Microwave Theory and Techniques
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Module - 05
Lecture – 21
Microwave Filters – I: Filters and Low Pass Butterworth Filter

Hello everyone. Today we are going to talk about Microwave Filters. You might have studied about filters in circuit theory course; where you would have heard about low pass filter, high pass filter, band pass filter, band reject filter and all pass filter. But today we will see how these filters can be realized at microwave frequencies so, let us start low pass filter.

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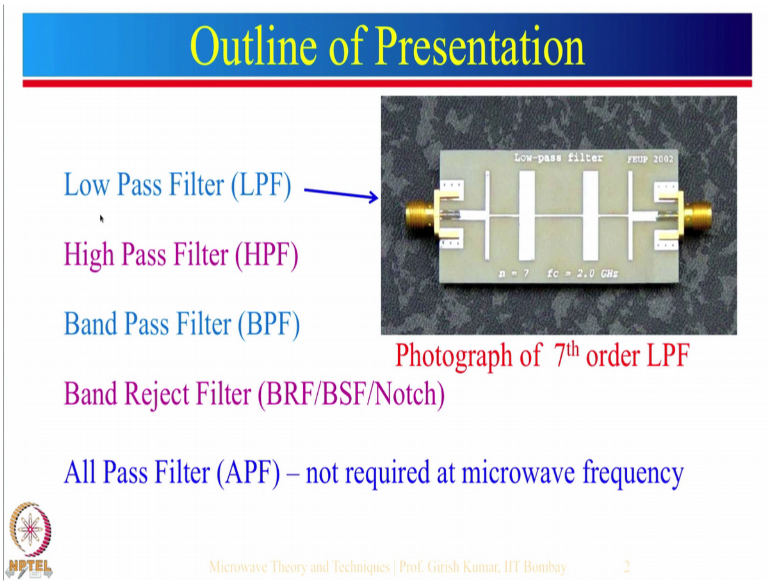

Basically a low pass filter is a filter which passes low frequencies. High pass filter passes higher frequencies. Band pass filter passes a particular band of the frequencies. Band reject filter which actually rejects a certain band. In fact, it is also known as a band stop filter. Sometimes you may think about this as a border security force, but that is a band stop filter here and then notch filter.

Then we have another one which is a all pass filter. Now, but in general I want to tell you we never ever design all pass filter at microwave frequency. The reason for that is in general all pass filters are designed for providing a phase delay. But at microwave frequency we can provide the phase delay by a simple line length. For example, if we want a 90-degree phase delay at a let us say f equal to 1 Gigahertz. So, at a 1 gigahertz wavelength is 30 centimeter. So, lambda by 4 will be 7.5 centimeter that will provide 90-degree phase difference.

But if you want to do the same thing at 1 Megahertz, at 1 Megahertz wavelength will be 300 meter. So, lambda by 4 will be 75 meter. And you do not want to have a PCB with 75-meter length, ok. So, that is why all pass filters are designed at lower frequency, but never ever at microwave frequency. So, I show one example of a 7th order low pass filter. So, here is a photograph of the 7th order low pass filter. You can actually see here there is a no inductor there is a no capacitor. Inductors and capacitors are realized by using transmission line concept.

You just recall when we were talking about transmission line, I had mention that a small transmission line if it is shorted it behaves like a inductor. And a small transmission line with an open circuit behaves as a capacitance. The same concept is used to realize a low pass filter, by using these transmission lines and hence you do not see any inductor and capacitor. So, we will see one by one how to do these things.

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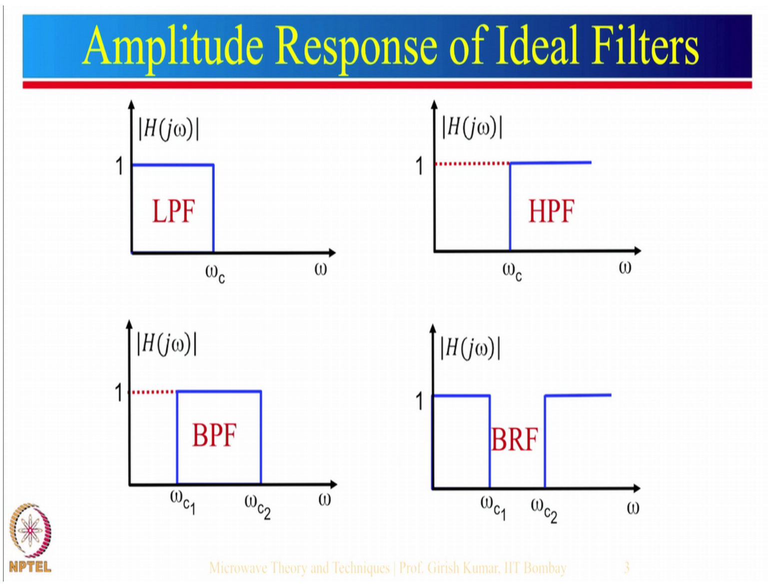

So, let just first look at the amplitude response of ideal filters so, here is an ideal low pass filter, which will pass all the frequencies up to frequency omega c and nothing will be passed beyond that. Now this is an ideal characteristic it does not really happen in a reality. So, ideal situation would be that there is absolutely no attenuation; that means, if input is let us say one output will be equal to 1, or if input is 10 output will be equal to 10. And there will be a shaft cutoff at omega c. And after that nothing will be pass, but as I mentioned this is only an ideal response. Practically if somebody wants to achieve this we would require a filter of the order of infinity. And you know that infinity does not exist in the real world.

So, from low pass filter we can just shift to the high pass filter, you can see that it is a reverse of that in fact, you can actually think about plotting as 1 by omega c. So, high pass filter passes all the higher frequencies beyond omega c and does not pass any frequency below that again it is an ideal characteristic. A band pass filter will pass the frequencies between omega c 1 and omega c 2 without any attenuation. And it will provide complete attenuation at other frequency band.

And a band reject filter or band stop filter or notch filter will actually provide of notch or you can say it will reject the frequencies between omega c 1 and omega c 2. And it will pass all the other frequency. So, if you just look at it from low pass filter we can see that the transformation is very simple. If we just make 1 s over here whatever is the value of s, and we know that s is equal to j omega. So, if we substitute s equal to 1 by s we will get a high pass filter. And a band pass filter you can see that the inverse of band pass filter will realize band reject filter. So, we will see these things one by one.

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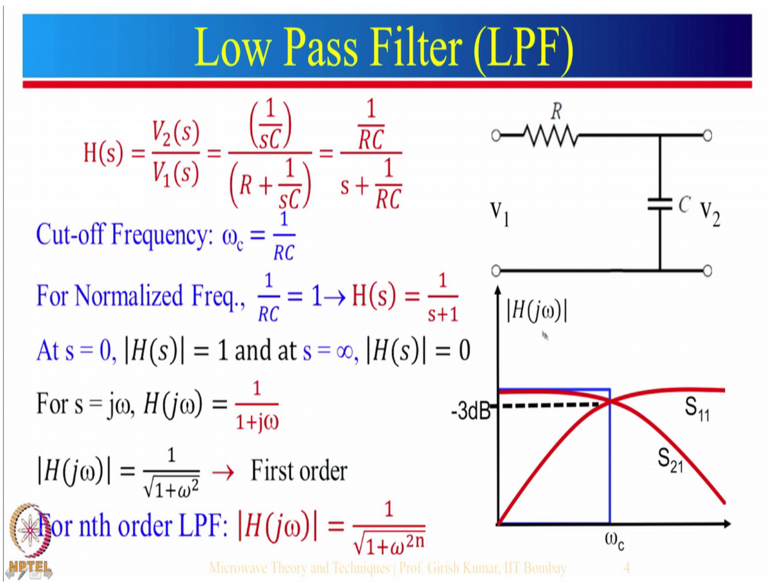

First let us start with the very simple low pass filter. This is not the circuit which is used at microwave frequencies, but I have taken deliberately this particular circuit, because you might be familiar with the RC circuit. RC circuit is nothing but a first order low pass filter. In fact, it is also known as integrator. Now to find the transfer function H s it is very simple a transfer function is given by V 2 s divided by V 1 s. So, this is the output voltage divided by input voltage in s domain.

So, we can find the voltage ratio in a simple way impedance of this which is 1 by s C divided by the total impedance which will be 1 by s C plus R. And if we multiply numerator and denominator by s divided by R so, if I multiply this s divided by R it will become 1 by RC. And again we multiply s divided by R, this will become s and we multiply this with s divided by R it will become 1 by RC.

So, if you actually look at the plot of this particular thing which can be done using Bode plot. So, we can actually say that the cutoff frequency is nothing but omega c is equal to 1 divided by RC; however, we will take a normalized frequency situation here, because once we deal with the normalized frequency, then we can design a filter at any desired frequency. So, in fact, most of the things as we will see that we have going to do for normalize frequency omega c equal to 1, and then we use the frequency transformation to design the desired low pass filter at the other frequency.

So, for normalized frequency we can say 1 by RC is equal to 1. And if 1 by RC is equal to 1, we can say H s is nothing but one divided by s plus 1. And sometimes it is very easy to actually find out whether this transfer function represents a low pass filter or a high pass filter or a band pass filter. You can do a quick check by substituting let us say s equal to 0. So, if you put s equal to 0, this will be equal to 1. And if you put s equal to infinity 1 by infinity will be 0.

So, you can actually get a little bit of an idea that at low frequency amplitude is nothing but one at very high frequency amplitude goes to 0, which is a characteristic of a low pass filter. So now, let us put the values of s equal to j omega. Or just to tell you were general cases s is equal to sigma plus j omega. Sigma represents the attenuation part, but here assuming that it is a lossless situation. So, sigma becomes 0. So, s is equal to j omega. So, if we substitute the value of s equal to j omega over here, we can say h j omega is nothing but 1 over 1 plus j omega. And if we now take the amplitude or magnitude of this particular thing, then we can say magnitude of this will be nothing but 1 divided by 1 plus omega square, square root of that.

So, this is a first order low pass filter. In fact, this type of the filter realization is also known as maximally flat low pass filter, or it was invented by your proposed by butterwort it is also known as butterwort filter. And for nth order low pass filter H j omega magnitude will be nothing but 1 plus omega to the power 2 into n, ok. So, that n is the order of the filter. So, let us see a typical response so, this is a H j omega magnitude versus frequency.

So, this will be the transfer coefficient plot, which is H j omega magnitude plot. But since we are going to design things at microwave frequencies so, we are going to represent these things in terms of s parameter. So, the transfer function from here to here can be represented by s 2 1, and the reflection coefficient at this particular thing can be represented by s 1 1. So, the cutoff frequency which is omega c over here at that point, you can say that the amplitude which was 1 or in terms of dB it will be 0 dB.

So, half power point will be minus 3 dB. So, this is where the cutoff frequency is defined. So now, let us see how we do it in microwave frequency and I am going to just tell you beforehand, at microwave frequency generally we do not use resistor, but we use inductors. So, instead of using a resistor here we will be using inductor, capacitor, inductor, capacitor. And why do we do that? The reason for that as because of this R; if you think about voltage V 1 and voltage V 2 see if this derivation, which has been done it assumes that the load is infinite. But the moment we put a load over here and at microwave frequencies most of the time input and output impedances are 50 ohm.

So, think about it if I put a 50 ohm, resistor over here and if this resistor is suppose 1 kilo ohm, then what will be V 2 by V 1 50 divided by 50 plus 1,000. So, the output voltage will be very very small it will never be equal to 0 dB or gain equal to 1, ok. And even if it is 50, and if this is 50, 50 divided by 50 will be still half value here, ok. But if we have a inductor, then the dissipation in the inductor will be relatively small.

So, let us see how we can realize a nth order low pass filter using inductors and capacitors.

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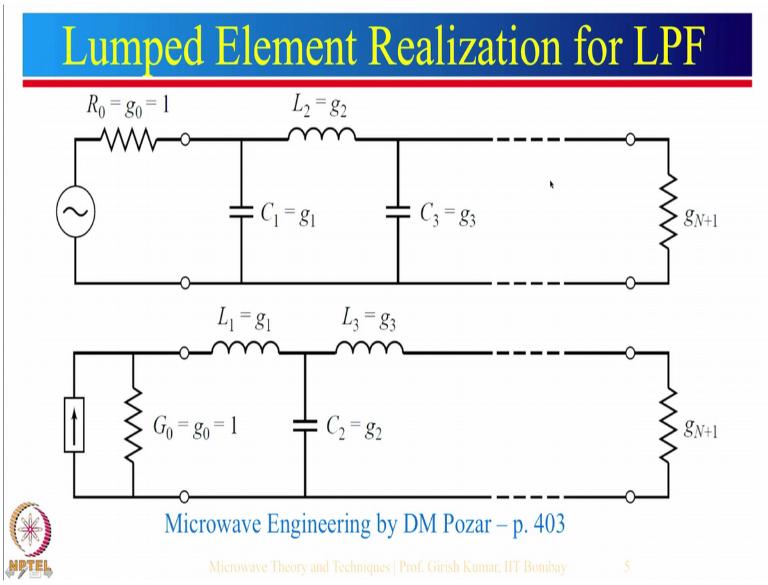

So, over here we have a source we are assuming in the beginning that the source impedance is equal to 1 and we will also take later on that load impedance is also equal to 1. Now 1 is just again a generalized case, or you can say it is a normalized impedance. Later on we are going to use impedance transformation technique also; to transfer this R equal to 1 or g equal to 1 over here to the desired impedance values.

So now let us see what is the difference between these two so, here the first element is capacitance then inductor then capacitance and so on. In this particular circuit here the first element is inductor, then capacitor then inductor. Now you can use either this configuration or you can use this configuration both of them will give you exactly the same result, ok. So, it does not matter you can start with either first element which is series element or you can start with the first element as a shunt element, ok.

Now, what do you see over here is, I have written g 1, g 2, g 3, ok. And again here starting point is g 1, g 2, g 3. And you know in Hindi g 1 also means g 1 [FL], right. So, you can actually you know anything which start with starts with g 1 right. So, you can think in the lighter sense also. So, we now need to find the values of g 1, g 2, g 3 to realize a low pass filter. Now there are several techniques to realize a low pass filter, and these have been proposed by different people. So, let us see what are these different techniques to realize these g parameters.

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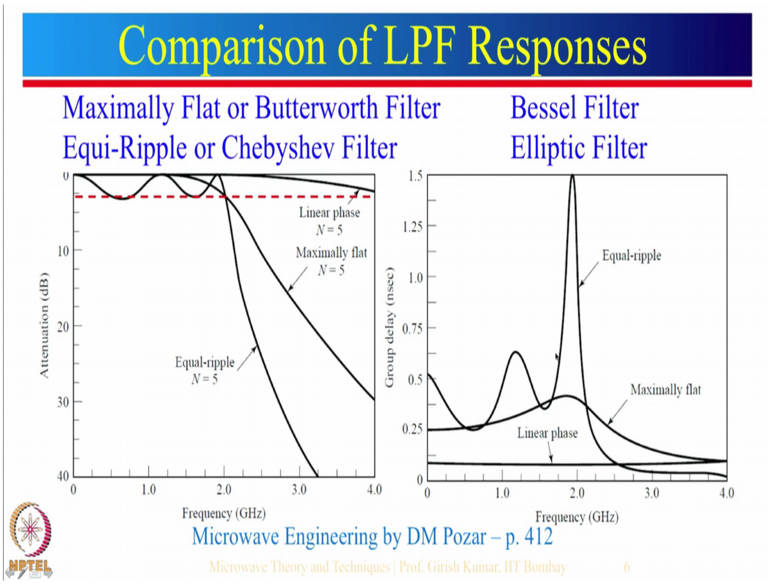

So, one which I had mentioned to you that was that the responses something like H j omega magnitude 1 divided by square root of 1 plus omega to the power 2 n. That response is known as maximally flat response or proposed by Butterworth hence also known as Butterworth filter. But here you see the responses of 3 different filters, but later on I will also talk about the 4th type of the filter also.

But first let us focus on these 2 types which is a maximally flat a Butterworth filter. The response of that is starting from here, you can see it is flat and then it is coming down over here. So, this is the response for a 5th order Butterworth filter. Let us see what is this Chebyshev filter. Chebyshev filter has a ripples in the pass band, and then it is going over here. Now what is the difference between these 2 then? So, in this case we have a maximally flat. So, this is the frequency range where you want to pass the frequency. But you can see that the attenuation in the stop band it is relatively slow, it is moving slowly and attenuating the frequencies relatively slowly.

Whereas for the same 5th order that equi ripple or Chebyshev filter. In fact, again Chebyshev I want to mention that this spelling in different books it is used slightly different because it was a Russian scientist. And in fact, the spelling given in some of the other books is starts with tsch, ok. But if you make that ts relatively silent in fact, now it is more known as che, so, Chebyshev filter. So, the main point of the advantage of this

Chebyshev filter is that the response from the pass band to the stop band the transition is relatively faster.

The disadvantage of maximally flatter Butterworth thing that the response is relatively slower from pass band to the transition band but the disadvantage of Chebyshev filter is that there are ripples in the pass band, ok. Now this is the amplitude plot now let us look at the phase plot. So, this is the phase plot or you can say that is a group delay just to tell you, the group delay is nothing but defined as minus d phi by d omega, ok. So, sometimes it is known as phase plot or sometimes known as group delay plot.

So, generally what we want we want group delay should be constant relatively, ok; that means, that all the frequencies in this particular region should reach the output with the same delay, ok. So, let us see what are the responses of these 2 cases first. So, here is the response you can see that this is the response for maximally flat. And this particular filter has been designed for a frequency of 2 Gigahertz. You can see that the cutoff frequency is 2 Gigahertz here. So, we have to see the phase response more from here to here, ok.

So, you can see that the group delay is not really 100 percent constant. But the variation is relatively small and then of course, it this is the response in the stop band. Let us say what is the response for the equi ripple or Chebyshev filter; just because you can see here that there are ripples in the pass band here also there is a ripple in the group delay also, ok. And then at the transition level you can see that the group delay increases significantly and then there is a large variation over here, ok.

So; that means, the Chebyshev filter also has a disadvantage of a relatively larger variation in the group delay compared to maximally flat. But still if you see what is the value of the group delay. These values are in nanosecond, ok. So, you can see that this delay is relatively very, very small. So, think about let us say if we are talking on let us say a mobile phone. So, if the signals are delayed by different time you will actually here different frequencies at a different time delay. But when you are listening to something a group delay of a nanosecond will not make any difference at all, ok.

So, even though there is a group delay over here it is not significantly bad; however, there are certain applications where even at this kind of a variation or if this variation is also not acceptable. So, to achieve relatively constant group delay of Bessel filters are there. A Bessel filters are also known as a linear phase or you can say a group delay is constant. So, as you can see for this particular filter the responses almost flat.

But if you now look at the amplitude response so, here it looks like a maximally flat, but you can see here that the transition is very, very slow. Even at this particular frequency you know at 4 Gigahertz. So, it was supposed to work at 2 Gigahertz, but even at 4 Gigahertz the attenuation is only 3 dB whereas, if you look at this frequency 4 Gigahertz, a Butterworth filter would actually do the attenuation by almost 30 dB. Whereas, a Chebyshev filter if you try to extend this further you can say that maybe close to 50 dB.

So, in fact, sometimes I actually say again in the lighter mode you can think about this particular fast response as something like a very fast sports car. So, which goes from 0 to let us say 100 kilometer per hour in, let us say a few seconds. And this is relatively as slow car which actually speaking goes from 0 to certain speed at a longer time. So, of course, the faster transition you pay penalty. So, very fast sports car would actually have you know larger petrol expenses. And even the car would be expensive. So, you pay the price over here in terms of larger ripple. Whereas, a slow vehicle over here will give you a very good thing; that means, maybe a better fuel efficiency or a lower price.

But the thing is you actually pay in terms of lower attenuation from here to here. In fact, I am going to show you later an example, that where if it is defined that we want certain attenuation. For example, just look at it at 3 Gigahertz, suppose we want an attenuation of more than 30 dB. So, here if I just look at even a line of 3 you can see that 30 dB will be somewhere here which is coming at 2.8 Gigahertz. But corresponding to 2.8 Gigahertz you can see what is the attenuation provided here maybe 12 to 14 dB.

So, if we want attenuation at certain desired frequency, then we do not have much choice if you want to use a smaller order filter it is better to use Chebyshev than to use Butterworth filter. But we will see later on that if you want this kind of a transition from pass band to the stop band, then we may have to use a much larger order of Butterworth filter, ok. So, what is elliptic filter? So, elliptic filter even though I have not shown the response, but let me explain you.

So, elliptic filter has this equi ripple response in the pass band. As well as it has the similar response in the stop band also. So, that means, there is a disadvantage over here also, but the advantage of the elliptic filter is for the same 5th order that response will be very, very sharp, it will come something like this here. And then it will have equal ripples like this here, ok. So, that means, basically you can actually attenuate certain undesired frequencies in this band very fast.

So, we will see these examples later on. So now, let us just look at the maximally flat Butterworth in more detail now.

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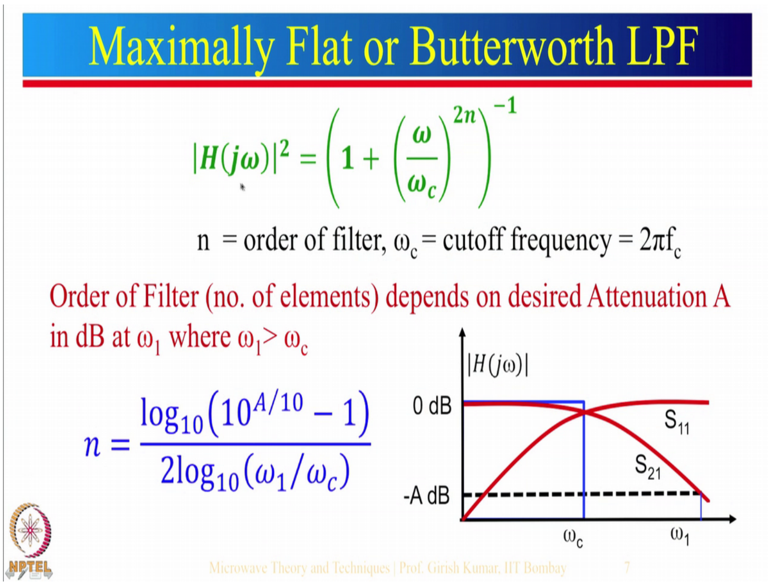

So, we have seen that H j omega square is nothing but I have just written this in a slightly different form. So, this is to the power minus 1 will be 1 divided by 1 plus omega by omega c to the power 2 n, ok. Square root term is not there because here we have squared at. So, here n is order of the filter omega c is cutoff frequency, but remember this is not really frequency in terms of hertz, it is actually the unit of omega is a radian.

So, omega c is equal to 2 pi f c. So, when you are designing your filter let us say at 1 Gigahertz. Please do not put omega c equal to 1, but Gigahertz, but you have to put f c as 1 Gigahertz multiplied by 2 pi. So now, order of the filter now the thing is many a times you may be just ask design a 5th order filter or 10th order filter, ok. But that is probably an analysis problem. But majority of the time people will actually specify that we want certain attenuation A in dB at omega 1 frequency.

So, this is the cutoff frequency omega c this is the frequency omega 1 and at omega 1 it is desired that attenuation should be A dB, ok. So, for example, this may be 0 dB, this may be minus 30 dB or minus 40 dB or minus 20 dB. Suppose if this is 1 Gigahertz, this may be 2 Gigahertz or 3 Gigahertz or depending upon the application to application. So now, this equation actually speaking can be simplified to this particular thing over here. I will give detailed example later on, but right now you can take this particular thing. In fact, all you have to really do it is take logarithm on both the sides and simplify and solve it for n.

So, n will give you order of the filter and what we have here 10 to the power a by 10 where A is now attenuation in dB. So, you can see here it is written as minus a. So, this will be minus a 20 dB or minus 30 dB but a is a plus number. So, if it is 20 dB you are just going to put 20 here. If it is 30 dB, you just put 30 over here. And this one here is the ratio of omega 1 and omega c. Omega 1 is the frequency where we want attenuation to be A and omega c is the cutoff frequency.

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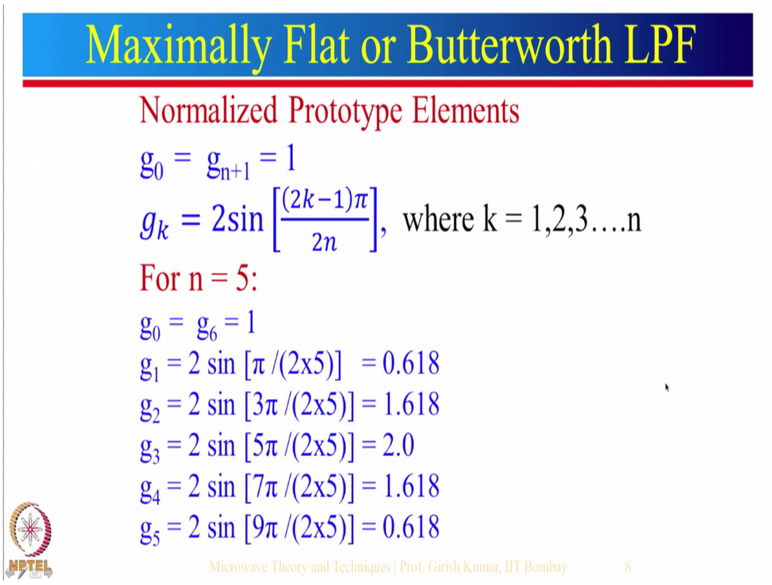

So now what we need to do we need to find out the g parameters, ok. So, for Butterworth low pass filter we can actually find the g parameters in a very very simple manner. So, here g 0 is equal to g n plus 1 is equal to 1, ok. So, g 0 if you recall it is a source resistance which we are taking as 1; g n plus 1 corresponds to the load resistance which is equal to 1. And these are the different g parameters given by simple expression which is g k. And if you know my name, my name is also g k Girish Kumar and of course, g k also stands for general knowledge.

So, g k actually is given by this very simple expression 2 sin 2 k minus 1 into pi divided by 2 n; where k is 1, 2, 3 and n is the order of the filter n will be the number of elements. So, let take a simple example of n equal to 5. So, if you put n equal to 5, 5 into 2 will be 10, and now all you do it is put first k equal to 1 so, this will be 2 minus 1. So, that will be pi divided by 10. So, put it over here simplify you will get 0.618, then in this particular case now all you have to do change the value of k.

So, 2 into 2 4 minus 1 will be 3. Then for the third one it will be 3 into 2 6 minus 1 so, 5 then 7 then 9. You can see that 0.618, 1.618, 2, 1.618 what I want to just mention here that the filter is symmetrical so; that means, g 1 is equal to g 5 g 2 is equal to g 4. Now of course, you can do the calculation by using this simple expression.

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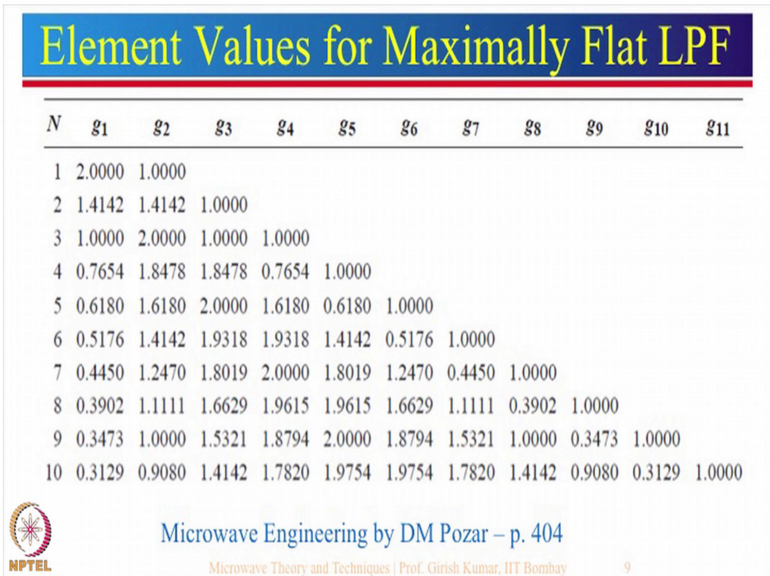

Element Values for Maximally Flat LPF

N	g_1	g_2	g_3	g_4	g_5	g_6	g_7	g_8	g_9	g_{10}	g_{11}
1	2.0000	1.0000									
2	1.4142	1.4142	1.0000								
3	1.0000	2.0000	1.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654	1.0000						
5	0.6180	1.6180	2.0000	1.6180	0.6180	1.0000					
6	0.5176	1.4142	1.9318	1.9318	1.4142	0.5176	1.0000				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450	1.0000			
8	0.3902	1.1111	1.6629	1.9615	1.9615	1.6629	1.1111	0.3902	1.0000		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	1.0000	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129	1.0000

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However, you can actually use this particular table. And what this table gives you? It gives you straightway the g parameters value for different values of n 1 to 10.

So, what we have just done? You can calculate g parameters using this particular expression or you can use the table to find out the g parameter. So, this table gives g parameters for n equal to 1 to 10. And g 1, g 2, g 3 all the parameters are given in this particular table.

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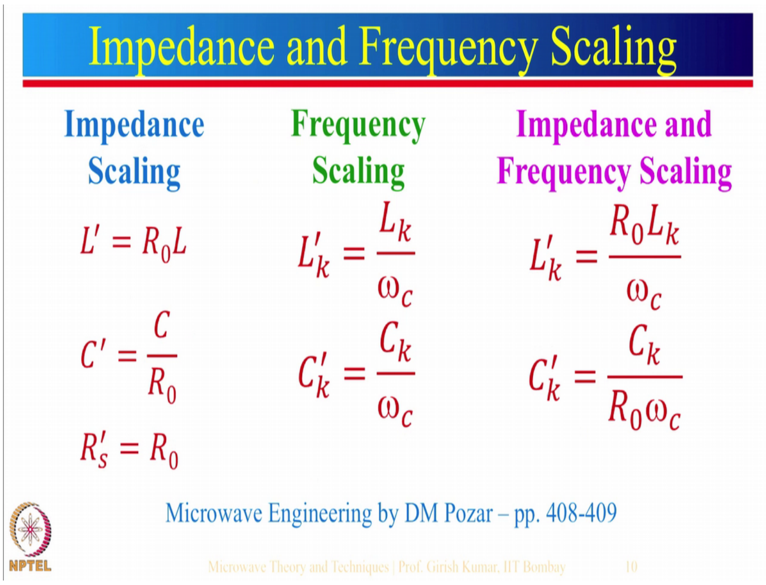

So now after we find the g parameters, now as I mentioned these g parameters are defined for normalized impedance which is R equal to 1 and normalized frequency which is omega equal to 1.

So, we have to do the impedance scaling as well as frequency scaling. So, what we do actually? So, we change all the components by same impedance factor. So, let us say instead of one we want the resistance to be equal to R 0. So, then all the impedances should be changed accordingly. So, for example, what is the impedance of L? That is equal to z equal to j omega L. So, if you want to impede so, if you want to increase the impedance by a factor of R 0 so; that means, the original value of L should be modified by a factor of R 0 so, that the impedance increases.

Now, for capacitance we know that the impedance is given by z equal to 1 by omega c, ok. So now, to increase the impedance we have to decrease the value of the capacitance. So, the capacitance should be decreased by a factor of R 0 so, these are the newer values. So, this we need to do for impedance scaling. Then we do frequency scaling in case of frequency scaling what we do we keep the impedance value same, but we only change the frequency.

So, for example, again what is the impedance of inductor, it is equal to z if it is given by z is equal to j omega L. Now since omega has increased we have to decrease the value of l by the same factor omega z. Similarly, the impedance for capacitance is given by z equal to 1 by j omega c. So, to keep the impedance same, but frequency is increased hence we have to decrease the value of the capacitance. Now we combine impedance scaling as well as frequency scaling it is known as impedance and frequency scaling.

So, we combine both the factors over here so, the new value of the inductor will be R 0 divided by omega c. The new value of the capacitance will be R 0 and omega c will be coming in the denominator. So, just to mention again so, the value of the new inductor will be obtained from the older inductor value or from the g parameters in a simple way by multiplying it with R 0, which is the load impedance or source impedance divided by omega c. Whereas, from the g parameters which are given by C k or g 1, g 2, g 3 which will be in the form of C k. And those things have to be divided by R 0 and omega c.

So, in the next lecture, we will take more practical examples. So, we will see that how to really design a low pass filter. So, just to make a quick summary of today so, we started with an ideal response of low pass filter, high pass filter, band pass filter, band reject filter. And I did mention that we do not design all pass filter at microwave frequency, because the phase delay can be obtained simply by using a smaller line length.

Then we looked at the comparison of different types of low pass filter. Say for example, we looked at maximally flat or Butterworth filter, or equi ripple also known as Chebyshev filter, Bessel filter and elliptic filter. After that we look at g parameters of Butterworth filter or you can say a maximally flat filter. And after that we looked into frequency scaling. So, in the next lecture we will look at some of the real design, and how to do the simulation of these kind of a filters and obtain the desired results.

Thank you very much. We will see you next time, bye.