let just substitute a various values. So, log 10, 10 to the power A what is A? 20 dB. So, you do not convert into numeric value, that conversion is done by this particular expression here.

So, 20 comes right over here then for omega 1 we want attenuation of 20 dB at one gigahertz. So, that is 1000 in terms of megahertz divided by omega c which is 400 megahertz. So, that comes over here. So, this comes out to be 2.51 of course, we come take real number for order of the filter we have to take the higher value compared to this one here. So, we choose n equal to 3.

So, the next step could be is to find out the g parameters. So, we know that g 0 is equal to g n plus 1, here n is 3. So, input and the output source impedances at equal to 1. We will do the impedance scaling as well as frequency scaling in the next line. The next step is now to find out g parameter. So, g 1, g 2, g 3, n here is equal to 3, you put the value we find out the corresponding g parameters which are 121.

Now, let us do the impedance and frequency scaling.

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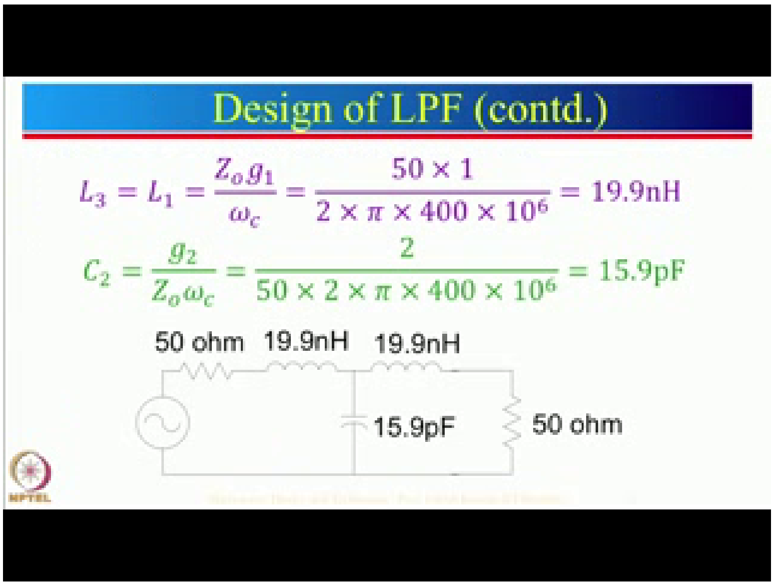


Here we have taken the first component as inductor, second capacitor, third inductor will also show you the design where we take first component as capacitor and then inductor and capacitor, but let just look into the, this design now. So, L 1 is equal to L 3. So, there I had written R 0, but here R 0 is equal to Z 0 which is given as 50 and we had seen that g 1 is equal to g 3. So, that is equal to 1. What is omega c? 2 pi into F c. So, 2 pi into 400 megahertz.

So, simplify this that comes out to be 19.9 nanohenry for capacitance again for the g parameter g 2 if you recall g 2 was equal to 2, Z 0 is 50 omega c is 2 pi F c which is 400 into 10 to the power 6 so, that comes out to be 15.9 picofarad. So, this is the actual circuit which will meet the requirement of the 400 megahertz cut off frequency and at least 20 dB attenuation at 1 gigahertz.

So, let us see the response of this particular filter.

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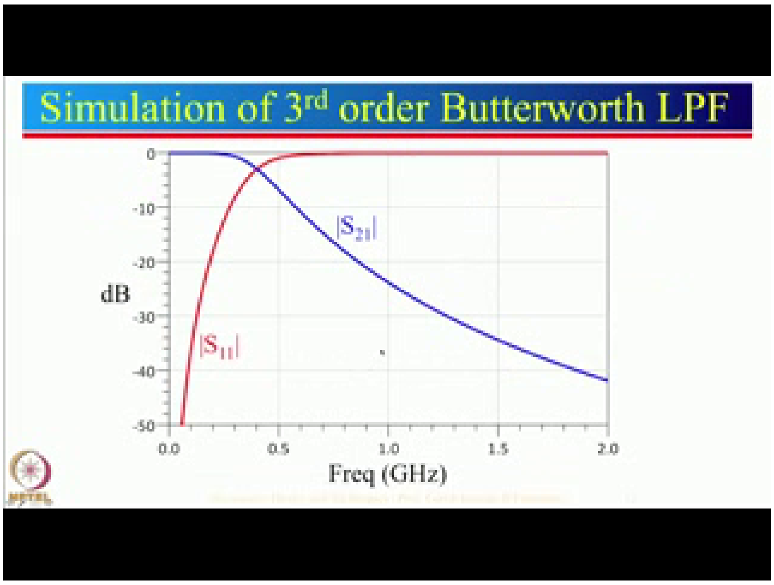


So, here is the response of S 21 which is a low pass filter and this is the reflection coefficient for this given low pass filter. So, let us first look at S 21. So, we had design this particular filter with a cut off frequency of 400 megahertz which is 0.4 gigahertz. So, if you look at from here 0.4 gigahertz we go up here. So, this is the cross over point and that corresponds to about minus 3 dB.

I just want to mention here if S 21 is equal to minus 3 dB correspondingly S 11 also will be equal to minus 3 dB and remember the circuit consist of only inductors and capacitors, hence there is a no insertion laws in this particular filter. Of course, this is an ideal response practically inductors and capacitors will have some losses. So, the response will not be exactly 0 dB, but it may be a 0.1 dB or 0.2 dB, depending upon the loses in the inductors or capacitors.

Let see what is the response at 1 gigahertz? So, at 1 gigahertz if you see this value, that is about 24 dB. Now, you might wonder we had actually designed it for minus 20 dB, but this is giving as minus 24 dB. The reason for that is minus 20 dB was obtained for a filter order n equal to 2.5, but you cannot have a 2.5. So, we took a higher value which means we took third order filter. So, for third order filter response will be little more steeper.

Had we take in suppose fifth order then this response will be steeper, here had we take n equal to 2 then the response would be like this here, you would not have got a 20 dB attenuation which was desired. So, if the attenuation desired is 20 and if you are getting 25 or 30 or 35 it is acceptable all the time.

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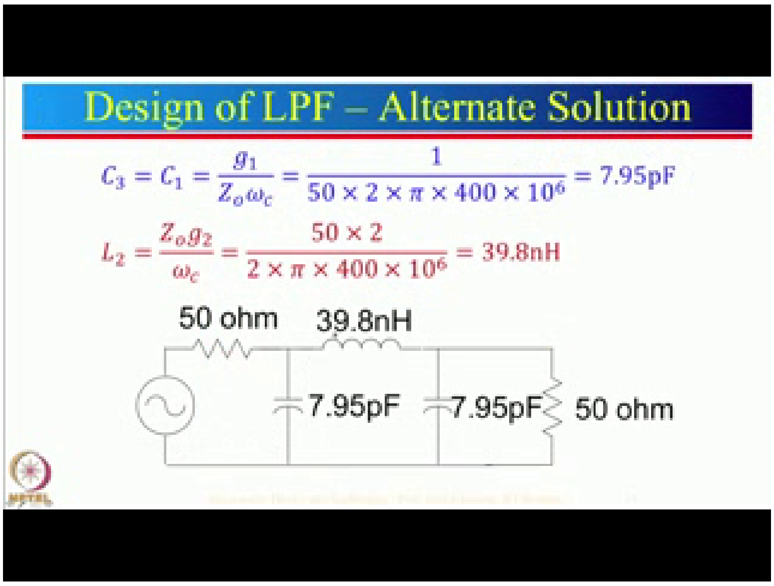


So, now let just look at alternate way of realizing; that means, here first element has been taken as capacitor, then inductor and then capacitor. Just recall previously we had taken first element as inductor then capacitor then inductor. So, in this case what happens now g 1 is equal to g 3 and we are now realizing capacitance. So, for capacitance what we do, we start with the g parameter we divide that by Z 0 and omega c. So, g 1 is equal to g 3 equal to 1. So, Z 0 is 50 to pi into F c. So, that comes out to be 7.95 picofarad.

Similarly, we find the value of the inductor we start with the g 2 parameter multiply this is Z 0 or R 0 divided by omega c. So, 2 into 50 divided by omega that comes out to be 39.8 nanohenry. In fact, I would like you people to compare this particular circuit with the previous circuit and you will actually see that if you look at the equivalent inductance and equivalent capacitance they are coming out to be similar ok.

So, I want you to check that let me just tell you for one case inductor we had seen that this was 19.9 and this was 19.9. So, 19.9 plus 19.9 is equal to 39.8 nanohenry. You can verify the same thing for the capacitance value, ok. So, let see the response. So, you can see that the responses exactly same there is no difference at all.

Now, I just want to also mention. So, how do we get these responses? Of course, this particular thing has been done using ADS software, but in the beginning we had used freely available software the name of the software is RFsim99 dot exe. You can download from the internet it is freely available and in fact, what you do in that particular software you can just give that just analysis and then you can look at the design per you can do the simulation of filter.

In fact, it would be very very easy you simply have to say which type of filter you want to design, low pass, high pass or band pass and then what you do you give the frequency what is the cut off frequency of course, there you have to define what is the order of the filter. That software will not calculate for you order of the filter and then it will give you the response which is very similar to this.

It will also give you the inductors and capacitors value. It will also give the circuit diagram. And, then you can actually go and added those components also. I just to mention to you for example, if you go to the market you would not get any capacitor which has a value of 7.95 pico farad, you may get at pico farad or 7 pico farad or may be 9 pico farad or 10 pico farad.

Similarly, you may not get 39.8 nanohenry you may get 40 nanohenry or may be 39 nanohenry and so on. So, what you need to do then these are the design values, but these are not the practical value. So, you go in that particular software you can click on that particular component double click on that and then you can change the value of the inductor or change the value of the capacitor and then simulate, you will know what is the response.

And, then check those things and depending upon whether it needs your requirement or not, you can keep on doing the modification and sometimes may be you can add inductors in series or in parallel to realize the desired value of inductors. Same thing you can do for the capacitor. You can use two capacitors in series or parallel to realize the desired capacitance value.

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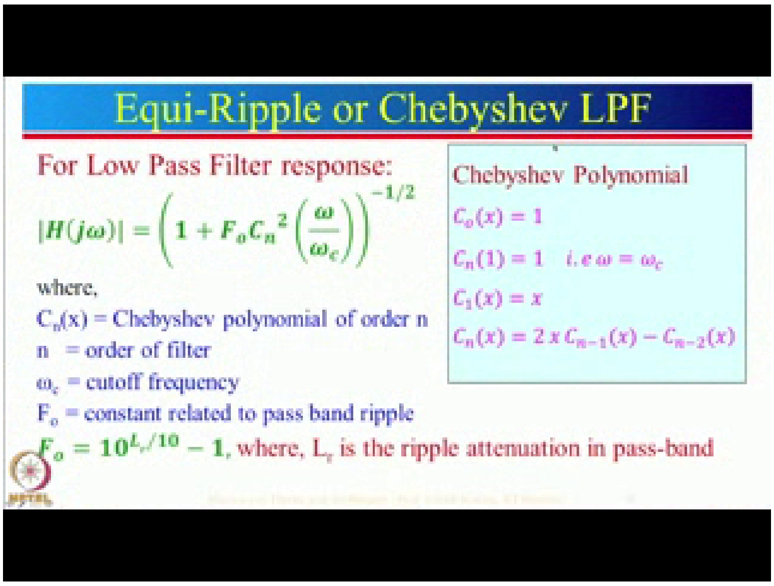


So, now let see the response for Chebyshev filter which is also known as equi-ripple filter. Now, for low pass filter the response of the Chebyshev filter looks something like this over here. Now, try to compare this with the previous configuration which was for Butterworth filter. So, for the Butterworth filter if you say this is the minus 1 by 2 this can be written as 1 divided by square root 2,. So, in the case of Butterworth this expression was 1 plus you can say omega by omega c to the power 2 n, but here we have a F 0 component as well as C n square component which is C n square of omega by omega c, ok.

So, this is not really coming as omega by omega c, but it is the function over here let just look at what are these different things here. So, C n x is Chebyshev polynomial of order n and let me just tell you this Chebyshev polynomial actually is defined in two different limits. One limit is for omega less than omega c, or if it is normalized then omega less than 1. So, in that particular region the variation is cosine, and in the region of stop band which is for omega greater than omega c or you can say normalize case omega greater than 1.

This Chebyshev polynomial is nothing, but cos hyperbolic function. So, n of course, is order of the filter as before omega c is cutoff frequency and what is F 0? F 0 is constant related to pass band ripple. In fact, you can actually choose how much ripple you want in the pass band. So, generally speaking if you look at the low frequency component most of the time they actually define minus 3 dB as pass band ripple, but I want to tell you please at microwave frequency never ever designed a Chebyshev filter for minus 3 dB pass band, ok. The reason for that is minus 3 dB for S 21 will also correspond to minus 3 dB for S 11 so that means, in the pass band half the power will reflect back only half the power will transmit.

So, majority of the time when we design microwave filter we never ever design Chebyshev response for higher than 0.5 dB pass band ripple. Why cut off of 0.5 dB? In fact, 0.5 dB pass band ripple corresponds to S 11 approximately equal to minus 10 dB or approximately equal to VSWR equal to 2.

So, here are is the expression for F 0 this is 10 Lr by 10. Lr is the ripple attenuation in pass band and as I mention please do not take Lr as 3 dB which is generally done at lower frequency at microwave frequency please never ever do. Majority of the time we take this Lr value to be may be from 0.1 dB to about 0.5 dB.

So, how these Chebyshev polynomials are simplified? In fact, this is the one of the simplified version of that which actually speaking does not even mention anything about cosine variation or cos hyperbolic variation. So, let see what is this very very simple things C 0 x is equal to 1. So, that is the C 0 value. Then, what is C n 1, which is equal to 1 which is corresponding to a general time again the last term and then let us see how are the other parameters here. So, C 1 x, what is x here? X is defined by this particular term over here and this simplification is please remember it is for omega equal to omega c. So, when omega is equal to omega c this term will be equal to 1. So, you can say x is equal to 1.

So, we know now what is C 0 which is 1 we can use C 1 x which is nothing, but equal to x and then all the other C n parameters can be obtained by using this particular expression. I will just make it simple for you people let us say we want to find for n equal to 2 let say we want to find what is C 2. So, for n equal to 2 let see what will be that this will be 2 into 2 minus 1 C 1. So, C 1 x, what is C 1 x is equal to x. So, this term will become 2 x minus C n minus 2 will be 2 minus 2 is 0. So, that will be 1. So, we can say that C 2 x is nothing, but 2 x minus 1. Then when you want to find C 3 what you do now for C 3 this will be 2 and this will be one and I have just mention how to calculate C 2. So, this is the way you can find out all the C n parameters.

Now, in this particular case again when you want to find out the g parameters it is now not as simple as finding g parameters for maximally flag. Here many more steps are required to find out the g parameters. So, let see what are these steps here.

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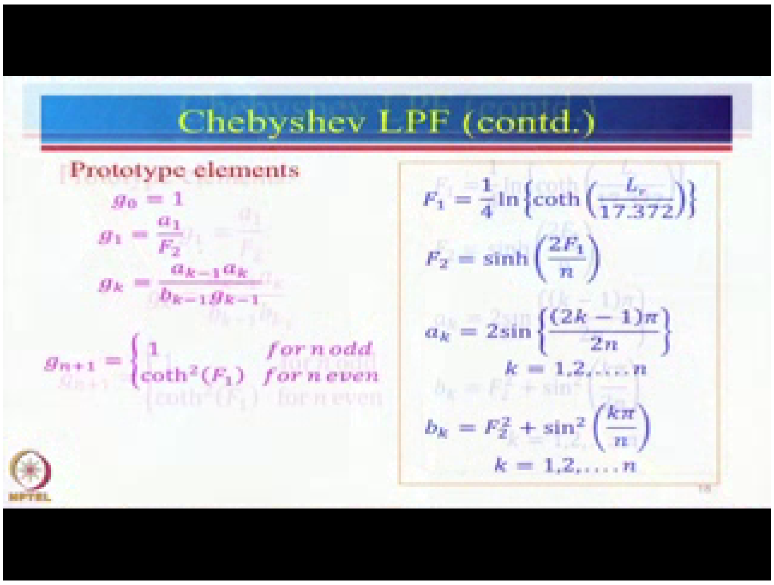


So, g 1 expression is given by a 1 divided by F 2. So, what is a 1? You can see from here you can find the value of a 1 from here. What is F 2? F 2 you have to use this particular expression, but in this expression there is a F 1, so you have to calculate F 1 and in F 1. You can see that there are losses in the ripple, or you can say attenuation in the ripple, and you can see that there is a cot hyperbolic function cot hyperbolic function is nothing, but cos hyperbolic divided by sin hyperbolic.

So, you have to do these calculations in general to find out the g 1 parameter as well as other g parameter. So, you can see from here if you want to find g k it requires a k minus 1 into a k divided by b k minus 1 into g k minus 1, ok. So, you can see here all these things can be found from here what is a k from here and then so, you can find the values of a k from here and b k from here.

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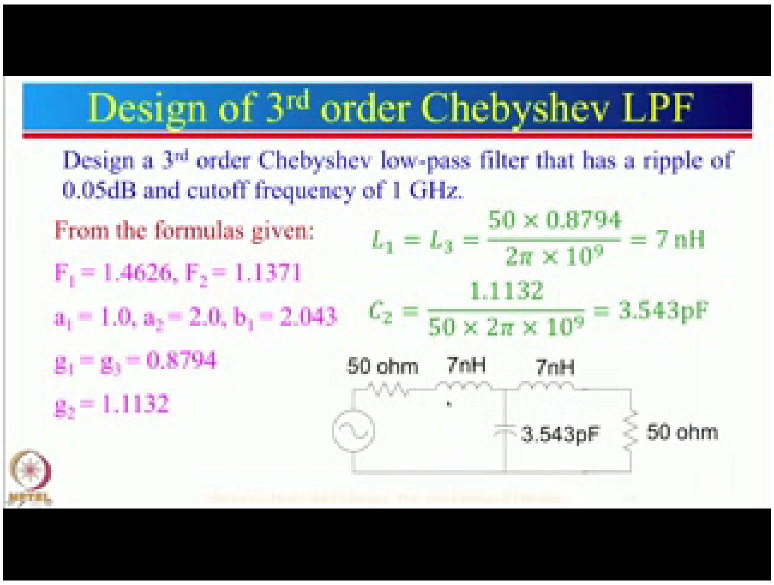


So, let just look at a quick design of a third order Chebyshev low pass filter. So, here the order is defined which may not be always given to you ok, but let say we want to design a third order Chebyshev low pass filter that has a ripple of 0.05 dB. You can see that it is a very very small ripple value the cut off frequency is 1 gigahertz. So, the formulas which has given in the previous slide you use that. So, first we calculate F 1, then we calculate F 2 then we calculate a 1, a 2, b 1 then g 1, g 3 which are equal and g 2.

Once you find these g parameters then after that we can use frequency and impedance scaling. So, we know that for inductor L 1 equal to L 3 which will be corresponding to g 1 equal to g 3. So, here this value is 0.8794 and to find the value of inductor we have to multiply with the impedance which is 50 ohm here, divide by the frequency, which is 1 gigahertz. So, that will be 2 pi into 1 gigahertz this comes out to be 7 nanohenry.

To find the value of the capacitance you use g 2 parameter. So, this is g 2 divided by R 0 or you can say Z 0 which is equal to 50 ohm multiplied by omega c which is 2 pi into to 10 to the power 9. So, that comes out to be this value over here. So, this is the actual circuit simulation for third order Chebyshev filter with the response of 0.05 dB ripple in the pass band and cut off frequency of 1 gigahertz.

So, let see the response of this particular filter.

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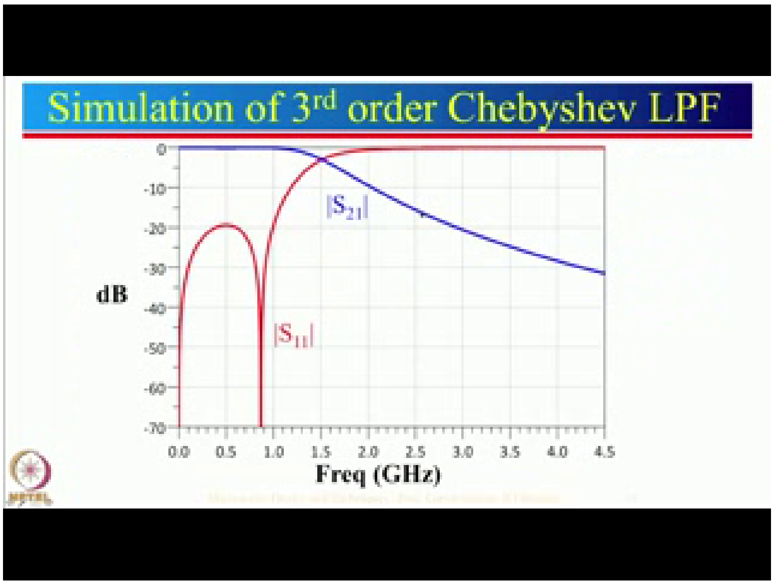


You can see that here this is the frequency versus s parameter plot here. So, this is the plot for S 21 now you can see here there is a very little flip over here actually speaking that flip here corresponds to 0.05 dB, it may not be even visible over here and then this is the response. So, that is S 21.

Now, this is the response for S 11 you can see that this response is slightly different than the maximally flat. In case of maximally flat S 11 actually looked like this here, but over here. In fact, this value corresponds to which is minus 20 dB. This minus 20 dB actually speaking corresponds to the ripple at this particular point over here.

So, corresponding to this little dip over here S 11 is about minus 20 dB. Now, just you tell you had we taken this as a minus 3 dB then S 11 will also what have been minus 3 dB, had this been 0.5 dB then S 11 would have gone up to about minus 10 dB. Now, again I want to mention here that we had design this filter at 1 gigahertz. Now, do not look at the cross over point because the cross over point corresponds to about minus 3 dB,.

Remember, this we had design for 0.05 dB ripple. So, corresponding to this 1 gigahertz, so, this is where that 0.05 dB ripple would come into picture and that is how the responses there, ok. Now, of course, you can do the calculations by using the formulas given to you or alternatively you can use these stables.

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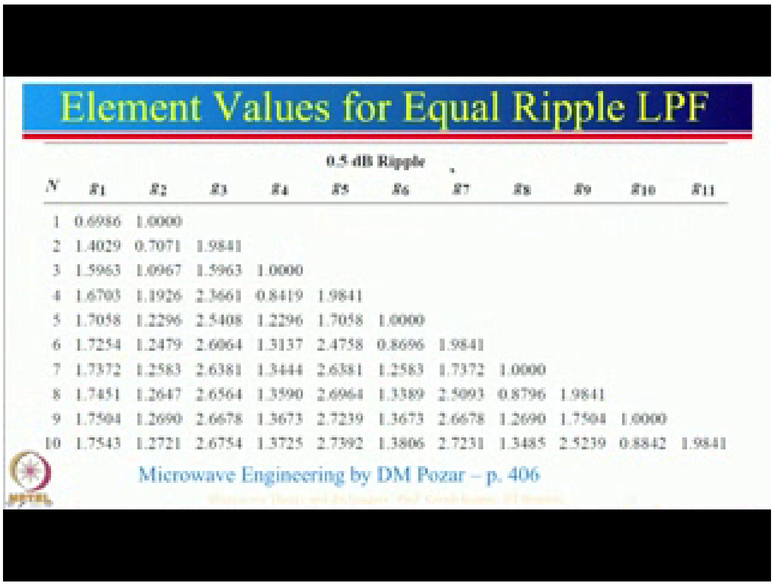

# Element Values for Equal Ripple LPF

0.5 dB Ripple

| $N$ | $g_1$ | $g_2$ | $g_3$ | $g_4$ | $g_5$ | $g_6$ | $g_7$ | $g_8$ | $g_9$ | $g_{10}$ | $g_{11}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.6986 | 1.0000 | | | | | | | | | |
| 2 | 1.4029 | 0.7071 | 1.9841 | | | | | | | | |
| 3 | 1.5963 | 1.0967 | 1.5963 | 1.0000 | | | | | | | |
| 4 | 1.6703 | 1.1926 | 2.3661 | 0.8419 | 1.9841 | | | | | | |
| 5 | 1.7058 | 1.2296 | 2.5408 | 1.2296 | 1.7058 | 1.0000 | | | | | |
| 6 | 1.7254 | 1.2479 | 2.6064 | 1.3137 | 2.4758 | 0.8696 | 1.9841 | | | | |
| 7 | 1.7372 | 1.2583 | 2.6381 | 1.3444 | 2.6381 | 1.2583 | 1.7372 | 1.0000 | | | |
| 8 | 1.7451 | 1.2647 | 2.6564 | 1.3590 | 2.6964 | 1.3389 | 2.5093 | 0.8796 | 1.9841 | | |
| 9 | 1.7504 | 1.2690 | 2.6678 | 1.3673 | 2.7239 | 1.3673 | 2.6678 | 1.2690 | 1.7504 | 1.0000 | |
| 10 | 1.7543 | 1.2721 | 2.6754 | 1.3725 | 2.7392 | 1.3806 | 2.7231 | 1.3485 | 2.5239 | 0.8842 | 1.9841 |

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So, here is a table given for 0.5 dB ripple. N parameters are given from N equal to 1 to 10 corresponding g parameters are given over here. So, you can actually do the checking. So, depending upon the order of the filter let say your designing for a fifth order filter you can calculate g parameters from here g 6 is equal to 1, which is the terminating impedance. Now, I just also want to mention that there is a book Mathai Young and Johns book which has given many of these tables for different values of ripple, and then either you can used that book took find out what are the g parameters or you can use the formulas given in the previous slides to find out what are g parameters,.

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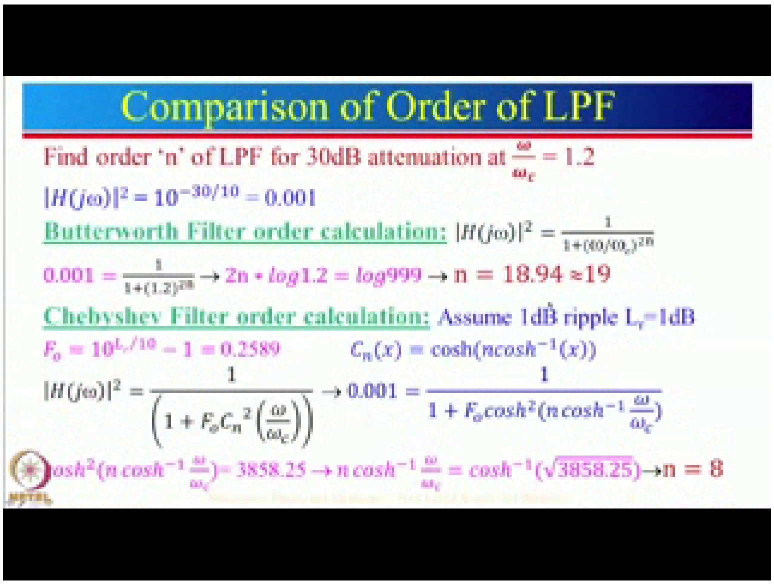


Now, we are going to look at a one comparison thing. So, for example, what is the order required for a low pass filter realized using Butterworth or Chebyshev configuration. So, here we are taking a little different problem in a sense that a larger attenuation is require at relatively closer frequency to the cut off frequency.

So, let say the desired value is 30 dB attenuation at omega by omega c equal to 1.2. I just to mention in our earlier slides we had mention this as omega 1, ok. So, please do not get confuse that is the same thing omega by omega c or you can say omega 1 by omega c. So, this is what is the frequency ratio equal to 1.2 you can think this way omega c may be 1 gigahertz then this will be 1.2 gigahertz if it is 2 gigahertz; that means, omega will be equal to 2.4 gigahertz ok.

So, for now 30 dB attenuation we can now look at step by step. Earlier I had given you the formula for n, but now will show you all the steps how that formula comes into picture and you need not remember the formula all the time you can use this particular simple expression to do the derivation.

So, let say 1 h j omega square to be equal to 30 dB, which is in reality minus. So, you calculate the value that comes out to be 0.001 and please note this is a square term. Now, this term here is given by this particular expression over here. So, now, substitute different values. So, this one here is 0.001, 1 divided by omega by omega c is 1.2 to the power to n. Now, what should you do you take log on both the sides, but before you take log you take this one this side, this divided over here. So, 1 divided by 0.001 will be 1000 and then this one comes on this side which will become triple 9 then take a log, ok. So, that will be 2n log 1.2 and the right hand side that time would be 999 take that and now you simplify this n comes out to be 18.94.

So, that means, to realize this particular thing you need a nineteenth order Butterworth filter. Let see what happens if you want to realize using Chebyshev filter here in this particular example we have assumed 1 dB ripple in the pass band. So, L r is equal to 1 dB. So, now, first calculate F 0. F 0 is given by this expression substitute the value of L r 1 you will get F 0 equal to this.

Now, in the stop band I had told you C n function is nothing, but cos hyperbolic function. So, this is the expression for that. So, what we do now? We write the same thing H j omega square is given by this particular expression is substitute the value 0.001, this is 1 divided by 1 plus F 0 just the expression of C n has been written over here which is write here.

Now, we try to simplify this particular thing over here, right. So, substitute the value of F 0 0.2589 and this term is transferred over here with simplification you get over here and then what should you do for this particular term next simplification is you take cos hyperbolic inverse of this particular term after taking a square root. So, this is what the term is that is on the left hand side, simplify for n you will get n equal to 8.

See you can actually see that to realize this particular response if we use Butterworth concept you would need and order of filter which is 19 whereas, if we use a Chebyshev filter we would require and order of only 8. Of course, I just to want to mention here we had assumed ripple to be equal to 1 dB. Now, if it is 0.5 dB you will see that the number of elements required will be more.

So, this is an exercise for you people please find out take this as 0.5 dB ripple and see what would be the value of n, but you can see the big difference if you use n equal to 8; that means, total number of inductors and capacitors required will be only 8 here total number of components required will be 19. So, you can say that the cost of this particular thing will be much larger even the circuitry realization will be larger.

So, in the next lecture I will show you what is the response of these two filters,. So, just to summarize today's lecture we started discussion about the Butterworth filter, we use that concept which we had discuss in the previous lecture of frequency scaling as well as impedance scaling and then we designed a low pass filter using Butterworth and we saw that whether you take first element as inductor or capacitance response remains exactly same.

Then we looked at the Chebyshev filter and for Chebyshev filter we saw that the finding g parameters required more steps, but however, Chebyshev filter as an advantage that for this particular specification given here you would require only 8 number of elements compare to 19 element. So, in the next lecture we will see what is the response, how the two responses compare with each other the till then thank you very much. Enjoy yourself, work hard, will see you next time. Bye.