Microwave Theory and Techniques
Prof. Girish Kumar
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Module - 03
Lecture - 15
S - Parameters

Hello everyone. So, welcome to today's lecture on S-parameters. In fact, it is a continuation of the previous lecture.

In the previous lecture we had seen a b c d parameters and what are its properties and we had seen that a is equal to d for symmetrical network and ad minus bc is equal to 1 for symmetrical network. And then we actually calculated a b c d parameters for a few cases and these were series z impedance or shunt y admittance. We also looked into the transmission line what are the a b c d parameters of that and then we looked at the cascaded network. And we had started our discussion on S-parameters so let us continue from where we left in the last lecture.

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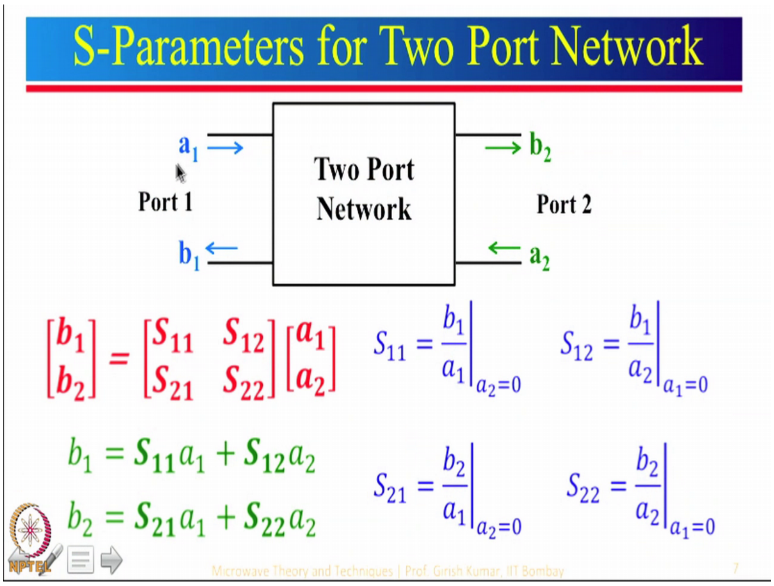

So, we actually define S-parameters in terms of incoming waves which could be a 1 and a 2 and outgoing waves b 1 and b 2. So, S-parameters are defined by this particular expression here or this was expanded over here. And then we took the cases where a 2 is equal to 0 or a 1 is equal to 0 and we had defined S 11 as b 1 by a 1. Now, b 1 by a 1 is nothing but reflected wave divided by incident wave. So, hence S 11 is reflection coefficient at port 1 and this is now reflection coefficient at port 2.

And in this particular case S 21 is you can say b 2 is the outgoing wave in this case we would say it is a transmitted wave divided by incident wave. And S 12 will be transmitted wave at the port 1 when the input is given at port 2.

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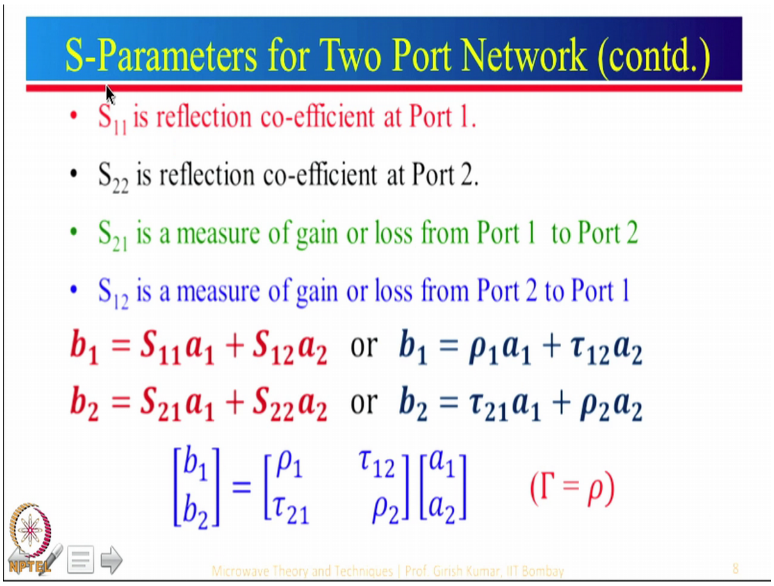

So, let us go through these things one by one again. S 11 is reflection coefficient at port 1, S 22 is reflection coefficient at port 2 and S 21 is a measure of gain or loss from port 1 to port 2. So that means, if we give a input at port 1 what is the output at port 2.

So, if it is an amplifier it will be again if it is some other circuit, it can be attenuator, it can be a power divider, coupler, filter and so on that will be then loss. So, this is now, from port 1 to port 2. The opposite of this is S 12 which is a measure of gain or loss again from now, port 2 to port 1 so that means. Now, the input is given at port 2 and we are looking at the output at port 1.

So, now, these S-parameters can be written in a slightly different way in a sense that S 11 is reflection coefficient. And I just want to mention here earlier we had used the symbol gamma for reflection coefficient, but in some books they use the expression for rho as a reflection coefficient. So, you just please remember rho is same as gamma depending upon which book you are going to read or which book you read. So, we can write here S 11 as reflection coefficient at port 1.

What is S 12? So, that is a transmission coefficient that 1 2. What is S 21? Transmission coefficient, 2 1 that means, input is given at port 1 output is taken from port 2 and this is you can say reflection coefficient at port 2. So, the same S-parameter matrix now can be written in the form of reflection coefficient at ports 1 and port 2 and transmission coefficients at port 1 and 2. So, it is just the another way of representation of S-parameters.

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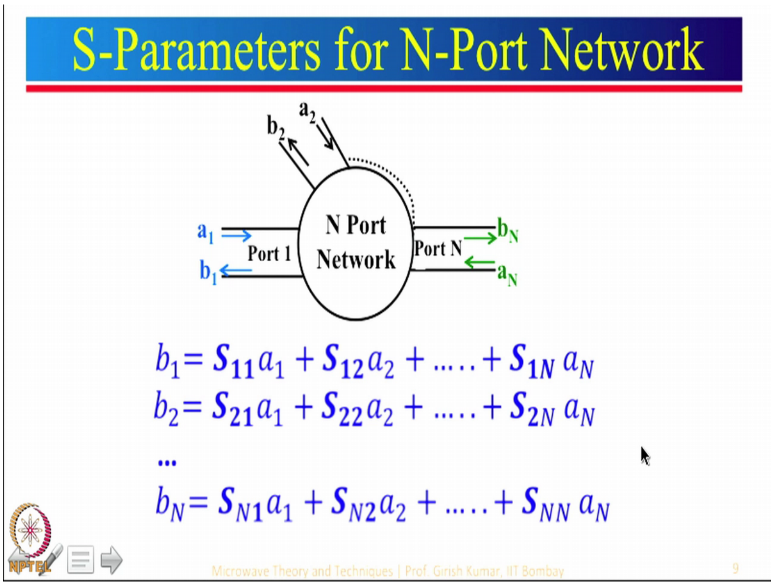

Now, let just look at S-parameters for N port network. Now, just to mention again a b c d parameters were defined only for two ports, port 1 and port 2. But S-parameters can be defined for N port so we can see here port 1 incoming wave outgoing wave then port 2 incoming wave outgoing wave.

Similarly you can follow so port N so incoming wave and outgoing wave. So, in this particular case now we can write b 1 as S 11 a 1, S 12 a 2 and all the way S 1N a N. Now, similarly b 2 b N, b N will be now, S N1 a 1 S N2 a 2 a 2 all the way S NN a N. In fact, just to tell you we will be looking at several examples where there may be a 5 ports or there may be a 4 ports or there may be 3 port in fact, we had designed at one time a power divider where input was only one port and there were 38 output ports. So, for that S matrix will have a size of 39 by 39.

Let us first look at what are the S matrix properties.

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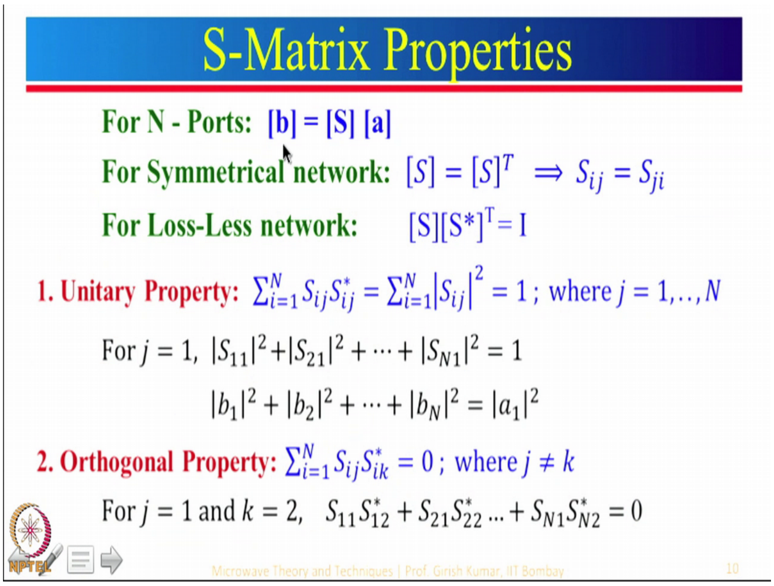

So, for N port we can now write in a very simple way instead of writing b 1, b 2 to b N and S 11, S 12, S 1N we can write in a very simple form which is b equal to S a.

Now, here just like in terms of a b c d parameters we had seen a is equal to d and ad minus bc is equal to 1 here we have a slightly different ways of expressing the thing. So, for symmetrical network if S is equal to S transpose that really means S ij is equal to S ji, where i and j can be anywhere from 1 to 3 up to N. So, if this property is satisfied that means, it is a symmetrical network.

And for lossless network just imagine first a lossless network will be a network where there will be no losses within the network. So, actually speaking it leads to two different properties we will just see one by one. But how we define lossless network? Basically S and that is S conjugate matrix and transpose of that that product of these two matrices will give a unique matrix over here which is given symbol as I. So, from here we can actually get two different properties one is known as a unitary property, another one is known as orthogonal property.

Let me explain one by one what is unitary property. Unitary property is that we will start with an example let us say S 11 square plus S 21 square plus S N1 square is equal to 1 so, that is a unitary property. Now, we will define this thing little later on but first let us just do this time we will use a bottom up approach.

So, let see if I give a input at only port 1 so that will be a 1 square. So, power is equal to a 1 square. So, if we give a power only at port 1. Then what will happen? If there is a no loss within the network then this power will get distributed to all the other ports. So, the power output at port 1 will be b 1 square at port 2 b 2 square and b N square. So, we can say that the input power at port 1 is equal to sum of all the power outputs at ports 1 to N so that means, there are no losses within the network.

So, now, from here divide everything by a 1 square. So, this will be b 1 by a 1 square which is equal to S 11. This will be now b 2 divided by a 1 square that is S 21. This one here will be b N divided by a 1 that will be S N1 square and a 1 square divided by a 1 square will be 1. So, this is what we get from here to here. Now, let us write this particular thing in this particular form or you can look into later on this here. So, what we have here this case was taken for j equal to 1. So, where j can be from 1 to n let just look at this term here.

What this term says? Summation of i equal to 1 to n S ij magnitude square is equal to 1. So, over here if we take j equal to 1 which is what is shown over here. So, what will happen now? i will vary from 1 to N, so 1 1 which is here, then i will be 2, 2 1, then 3 1, and then N 1. So, this particular thing is actually same as this which is same as this over here.

And this whole thing can also be written in the form of S ij multiplied by S ij conjugate. We know that complex number multiplied by its conjugate will give the magnitude square. So, that is the unitary property which basically is nothing, but you can say that summation of all the S-parameters for given j equal to 1 will be equal to 1 which is unit.

What is the orthogonal property? Orthogonal property is something similar to which is a basically 0, what it really means is that if this is a lossless network. So, if a network is lossless what will be the power dissipated in the network, it will be equal to 0. So, that is what this term comes from and here what we have S ij you can say i and i common, so i will vary from 1 to N.

And this is j, this is k, and in this case condition is j should not be equal to k, ok. So, that is very important if j is equal to k then actually speaking we are getting this condition, ok. So, j is not equal to k. If you take k equal to j this becomes unitary property. So, over here orthogonal property is nothing but since a network is losses within the network will be equal to 0. So, just to give you an example here, we have taken here let say j equal to 1 and k equal to 2.

And if we take j equal to 1 and k equal to 2 expand this whole thing so we can say i is going to vary from 1 to N. So, 1, j is 1 so i is 1, k is 2. In the second case now, i is equal to 2, j and k remain 1 and 2 and in this particular case now, i is equal to N and j and k will be 1 and 2. So, these are the two properties which are useful and even this one is useful to know whether the network is symmetrical or not.

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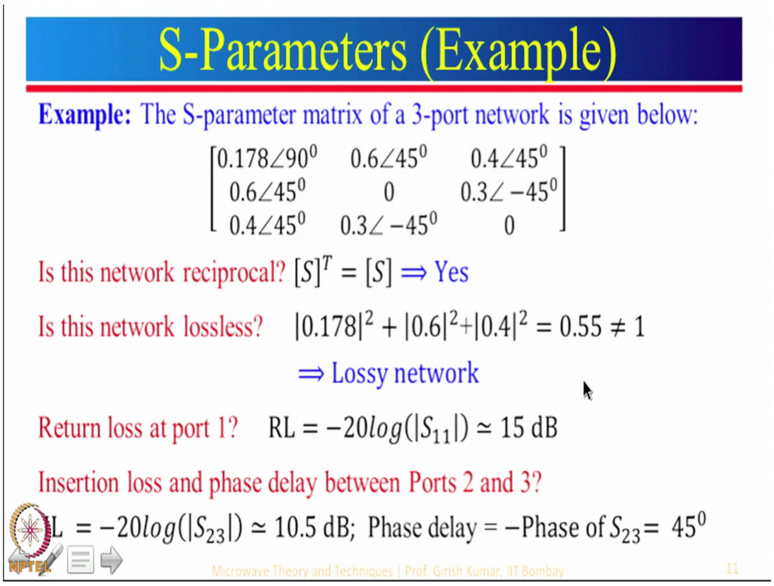

Now, let just take an example. So, this is the example where it is given that S-parameter matrix of a 3 port network is given below. Let us just look into it carefully. So, what is this here? This is S 11, this is S 12, this is S 13. So, this is S 21, S 22, S 23, S 31, S 32, S 33. So, there are now, several questions, the first question is this network reciprocal.

Now to check whether the network is reciprocal or not; we have to check whether S transpose is equal to S or not. So, let us see whether that is valid or not. So, you can see that S 12 is same as S 21, S 13 is same as S 31 and this term here S 23 is same as S 32 so which means that this property is satisfied. So, we can say that this network is reciprocal.

Next question; is this network lossless? Well, in order to find out whether the network is lossless or not we can use the unitary condition also or we can use the orthogonal property also. So, let just apply only unitary condition. So, what unitary condition says, S 11 square plus S 12 square plus S 13 square should be equal to 1. Let us first check whether it is 1 or not. So, 0.178 square plus 0.6 square plus 0.4 square you add up all these things it comes out to be 0.55, ok. So that means this particular network is not lossless because it is not equal to 1 hence this is a lossy network.

Now, we would like to know what is the return loss at port 1 ok. Now, I just want to mention here because the term return losses mentioned, see there are two terms which are used in the literature, sometimes they say return loss if they say return loss that is minus 20 log S 11. First of all why 20, why not 10? Because S 11 is not a power, see had this been S 11 square then it will be 10, ok. But for power we always take 10 log for a square root of power unit we take 20 log.

So, this minus sign is coming over here because we are calculating return loss. Now, many a times they also say reflection coefficient. Now, reflection coefficient you do not put minus reflection coefficient is only this much here, ok. So, please remember the difference between reflection coefficient and return loss in case of return loss you add a minus. So, now, minus 20 log S 11 we know what is S 11 0.178 you take the log of that that comes out to be approximately 15 dB. Now, the question is now, insertion loss and phase delay between ports 2 and 3.

Again I want to mention if we say insertion loss insertion, loss is minus 20 log of 23 why. Because we want to find out between ports 2 and 3, if it was just the transmission coefficient then this minus sign will not come. So, please read the problem carefully whether they are asking for transmission coefficient or whether they are asking for insertion loss insertion loss will have minus. So, now, S 23 we know what is the value of S 23 which is 0.3 you take the log of that that comes out to be 10.5 dB.

Now, again here what is phase delay? So, when we talk about again phase delay, so delay has already built-in because we are talking from point a to b what is the delay. So, here this is S 23 parameter, but when we talk about the delay will be negative of that so which is going to be plus 45 degree.

So, please take some different examples and do some own calculation. You can try also applying orthogonal property to this also and you will see that orthogonal property is not valid for this particular case. You can just see there multiply this with this plus this plus this plus this plus this term will be 0, this term will be 0, but this is a nonzero term so hence that product is not equal to 0, ok. So, please apply these things carefully you can solve the problems in a very simple manner.

So, now, we want to actually see how a b c d parameters can be related with S-parameters. So, for that we are going to define these waves in terms of voltages and currents.

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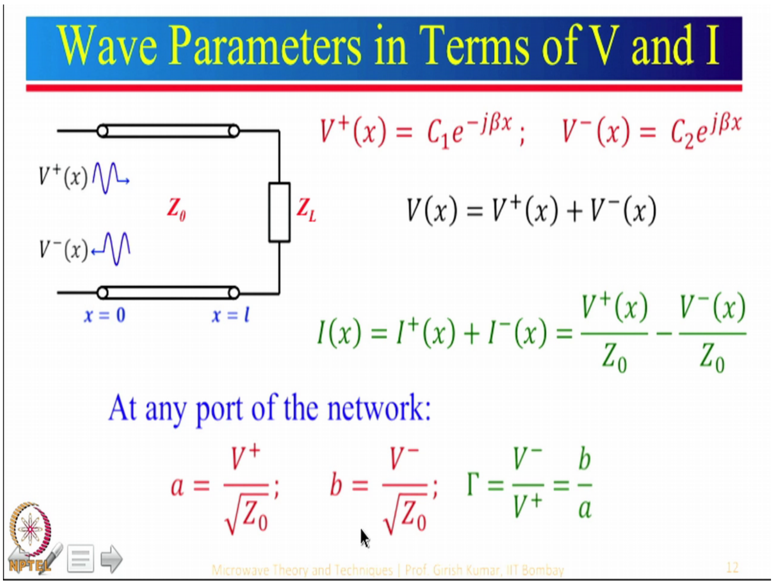

So, let us just see first a simple transmission line which has been terminated with a load impedance seidel. This particular thing we have done earlier also, when we were talking about transmission line. So, here we can say that this is the incident wave which is shown by the sign plus and this is the reflected wave which is shown by a sign minus, ok. So, you can say incident wave reflected wave, ok. So, now, this particular term is defined in this particular form here so that voltage is some constant and e to the power minus j beta x term comes because x is equal to 0 over here.

Now, you can see that further reflected wave from here it will be the opposite. So, this is a plus sign here. Now, I just want to mention. So, plus sign does not mean over here that it is going to be positive, no, x is negative when you are going from this particular direction. And so what will be the total voltage total voltage at this particular point will be summation of these two voltages which is incident voltage and reflected voltage. So, that will be the summation of the voltage. So, which is something similar to we are talked about V 1 for port 1.

Now, similarly now, the current we have not shown it over here, but current in the same fashion is defined. So, current will be again I plus and I minus and the currents are related to the voltages. So, V plus divided by Z 0 and divided by Z 0. So, now, at any port of the network, so we have just shown here the generalized form a b, but suppose if it is a port 1 then this will be a 1, this will be V 1, if it is port 2 this will be a 2 and this will be V 2 and so on.

So, here a is defined in terms of V plus divided by square root Z 0 and b is defined in terms of V minus divided by square root Z 0. So, now, from here you can actually see that if we take the ratio of the two, so we know that reflection coefficient which is let us say S 11 or S 22 S 33 depending upon which port we are looking at. So, reflection coefficient is nothing but reflected divided by incident. So, which is b by a we had seen earlier but now, let us just look at over here also if I take the ratio of b divided by a square root Z 0 will get cancelled. So, what we are left with this? V minus divided by V plus.

So, now, the difference is simple reflection coefficient let say port 1 will be gamma 1 which will be b 1 by a 1 or gamma 2. Now, from here you can also see if we try to define these things you come over here and look at the terms here. So, we have seen that voltage. So, V plus will be a times square root set 0. What will be V minus? b times square root of 0.

Now, what will be I? You can say that we substitute the value over here. So, we can say that now, V plus is nothing but square root Z 0 multiplied by a divided by Z 0. So, the term will be nothing, but now, this divided by square root Z 0 and that will have a negative term. So, this will be nothing, but now, in the form of a and b in between we will have a negative term. Whereas, over here it will be a and b term but in between the term will be positive term. So, now, I am going to show you the relation which is ABCD to S-parameters.

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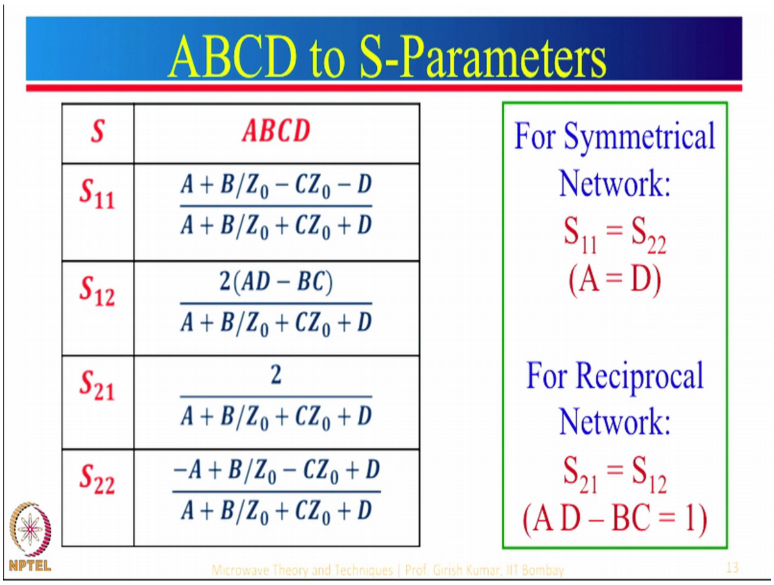

Now, the detail derivation you can find in any one of the books which I had mentioned for example, you can see Pozar book or you can see Collin's book and other book. So, I am not actually showing you the detailed derivation of that but what we are interested now, is ABCD to S-parameter conversion. Why we are looking into that? Because we had seen earlier how a larger network can be divided into smaller network for which we know ABCD matrix individually. By multiplying ABCD matrix of individual network we can find overall ABCD matrix.

But our objective is not ABCD when we are talking about microwave. At microwave we are talk about incident wave reflected wave. So, we are more interested in finding S-parameter. So, you can say that all that ABCD we discuss about that was an intermediate step to reach the final goal of finding S-parameters. So, here you can say S 11 is given by this particular expression but I will try to make things little simpler for you. So, just recall now, what was the unit of A? It was a dimensionless. What was the unit of D? It was dimensionless. What was the unit of b? It had a unit of impedance. So, you divide that by Z 0 so that become dimensionless.

What was that unit of C? It had (Refer Time: 21:57). You multiply with this impedance, now this become dimensional so in fact, it is sometimes easier to remember instead of a writing capital ABCD if you write in terms of let us say normalized value. So, if it all these are normalized then this will become let us say normalized value if you represent as small abcd, then this will become small a plus b plus c plus d. So, you do not have to worry about remembering these Z 0 or you just think about these are all normalized thing.

Now, if you look for all these S-parameters that denominator is exactly same which is nothing, but small a plus b plus c plus d. Now, let us look at the numerator. So, first we will look at the numerator for S 11 and S 22. If you see here this is like if we again think normalized A plus B minus C minus D.

Now, you look over here, here A has become minus A, minus D has become plus D, otherwise these two terms are exactly same. So, we can now, say if A is equal to D, if A is equal to D and we have seen that is the condition for symmetrical network. And if A is equal to D this term will get cancel this term will get cancel so that means, S 11 is equal to S 22. So, we can define either way you can say for symmetrical network you can say S 11 is equal to S 22 or A is equal to D you can drive either from here or from there.

Now, let just look at S 12 and S 21. See the numerator this is two times AD minus BC and this is only 2. And we have seen that for reciprocal network AD minus BC is equal to 1. So, if I put AD minus BC equal to 1. So, this will be 2, this will be 2. So, you can say that S 12 will be equal to S 21. So, if the network is reciprocal then 21 will be equal to S 12 or we can say AD minus BC is equal to 1, ok. Now, of course, in the books they have also given how to convert from S-parameter to ABCD but actually speaking most of the time that is not required but if at all you require for some other purpose you can always see the textbooks which I have mentioned to you, ok.

So, now, let just take an example how to find S-parameters.

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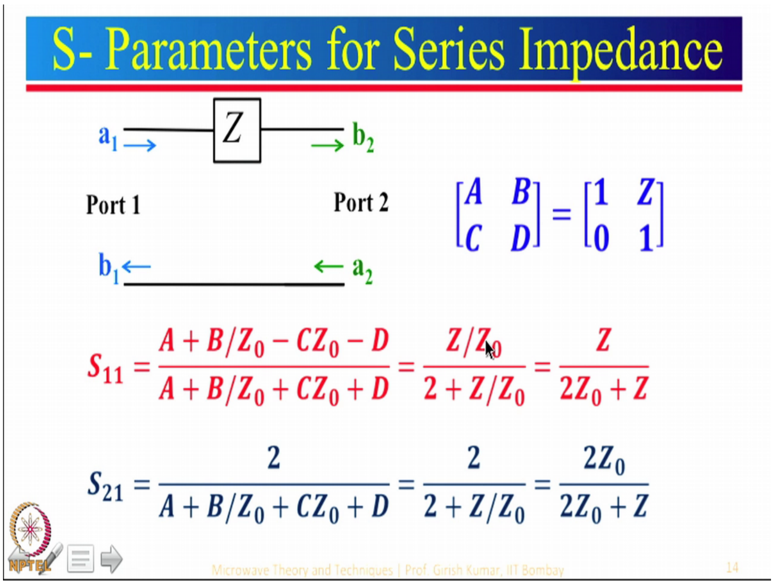

So, we are going to actually solve this problem in two different approaches. So, first approaches we are going to actually find out first ABCD parameter and then we use the formulas to get S-parameter. So, let us see what is the simple problem we have taken.

So, this is a series impedance. So, we had seen that a 1 and a 2 incoming waves, b 1 and b 2 are the outgoing waves ok. And for this particular network we had actually done the derivation for ABCD parameter for a series impedance it was 1, Z, 0, 1.

Now, we are going to use ABCD to S-parameter conversion formula. So, this is the ABCD to S conversion formula. So, let us see what is A. So, A is 1, D is 1, so 1 minus 1, 0 so that term will not be there and what about here? B by Z 0. What is b? Z. So, Z by Z 0 will come here. And what is C? C is equal to 0. So, there is a nothing else there. So, now, in the denominator A is equal to D, and that is equal to 1. So, 1 plus 1 will be equal to 2 and this term here B by Z 0 plus C Z 0. So, B is nothing, but Z so Z by Z 0, C is equal to 0. So, S 11 is given by this particular expression and we can multiply numerator and denominator by the term Z 0. So, what do we get? Z divided by 2 Z 0 plus Z.

Now, we will try to find out S 21 using the same concept of the ABCD parameters to S-parameter transformation. So, the formula is given by this here. So, this two remains two and the denominator will be same as what we had over here which is 2 plus Z by Z 0. If you know multiplied by Z 0 that will be two Z 0 divided by 2 Z 0 plus Z. So, these are the S 11 and S 21.

Since the network is symmetrical as well as reciprocal. So, we can say that S 11 will be equal to S 22 and S 21 will be equal to S 12 which will be same as this here. Now, this is problem which we solved using ABCD parameter. In fact, this was very simple in this simple problem can be also directly solved also. So, let us look at the direct solution which is the another approach. So, let us say what is reflection coefficient. Is nothing, but equal to S 11. So, we can find reflection coefficient equal to S 11 by this particular expression.

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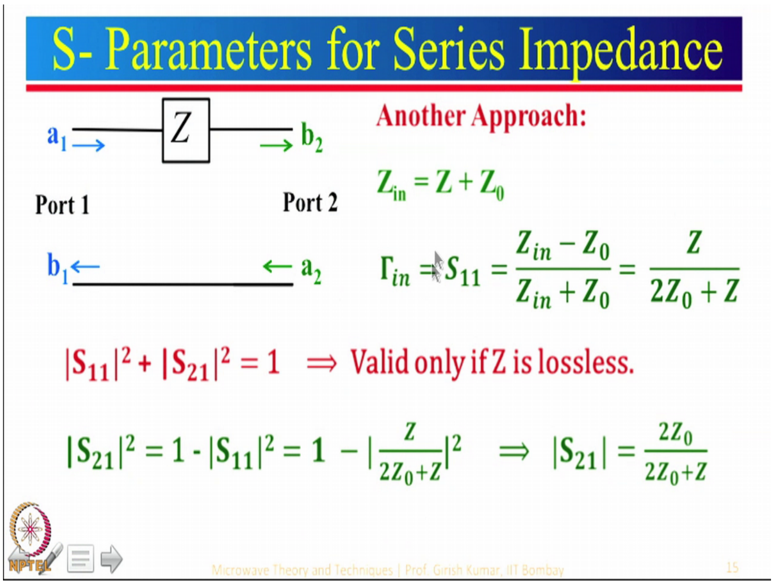

So, what we need to know? We need to find what is Z input. So, Z input looking from here that is this impedance Z. And remember S-parameters are defined when this port is terminated into the characteristic impedance which is Z 0. So, this is 0, this is Z. So, looking from here Z input will be Z plus Z 0.

So, now, we substitute this value over here. So, Z in is Z plus Z 0. So, if you look into this term here Z divided by 2 Z 0 plus Z you will see that this term is exactly same as we had defined earlier.

Now, be careful when you are going to use this particular expression. See this is S 11 square plus S 21 square equal to one, but this is valid only if Z is lossless. Please do not use this particular concept if Z is lossy that means, if Z is a resistor you cannot use this particular thing because for lossy network this is not at all valid, ok please remember that. However, you can apply this for inductor, capacitor or a transmission line which is lossless you can apply this particular condition. So, let us see how we can solve this assuming that Z is lossless. So, S 21 square is equal to 1 minus S 11 magnitude square. So, S 11 we have already done the calculation you put it over here and now, if you solve this you will get this expression which is same as before.

Now, I just want to again warn you one more time this you will get the derivation only when Z is not resistive, ok. So, please apply these things little bit more carefully and that way you can solve the problem. Of course, we can solve it directly if it was simple series element but now, imagine if it had a another thing here another here, another there and multiple elements are there then solving using this particular approach will become very complicated.

So, for all these cases it is better that you use this particular approach where you find ABCD matrix of the cascaded network. So, let there be n number of terms over here multiply all of those find overall ABCD matrix and then yes just use this particular formula to find out overall S-parameter.

So, we will conclude today's lecture at this particular point. So, today we talked about S-parameters and what are its various parameters we looked at how to convert from ABCD parameters to S parameter.

Now, in the next lecture we are going to take several practical examples of how to realize different circuits. For example, power dividers, couplers, filters and so on and where we are going to apply the concept of ABCD matrices of different smaller components multiply them and get the overall S matrix. So, till then steady hard and we will see you in the next lecture; bye.