Microwave Theory and Techniques
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Module - 04
Lecture - 16
Power Dividers – I: Two-way, Three-way and Four-way Equal Power Dividers

Hello. In the last few lectures we have been talking about transmission lines, impedance matching, smith chart, ABCD parameters, and S parameters. Now we are going to talk about applications of all those things. So we will talk about the first application today which is power dividers and combiners. So, let us see where we need power dividers and when we need combiners. So, first we will actually going to look at 2 way equal power divider.

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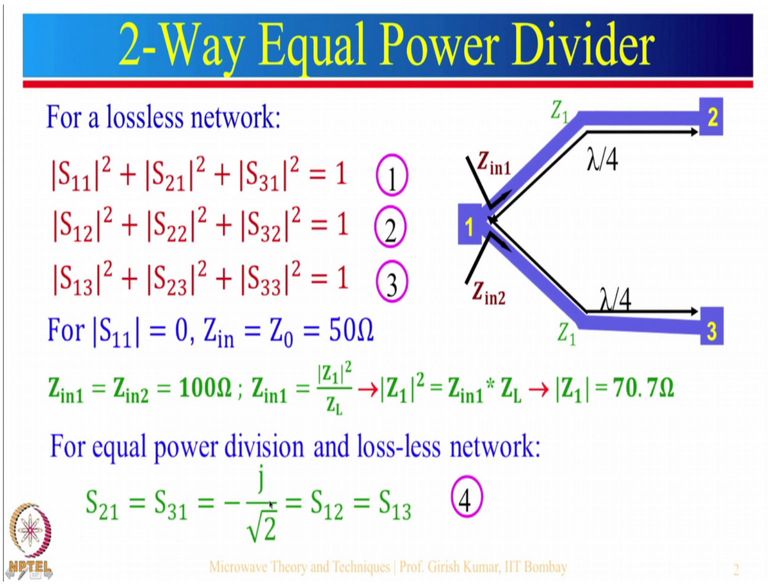

So, let us say if we are giving a input at this port here, then the half power should go here ideally and half power should go over here ideally and nothing should reflect back. So, where we need this kind of a power divider; for example just to mention one of the applications that if you want to feed let us say 2 element antenna. So, what will you do so we actually gave input at one place and then the power is divided equally and that is given to the 2 element.

Now, there may be a element array could be 8 by 8 array or 16 by 16 array. So, in that case we will need large number of power dividers and majority of the time we feed these arrays using equal power division. So, that is one of the simple application, but this power divider may be required at many other places which maybe a equal power divider or even un equal power divider. So, let us start with the simple most thing which is a 2 way equal power divider.

So, let us see what we have shown over here. So here is the input port, this is the output port, this is the output port, and here we are assuming that all the 3 ports have a input impedance which is equal to Z0 and majority of the time we take this as 50 ohm. I mean just to tell you most of the microwave generators really use 50 ohm impedance source impedance or in fact we use load impedance also, so that there is a standardized process.

So, accept in Russia where they do not use 50 ohm, almost all the other countries in the world use 50 ohm as standard for microwave. So, let just see what we had studied in the previous lecture. That we had seen that for a lossless network this is a unitary property and we have written it for all the 3 ports. So, S 11 square plus S 21 square plus S 31 square is equal to 1 and the same thing you can write it over here for S 12 S 22 S 32 and then S 13 S 23 S 33.

So, this is the lossless network property, and here is this a lossless network when we are going to assume that these transmission lines are lossless or in general also the transmission lines have very very low loss. And what we have shown over here this is a micro strip realization of a 2 way equal power divider. We will take real simulated examples also, but let us first look at the concept part.

So, ideally what do we want we want S 11 to be equal to 0; that means, the reflected power at this port should be equal to 0 and the half power should go here half power should go over here. So, for S 11 to be equal to 0, Z input should be equal to Z 0 and that should be equal to 50 ohm. So let us say the input impedance here should be 50 ohm. Now we can see that a one branch is coming from here, another branch is coming from here, and we can see that these 2 branches are coming in parallel at this particular point.

So, if the two things which are coming in parallel and we want equal power division. So, what we really want that the input impedance looking from here, and input impedance looking from here should be equal and that should be equal to 100 ohm because, then we can say a 100 ohm in parallel with the 100 ohm will be equal to 50 ohm. So, now let us see how do we get 100 ohm over here, we know that this is 50 ohm impedance we need to transfer this impedance from 50 ohm to 100 ohm. And we had studied when we talked about transmission line that a quarter wave transformer can transform one impedance to the other impedance. And the formula for that is Z input 1 looking from here is given by Z 1 square divided by Z L. So, Z 1 is the characteristic impedance of this particular line. And we have taken these 2 equal because, we want equal power division. If we do not want equal power division then Z 1 will not be equal to this particular impedance which maybe let us says Z 2. We will see that later on let us first complete this particular thing now.

So, Z input 1 we want that to be 100 ohm ZL we know is 50 ohm all the 3 ports are terminated with 50 ohm. So, from here we can calculate the value of Z 1 square which is Z in 1 into Z L. So, Z input 1 we want this to be a 100, so 100 multiplied by 50 and then we have to take square root of that which comes out to be 50 square root 2 and that comes out to be 70.7 ohm. And this length should be lambda by 4.

Now, if you recall we did mention about single quarter wave transformer and 2 quarter wave transformer. So, here this part is only single quarter wave we will take example of 2 quarter wave also and we have seen that the single quarter wave provides impedance matching only at a single frequency. So we will see these results one by one. But now what we have here so 4 equal power division and lossless network. What will happen now since nothing is reflected back and this is a lossless network.

So, half power goes here half power goes here, so that means S 2 1 which is power going from 1 to 2 and then from 1 to 3 which is S 3 1 that should be equal to 1 by square root 2 I will talk about this term little later on. So, 1 by square root 2 means that square of this will be 1 by 2. So that means half power goes here half power goes here; now what about this term here so you can see here that from here to here the length is lambda by 4.

So, length lambda by 4 if you look at the phase delay because of this length will be equal to beta, which is equal to 2 pi by lambda into lambda by 4. So, 2 pi by lambda into lambda by 4 will give us 90 degree. And since the delay is from here to here we have a minus sign. So, minus comes because of the phase delay and a 90 degree can be represented by a j term. So, we know that 0 plus j 1 has nothing but you can say that will have an angle of one angle 90 degree, so that is how this term minus j comes over here and since the network is reciprocal as you can see that there are no active components over here. So, whatever we give if I give the same input it will come over here. So, we can say that S 21 equal to S 31 will be equal to S 12 equal to S 13. So, we have now found 1 2 3 4 and this is 5 so out of this 9 component we have now found 5 components. Now, we need to find other 4 components also, so let us see how we can do that.

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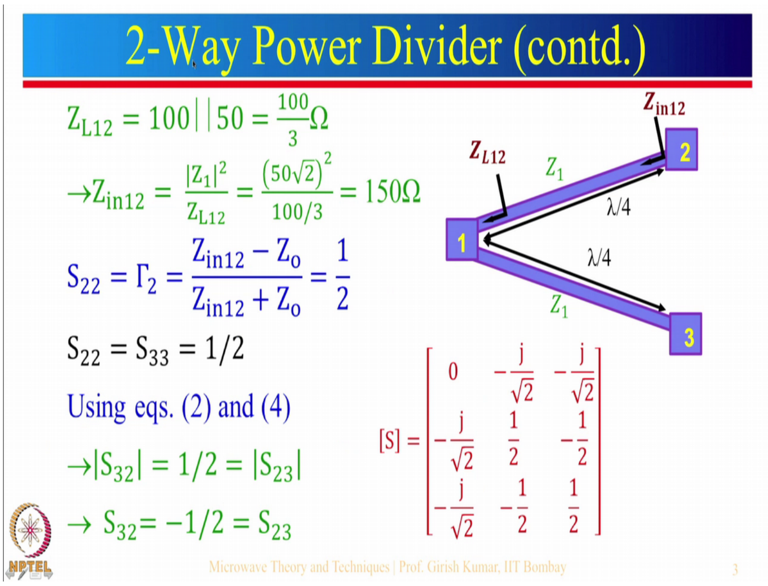

So, this one here we are now going to try to find out the input impedance at this particular port over here. So, to find the input impedance at this particular port we have to look from here, so then we come at this point and then we have to look at this here. So, let us first calculate what will be this value of the input impedance. So, we can see that from here if I look at the input impedance, so this is 50 ohm and that impedance in parallel with. So, we had a 50 ohm quarter wave transformer had transform this impedance to 100.

So, what this one sees here is a 100 in parallel with 50, so we can actually find out Z L12 which will be 100 in parallel with 50 and that comes out to be 100 by 3 ohm. Now we need to find Z in 12 at this particular thing here, so that can be found as Z 1 square divided by Z L12. So, that is Z 1 square divided by Z L12 and Z 1 square we know is 50 square root 2 which is 70.7 ohm. So, square of that divided by 100 by 3 we simplify that comes out to be 150 ohm.

So, now we can find out what is the value of S 22, so what is S 22 same as reflection coefficient at port 2 and that can be written in this particular form here. So, if we now substitute the value Z in 12 is 150 minus 50. So, the numerator will be 150 minus 50 100 the denominator will be 150 plus 50 which will be 200, so 100 by 200 will be equal to 1 by 2. So, if we are look from this side we can say that S 22 is equal to 1 by 2 and same way we can do the derivation or by symmetry we can say S 33 will be equal to S 22 and that is equal to half

So, now we can use the equations 2 and 4 and equations 2 and 4 basically are related with you can say here S12 square plus S 22 square plus S 32 square is equal to 1 and this is the equation 4. So, if we now substitute these values so what we can find out is that S 32 magnitude is equal to half and that is equal to S 23; so that means, the power from here to here is equal to you can say 1 by 4 because, power will be square of this and from here to here will be 1 by 4.

Now there will be a phase difference from here to here we have seen that this phase difference is 90 degree, so this will be 90 degree and this will be another 90 degree. So, the phase difference between these 2 ports will be 180 degree which is represented by minus sign. So, now we can complete the S matrix, so S matrix when we have feeding at input port 1 so that is 0, half power goes here half power goes here. So, that is minus j by square root 2 minus j by square root 2 1 by square root 2 square will be half minus j is coming because of the minus 90 degree phase delay.

Now, because of the symmetry this component will be same as here this component will be same as here. Now S 22 we had seen that is equal to half same as S 33 and then S going from 2 to 3 and 3 to 2 they are equal to minus half over here, so that is the S matrix of 2 way power divider. Now let us see whether this particular thing can be use as a power combiner, so we can give input here half goes here half goes here.

Now, is this a good power combiner, so suppose if I give a input at port 2. So, what will happen if I give a input at port 2 you can see that S 22 is half, so that means one-fourth of the power will reflect back that is not a very good thing and out of that rest of it. So, one-fourth goes over here and over here again this is 1 by 2 square. So, that will be 1fourth so one-fourth reflects back one-fourth goes over here and half power goes over here. So that means this is not really a very good combiner it is a good power divider but not a good power combiner. So, what we need to do as a power combiner we will see that little later on.

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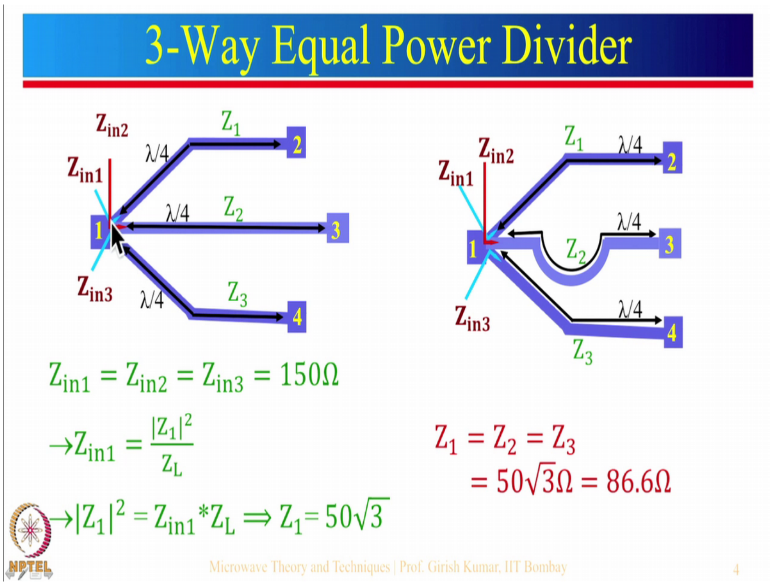

Now, let us first look at if you want 3 way equal power divider what do we need to do. So, let us see in order to design a 3 way equal power divider. So, we have 1 lambda by 4 section another lambda by 4 section another lambda by 4 section and just we have written here Z1 Z 2 Z 3, but we know that for equal power divider Z1 should be equal to Z 2 should be equal to Z 3.

Now, you can see that this length is lambda by 4, but since it is bent. So, you can see that the length will be same lambda by 4, but it is now little bent over here. So, you can say that all these 3 ports are not properly aligned, so these things can be aligned by using this particular concept also. So, where you can see that this is bent here this is bent like this here and this one over here, so the all the 3 ports are at the same level ok. This will be required more when you are actually going to have a physical PCB layout and you want to put a connector. So, you do not want a PCB to have a something like this kind of a shape majority of the time let us say a nice PCB will be more like let us say a rectangular shape over here and then you can put a connecter here connector here and connector over here.

But now let just look at the design since we want 3 way equal power divider. So we will say that Z 1 is equal to Z 2 is equal to Z 3 and in that case we can say a Z in 1 Z in 2 Z in

3 should be equal and these should be equal to 150 ohm why 150 ohm because now 3 of these are in parallel. So, if there are 3 things in parallel which are equal then the net input impedance at this port will be a 150 ohm divided by 3 which will be equal to 50 ohm. So, now we want desired impedance at this point is 150 ohm this impedance is 50 ohm, so we need to use a quarter wave transformer we know the formula. So, Z in 1 is given by Z 1 square divided by Z L Z 1 desired is 150 ZL is 50.

So, from there we can calculate Z1 as 50 square root 3 and we have Z 1 equal to Z 2 equal to Z 3 which is equal to 86.6 ohm; so that means, we can design a 3 way equal power divider in a very simple manner that we design these micro strip lines for an impedance of 86.6 ohm and this will work as a equal power divider again this is not a good power combiner ok. So, we will tell you later on how to design power combiner.

Now, suppose if we had not use these quarter wave transformer then what would have happen, let us say that these 2 3 4 are connected right here at this point without using a quarter wave transformer. Then what would have happen then 50 ohm then this 50 ohm then this 50 ohm the 3 50 ohms will be in parallel. So, what will be the equivalent impedance at this point if they are combined together and we say that the port one is here then it will be 50 divided by 3 and that will be 16.67 ohm and that means that input port will not be matched, so lot of power will get reflected back. So, it is required that we design these things properly choose the impedance values carefully, so that we can design a equal power divider.

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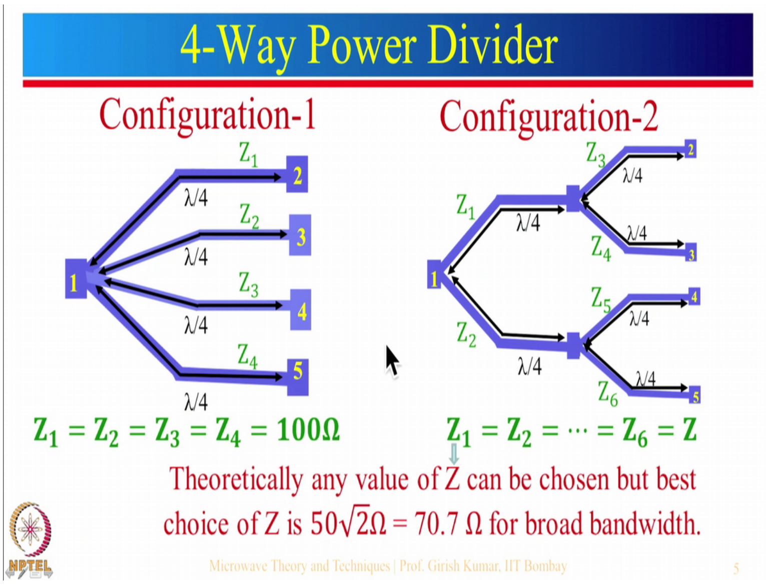

So, from 3 way let us go to another thing which is a 4 way power divider I have shown here 2 different configurations, let us first look at the configuration 1. So, what is the configuration 1 well we have a lambda by 4 section another lambda by 4 another lambda by 4. Now, this is a general case; we have written Z1 Z 2 Z 3 Z 4, but for equal power divider these should all be equal and you can see here I have written here as a 100 ohm why 100 ohm? Well, let us see now Z 1 square by 50 will be 100 square divided by 50, so that means input impedance here will be now 200 ohm. So, this is 200, 200, 200, 200: 4 impedances of 200 ohm in parallel will be equivalent to 200 divided by 4, so we get 50 ohm. So that means S 11 will be equal to 0 and since all the 4 impedances have been taken equal, so equal power division will take place and as I mentioned earlier these outputs can be connected to 4 different antennas and that will form an array of 4 different antennas.

Now let us try to do the same thing in a slightly different way so this is a configuration two. So, let us see what this configuration 2 is so you can see here that we have got a 2 way power divider, we have written a general case where Z 1 Z 2 Z 3 Z 4 Z 5 Z 6. So that means this configuration can be utilized for unequal power divider also. But here we will first take the cases where the power divisions from here to all the 4 ports are equal.

So, let us see what happens now so we are giving a input at port 1, so half power goes here half power goes here. So, from here then half power goes here half power goes here and half power goes over here half power goes over here and in fact if you just do little bit of a simplification I will just give you a little hint here. So, now what is the input impedance at this port, so let us say if you take all the impedances equal as we will see it works out fine so which is equal to Z. So, this will be now Z square divided by say 50 then this will be now 2 are in parallel. So Z square by 50 this is Z square by 50, so in a reality it will be Z square divided by100.

So, now if we look at this here so this impedance is again Z. So, the input impedance looking from here will be this Z square divided by the load impedance and what is the load impedance Z square divided by 100, so Z square Z square will cancel and we will get 100 ohm over here. So, 100 in parallel with a 100 and that will be equal to 50 ohm. So that means, this particular configuration you can choose any value of Z and this will work and this is where the design concept comes into picture. So, what should be the value of Z, shall we take all these lines 50 ohm that will do the job shall we take all these impedances as 10 ohm that will also do the job, shall we take everything to be 100 ohm that will also do the job. So, now the question is which is the best choice ok, so for that you have to remember few things see what is we are really doing. So, on 50 ohm let us say we are going to some value, but ultimately we are going to 100 ohm and these 100 in parallel with 100 will be 50.

So that means, from here 50 ohm we are going to 100 ohm. So, it is always better that let us say we are going from 50 to 100, say it is better that we take a intermediate step and then go over here ok. Suppose if you have taken this as 50 and 50, so this 50 and 50 we will get here as a 25 ohm and if this is 25 ohm this is 50, so 50 square divided by 25 I still get a 10. But in reality we have not really use the transforming property of this thing which we had done for a 2 way. So, by using this 70.7 what we have did from 50 we went to 100, then 100 in parallel with 100 is 50 then from 50 we went to 100.

So, we are moving between 50 to 100 rather than going to some different values because, if we had taken 50-50 from 50-50 this would have become 25. So that means, objective was from 50 to go to a 100 but what we did from 50 we went first down to 25 ohm and then we are going to 100. So, whenever there are larger variations it will give rise to smaller bandwidth. So, if you use this kind of a value here 70.7 ohm for all the impedance value we will actually get the relatively broader bandwidth compare to if you take any other value of Z.

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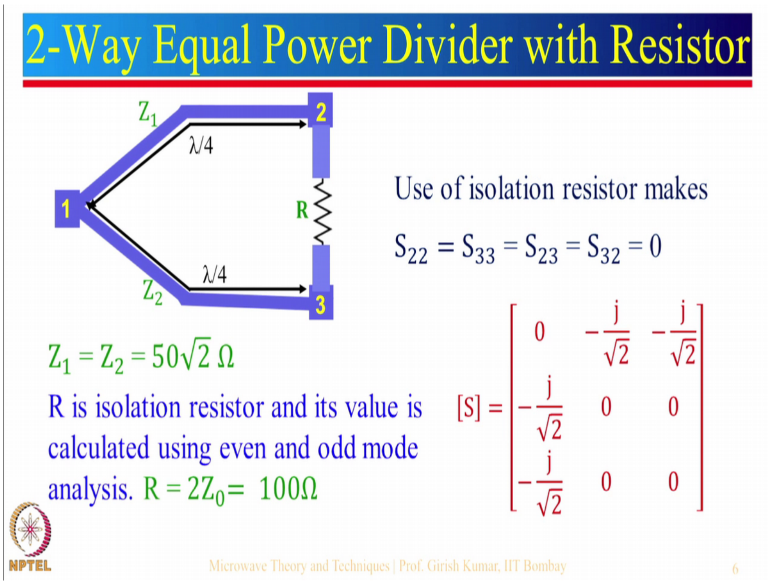

Now, let us see how we can use this equal power divider as a combiner ok. So, we are actually put over here a resister R and that is also known as isolation resistance ok. We will look at the concept little later on, but first just see what really this isolation resistance really do, what it actually does it makes S 22 equal to S 33 equal to S 23 equal to S 32 equal to 0 provided we take this isolation resistance value equal to 2 times Z 0 which is 100 ohm.

If we take this particular thing we will get this particular equation and you can actually see that all these parameters are not equal to 0. What is this really mean first let us just look at the concept point of view. So, now if we feed the input at this port here, so what will happen you can see that nothing is reflected back and what is happening from here to here well nothing is going there because S 32 is equal to 0. So that means, if I am giving a input here nothing is reflected back and nothing is going over here. So that means, that isolation between the 2 ports is very good and also the ports are matched.

So, if you look at the S matrix here 0 0 0, so that means all the 3 ports are now matched and let us see another thing, so from here if I give the input half power goes here half power goes over here. So, which is same as before accept the modification is that these components are become 0. Earlier if you recall these components had the magnitude of half which really meant one-fourth of the power was reflected and one-fourth of the power was transmitted to the other port ok.

So, now the question is what really this is and how that really happens. So first again let us go to the power divider concept first, so from here we gave input power here. So, let us say half power comes here half power comes here. So, if you think in the form of the voltage suppose whatever is the magnitude and the phase at this particular port let us say V 2 that will be exactly same as V 3. So, if the 2 voltages are exactly same and they have the same phase difference, what will be the current flowing through the resister there will be no current so hence there will be a equal power division.

So, now how these things are happening why these things are becoming 0, so let me just explain in a very simple manner. Now let us say we are giving a input at this particular port here, so what happens from here to port 3 there are 2 paths are there is a one path from here and there is a another path from here. So, this path you can see that this will provide a phase difference of one 180 degree and assuming that this path is very very small the way I have shown it looks little larger but we will show you how the practical realization takes place.

So, this path length is generally taken very very small if we do not take small, then also I will show you what really happen. But assuming that this path length is very small or negligible in comparison to this, so this will give me 180 degree phase shift this will give 0 degree phase shift. So, what will happen in that case this power will be outer phase of this particular thing, these 2 will cancel and nothing will come over here ok. So, now this whole thing can be used as a power combiner, so what is a power combiner? Now let us say if I give 1 watt over here and if we give 1 watt power over here. So what will happen now do not look at always the S matrix ok, because see if the way S matrix is defined the S matrix is defined that when we are feeding it here all the other ports are matched, when we feed here all the other ports are matched when we feed here all the other ports are matched.

So, here the situation is slightly different, we are giving a input here also and we are giving input here also. So, if you apply the S matrix blindly is not going to do the job you have to now apply the logic. So, let us say if I am giving a 1 watt power here and a 1 watt power here, you have to ensure that these 2 powers are exactly at the same phase. So, assuming that again if they are in the same phase so this voltage V2 let us say that V 2 has a phase angle V 2 angle 0 this is also V 2 angle 0.

So, if the 2 voltages are same and they have a same phase, so what will be the current flowing through this there will be a no current flowing through that. So, what is the coupled power there is a no coupled power, so where the power will go from here. So, power from here nothing is reflected nothing is going over here and these are relatively lossless line. So, this 1 watt will come over here and this 1 watt will also come over here, so we will get affectively 2 watt of power. So in fact you can actually feed let us say a 10 watt of power 10 watt of power you will get 20 watt of power. So, this is the 1 of the effective way of designing a power combiner circuit.

Now, over here you have to ensure that the 2 inputs here are at the same phase, just look at the extreme case here suppose the input of this one watt power is let us say a 1 angle 0, but the input at this thing is 1 angle 180 degree. So, if I will look at the voltage here so this will be V 2 and this will be minus V 2 even though the power is same V 2 square and minus V 2 square ok. So, power remains same but, because now there is a phase difference if this is V 2 and this is minus V 2, you can now see that there will be a large current flowing through the resistor and there will be a huge power loss and when there is a power loss you will not get 1 watt input 1 watt input and 2 watt output over here it will not really happen ok. So, as a power combiner you have to ensure that the 2 inputs are exactly at the same phase ok.

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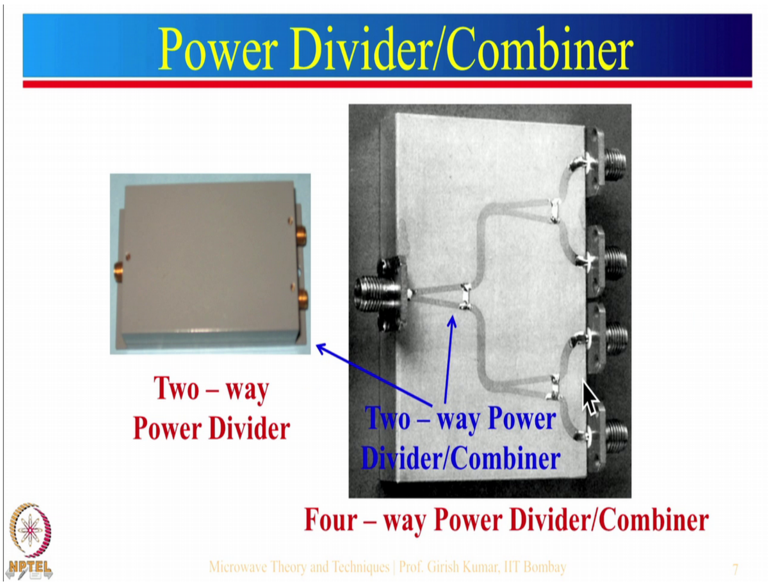

So, I am just going to show you one practical example here and that is you can see here, input is given over here this is actually a 2 way power divider there is a isolation resistance is connected over here and then this one over here is divided equally further and there is a isolation resistance here there is a isolation resistance over here and this is now you can say connected to the 4 connectors.

So, from here one way to 2 way and then 2 way to 4 way and these are the SMA connectors as I am shown you earlier you can use SMA connector or n type of a connector. Now eventually you are not going to buy a component which has a open PCB like this, this has to be put inside a box. So, just you give you so basically what this box shows only this particular portion over here, of course this is designed for slightly different frequency.

So, you can see that there is a 1 input here and there are 2 output ports over here. So, this is the simple way you can do, but I also just want to mention now see what will be the cost of this particular thing, it is going to be very very small. And yet if you want to import these 2 way power dividers or 4 way power dividers it may cause you anywhere between 50 dollars to 100 dollar.

So, in the next lecture we will actually see how to practically realize these power dividers combiners. We will also look at unequal power divider and other cases till then.

Thank you. And we will see you again next time. Bye.