Microwave Theory and Techniques
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Module - 03
Lecture – 14
ABCD – Parameters

Hello, in the last few lectures we have been talking about transmission line. We saw different types of transmission line like coaxial line, microstrip line, strip line and waveguides. After that we looked into transmission line loaded with a load impedance Z L, then we took different cases of Z L for example, Z L equal to 0, Z L equal to infinity, Z L equal to Z 0, and then we took 2 special cases of length equal to lambda by 4 and lambda by 2. And we saw that when the length is lambda by 4 it acts as a quarter wave transformer.

After that we discuss about smith chart, and we actually saw how easily we can plot complex impedance values and also how impedance matching can be done using smith chart. So, we looked into two different techniques one was lumped element technique and the second one was single step matching. Today we are going to talk about ABCD and S parameters.

So, now we are not going to learn what you studied in your very low class which is ABCD, but we are actually going to look at the ABCD parameters which are defined in terms of voltages and currents. And then we will talk about S parameters which are defined in terms of wave parameters because at microwave we actually are more interested in the waves then just in voltages and currents. Then you might wander then why we are discussing about ABCD parameters which are mainly concerned with voltages and currents. So, I am going to show you how ABCD parameters are very convenient to solve a complex network, and then we will talk about how ABCD parameters can be converted to S parameters.

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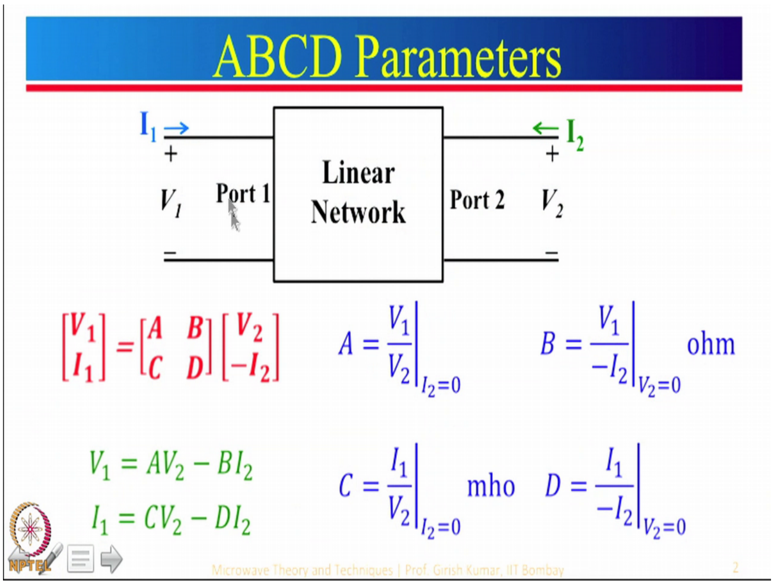

So, let us first look at what is the definition of the ABCD parameter. So, here we have a two port network, so port 1, port 2 and this is a linear network and let us see that the input voltage here at port 1 is V 1 and at port 2 voltage is V 2.

Current is coming into the port 1, and current is also coming into the port 2. In fact, this has been a general definition you might have studied Z parameters or Y parameters. In case of Z parameters just to refresh your memory. What we have generally for Z parameters? V 1 and V 2 are on one side, I 1 and I 2 are on the other side and in between there is a Z matrix Z 11, Z 12, Z 21, Z 22. In terms of Y matrix or Y parameters we have I 1, I 2 on the left side and V 1, V 2 or on the right side and then there is a Y matrix.

In fact, there are hybrid parameters are also there which actually consist of voltages and currents and you might have studied those hybrid parameters for transistors. Here we are going to look at ABCD parameters and the way ABCD parameters are defined which actually becomes easier later on as we will see to solve a complex network. So, let us see what ABCD parameters are related to.

So, voltage at port 1 and current at port 1, relate to voltage at port 2 and current at port 2. Now, you might see there is a minus sign over here this minus sign is coming because we have taken this current as in coming at port 2. If we take this current I 2 as outgoing then this will become plus. In fact, many a times you can just think about it that if this is I 2 which is coming in just think about this is current let us say I 2 dash and which is leaving here, then I 2 dash will be nothing but minus I 2 and that will be here then plus I 2 dash, ok. So, it just says matter of notation.

So, how do we define ABCD parameter? Let us open the matrix first. So, V 1 is nothing but a multiplied by V 2 then minus B times I 2. And what is I 1? I 1 is nothing but C multiplied by V 2 minus D times I 2. So, these are the ABCD parameters. Now, by using these 2 equations we can find out the ABCD parameter. So, let us see what is A? We can say from here a will be equal to V 1 by V 2 provided I 2 is equal to 0. So, if we put a boundary condition is I 2 equal to 0 which is what has been put over here, then what will be A? V 1 divided by V 2. So, you can actually think about A is nothing but voltage ratio provided I 2 is equal to 0. And in what condition I 2 will be equal to 0? If this particular port is open circuit then I 2 will be equal to 0.

Now, let us see how we define B. B is nothing but V 1 divided by minus I 2 provided V 2 is equal to 0. So, if we put V 2 equal to 0, so we can see here if V 2 is put 0 here then V 1 divided by minus I 2 will be equal to B. Now, as you can see this is voltage divided by current. So, the unit of this will be ohm or you can say this will represent the impedance value.

Now, let us see what is C. So, to define C you can see from this particular expression, if we put I 2 equal to 0. So, what I 2 equal to 0 would mean? This term will become 0. So, we can say from here C 1 is nothing but I 1 divided by V 2 provided I 2 equal to 0. So, I 2 be equal to 0 means open circuit, and this unit will be now I divided by V. So, that will be equal to mho. So, the unit of the C is mho. What is D? So, again from here we can find the value of D. If we make V 2 equal to 0, if V 2 is 0 then this term will become 0. So, then I 1 divided minus I 2 will be equal to D. So, you can see that this is the expression provided V 2 is equal to 0 and what V 2 equal to 0 would mean that you actually short circuit the port 2. So, this is how we can actually define ABCD parameters.

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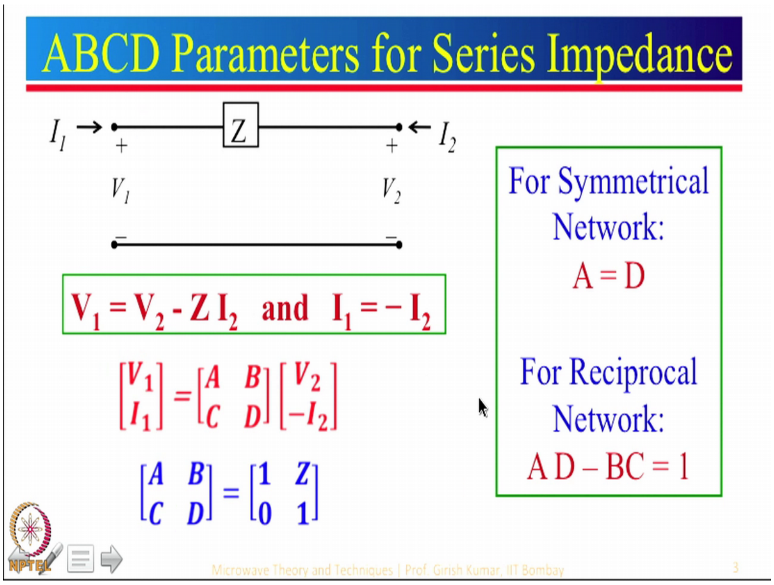

Now, let us just take some example and see how we can find out ABCD parameter. To start with will take a very simple problem and then we will built up on these small small little little things to analyze a more complicated configuration. So, ABCD parameters for series impedance, so you can see here those are basically port 1 you can say this is port 2 and there is a only single element which is a series impedance.

So, now for this particular case we can write the voltage equation and current equation and you have to remember basically that we have to write V 1 and I 1 on one side and V 2 and I 2 on the other side. So, with that objective in mind let us say what is V 1. So, V 1 is nothing but we can say this voltage plus voltage drop over here. So, V 1 will be equal to V 2 plus you can say I 1 Z or I 1 Z is equal to minus I 2 Z. So, we can say that V 1 is nothing but V 2 minus Z times I 2. So, that will be this voltage equal to this voltage drop plus this voltage over here. And what is I 1? Again why we are writing in this particular form because, we have to write in this particular fashion.

So, what is I 1 equal to? I 1 is nothing but equal to minus I 2. So, from here you compare this particular thing with this particular equation over here. So, we can say that V 1 is A V 2. So, what is coefficient of V 2? 1. So, that is 1 here. What is now V 1 is equal to B times minus I 2 and what will be B here then? Z. So, now, let us see I 1, I 1 is nothing but C times V 2 well there is a nothing corresponding to V 2 that means C is equal to 0.

And then from here we can say I 1 is equal to then D times minus I 2 the term. So, from here we can say compare this. So, D will be equal to nothing but 1. So, we can say that ABCD matrix for this particular case is nothing but 1 Z 0 1. Please try to remember this because we are going to use this later on, few properties of ABCD parameter. So, one of the property is that if the network is symmetrical, then A will be equal to D. As you can see if you look on this side or you look from this side the network is nothing but a symmetrical network. So, A should be equal to D, you can see that A and D are 1 and 1. So, they are equal.

Now, for reciprocal network it is AD minus BC is equal to 1. Actually this term really corresponds to the determinant of matrix ABCD. So, what will be the determinant? AD minus BC. So, the determinant of the matrix if that is equal to one this is a reciprocal network. What reciprocal network means? That if I give a input on this side whatever the output I get if I get this same input on this side then I should get the same output here. So in fact, all these networks will be reciprocal which have let us say resistor, inductor, capacitor, transmission line other thing. So, of course, if it consists of active network then it will not be reciprocal.

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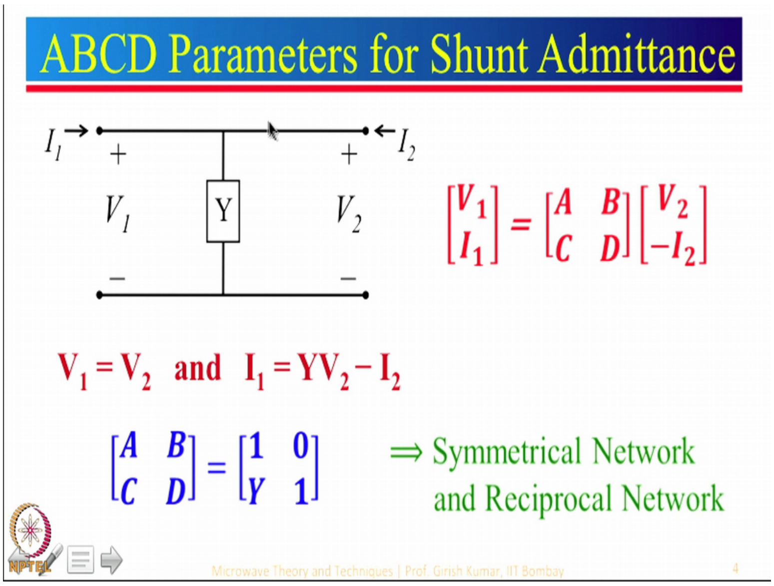

So, now let us take another example. So, this time will take an example of ABCD parameters for Shunt Admittance. So, we have here again as before port 1 here, port 2 here, now instead of having a series impedance in the previous case, now we have a shunt admittance.

So, now we want to write ABCD parameters of this I have just written it again. This is how we have define ABCD parameter. So, again we want V 1 I 1 on the left side. So, what is V 1? This voltage is same as this voltage which is equal to V 2. So, V 1 is equal to V 2. And, what about the current I 1? So, we can say that just general you think about, this current and this current these 2 currents are going into this. So, we can say that I 1 plus I 2 will be nothing but this particular current and that current will be nothing but V 2 times Y and then we can write I 1 plus I 2 was written there I 2 goes to the other side.

So, now this is similar to the form which we wanted, V 1 I 1 on the left side, V 2 I 2 on the right hand side. So, again now comparison let us say now ABCD. So, you look from here. So, V 1 is nothing but A V 2. So, the coefficient of V 2 will be 1, so it is write here. Now, V 1 is B times I 2 when there is a no term of I 2 that means, B is equal to 0. For now C let us say what is I 1 is C times V 2, so C will be Y and then I 1 is this expression here from here then I 1 is C V 2 minus D I 2. So, we can say from here D is equal to 1. So, this is ABCD matrix for shunt admittance which is nothing but 1 0 Y 1. So, again you can say that this is symmetrical network why because A is equal to D; it is also reciprocal network because AD minus BC will be equal to 1. So, 1 into 1 is 1 minus 0. So, AD minus BC is equal to 1.

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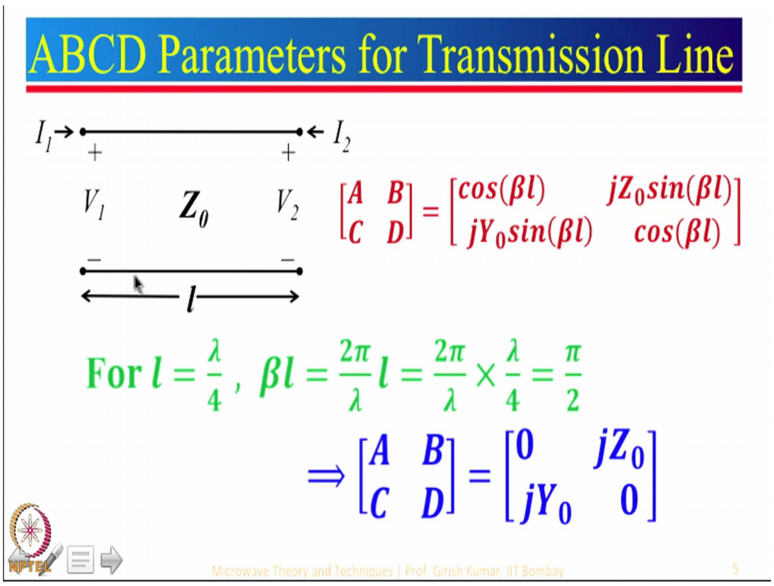

Let us just take now another case of a transmission line. You might have studied about transmission line and you know that let us say if we want to define a voltage V 1 then voltage V 1 is nothing but equal to V 2 cos hyperbolic gamma l plus Z 0 sin hyperbolic gamma l, similarly one can write expression for I 1. Now, in this particular case here we are assuming that this line is loss less or we can say most of the time line losses will be very very small. So, that cos hyperbolic gamma l, what is gamma? Gamma is alpha plus j beta. So, alpha will be negligible or alpha is equal to 0 for ideal transmission line. So, cos hyperbolic gamma l will become now cos beta l, and sin hyperbolic gamma l will become j sin beta l. So, this is what is the ABCD matrix for a transmission line.

We will take one of the special case as we will see later on when we take on more circuit examples that this will be a 1 of the special cases which will come several time. So, let us look at that right now, when length is equal to lambda by 4. And if we substitute l equal to lambda by 4 what will happen to beta l? So, beta is 2 pi by lambda so that is 2 pi by lambda l is equal to lambda by 4 we substitute that. So, beta l becomes pi by 2.

So, cos pi by 2 is equal to 0, sin pi by 2 is equal to 1, sin pi by 2 will be 1, so we will be left with j Y 0 and cos pi by 2 will be equal to 0. So, this will be the ABCD matrix for transmission line of length l equal to lambda by 4. I just want to mention of you additional thing here many books instead of writing beta they write k. So, beta is equal to k which is equal to 2 pi by lambda.

Again one more thing I want to mention here. So, this lambda will be equal to lambda 0 in free space, but otherwise say for coaxial line lambda will be equal to lambda 0, by square root epsilon r for microstrip line it will be lambda equal to lambda 0 divided by square root epsilon e. And we have given you the formulas how to calculate epsilon e for microstrip line. So, please use that when you are designing something for microstrip line.

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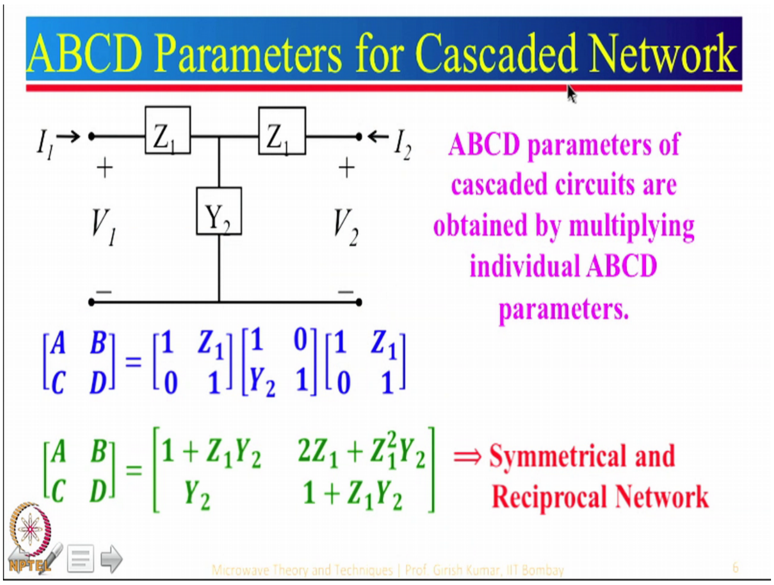

Now, let us just take a another case now here it is ABCD parameters for cascaded network. So, what we have here? We have two series impedances Z 1 and Z 1 here and we have a 1 shunt admittance which is Y 2. Now, this particular problem can be solved if we want to write voltage equation and current equation you can solve that. So, you can actually write here let us say here node equation here so which will be let us say some node 3. So, we can say that V 3 times Y 2 V plus V 3 minus V 1 divided by Z 1 plus V 3 minus V 2 divided by Z 1, and then you can solve the equation or you can write a loop equation here loop equation here.

However, we are not going to use that technique that can be done there is no problem at all, but since we have already studied ABCD parameters for series elements we have also studied ABCD parameters for shunt element. So, now, we are going to use the concept of ABCD parameters of cascaded circuit which can be obtained by multiplying individual ABCD parameters. What it really implies?

First let us just see, we have 3 different circuit element. So, what we do? If you recall Britishers used to say divide and rule. Well, I have modified that particular statement I say divide and solve. So, you divide this particular network into 3 different networks. So, this will be from here to here, one network which consist of only series impedance, then from here to here it will be the shunt admittance network and then from here to here we will have a series impedance network.

So, what is this cascaded business? So, let just see from here just recall now we had taken I 1 as here and just imagine that the current was entering here. Now, the current which was entering here which was minus, but that minus current can be thing about positive current over here. And then the current which is leaving here which was actually minus which will act as a entering here. So, if you look at just this particular network. So, V 1 I 1 for this let us say I did mention a V 3 node here. So, that will be V 3 node over here.

So, we can actually say that ABCD parameter of this, whatever is the current going out from here will become input current and input voltages over here so that means, this ABCD matrix can be multiplied with this. And then again whatever is the current coming in the same current is going just the directions are different. So, you have to take care of that part which is inherent of ABCD matrix. So, what we can do actually? That to find out the overall ABCD matrix what we simply need to do it is multiply ABCD matrices of these 3 separate elements.

So, let us see for the series impedance, what we had seen? ABCD parameters are 1, Z 1, 0, 1; for shunt we had 1, 0, Y 2, 1; for series this 1, Z 1, 0, 1. So, now, all you have to do it is multiply these 3 matrices of course, you do one by one. So, first you multiply this matrix with this matrix here and then after that this resultant matrix is to be multiplied by this matrix here. So, just one step in between, you simplify this is the expression you will get. And just you see whether you have done any mistake or not you can make a quick check whether the network is symmetrical or and reciprocal or not. You can see from here if you look from this side or we look from this side, network is symmetrical from both the sides.

So, hence A should be equal to D. Let us see whether we have got that or not. So, this is 1 plus Z 1 Y 2, this is 1 plus Z 1 Y 2. So, we can say A is equal to D. Now, this network is reciprocal because all these are nothing but RLC component. So, that is reciprocal network. So, let us see AD minus BC is equal to 1 or not. So, now, you multiply this with this here what we will get? 1 plus 2 times Z 1 Y 2 plus Z 1 square Y 2 square. Now, minus this term here 2 Z 1 Y 2.

So, you can say that 2 Z 1 Y 2 will get cancel by these two terms combined together and this is now Z 1 square Y 2 square. So, we had also got Z 1 square Y 2 square. So, subtraction from here to here will lead to a value which is equal to 1 so that means, this is a reciprocal network and that really means that so far we have done the current calculations.

Now, we are going to take many many more examples later on about ABCD parameters, especially just to mention that I did take an example of transmission line. So, we will see many cases later on where this particular component may be a transmission line or this component may be a transmission line or there may be some lump element. So, you have to wait for few more lectures, and then we are going to take several examples where we are going to use these properties of ABCD parameters and specially we will use the concept of divide and solve the problem.

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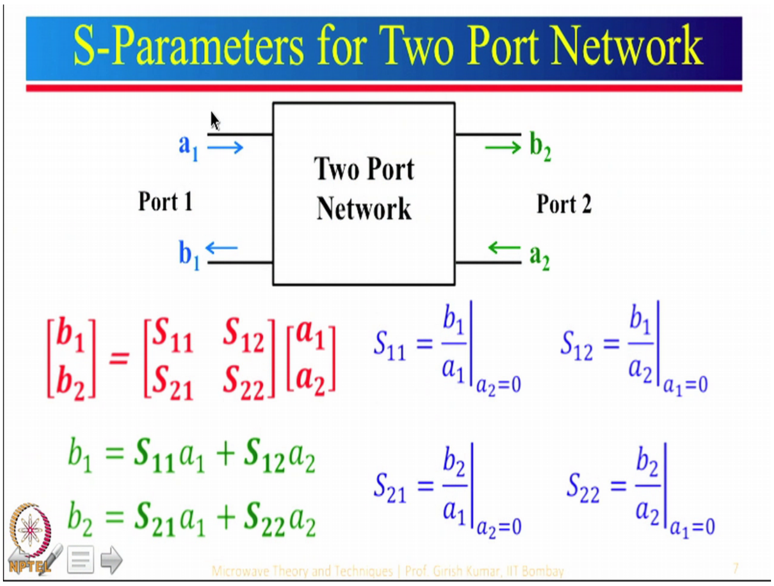

Now, we are going to the next particular topic which is S parameters. Here we have defined S parameters for 2 port network, but by the way S parameters are valid for n port network also. Just to tell you that ABCD parameters are defined only for 2 ports, but S parameters are defined for n port, but for simplicity let us just take 2 port and then we will extend this concept to n ports.

So, let us see what we have here. So, here a 1 is the incoming wave. So, it is not same as current it is a incoming wave and b 1 is reflected wave. So, we are talking about now microwave frequency, where we do not talk about voltages and currents whereas, there we call let us say an antenna which will receive the signal and through the antenna we can transmit the signal. So, there are incoming waves or outgoing waves. Similarly for an amplifier there will be a incoming wave there will be outgoing wave. So, generally speaking when we deal with S parameters they are dealing with waves.

So, a 1 is incoming wave similarly a 2 is incoming, so a parameters are incoming. So, over here b 1 is outgoing sometimes just say or leaving wave. So, this is the outgoing wave or leaving wave, ok. Sometimes in some books they also talk this as a transmitted wave also or over here they use the term reflected wave. So, in this case if this is incident this is you can say reflected and if this is incident this can be transmitted wave, ok. So, it all depends how you look at it, but basically we just say that a 1 or all a's are incident wave all b's are reflected wave.

Now, we will define the S parameter. So, we can actually define b 1, b 2 on this side that means, there are the reflected wave these are the incident wave and this is how we define S parameters, ok. We can open the matrix let us look at here. So, b 1 is equal to S 11 a 1 plus S 12 a 2, we can write b 2 as S 21 a 1 plus S 22 a 2. So, now, from here just as we did in terms of ABCD parameter will just exactly the similar thing. So, let us say from here how we can find the value of S 11? So, we can find S 11 by making let us say a 2 equal to 0. So, if a 2 is equal to 0 then S 11 will be given by b 1 divided by a 1. So, we can see that S 11 is nothing but b 1 divided by a 1 provided a 2 is equal to 0.

Now, here unlike the previous case where voltage equal to 0 implied short circuit current equal to 0 implied open circuit here a 2 equal to 0 implies match load termination. If you recall when we had mentioned about it that if something is going from here and over here, so nothing should reflect path and if nothing will reflect, but only when it is terminated into a match load, ok. And here we are defining everything in terms of characteristic impedance Z 0. So, if this particular port is terminated with a characteristic impedance of Z 0 that means, load is equal to Z 0 in that case nothing will reflect. So, a 2 will be equal to 0.

So, please remember when we define S parameter we are not dealing with short circuit or open circuit we are more dealing with the match load termination. So, now, let us see from here how we define S 12. So, S 12 will be equal to b 1 divided by a 2 provided a 1 is equal to 0. So, we can see that S 12 is nothing but b 1 divided by a 2 when a 1 is equal to 0. So, what a 1 equal to 0 really means that means, now port 1 is terminated into match load and what it really means here. So, this means that when we are giving input at port 2 what is the output at port 1 or what is the wave going out here.

Now, let us see what is S 21, S 21 is nothing but b 2 divided by a 1 provided a 2 is equal to 0. And similarly we can find out S 22. What is S 22? S 22 is nothing but b 2 divided by a 2 provided a 1 is equal to 0. So, please remember these thing. So, S 11 actually can also be termed in a slightly different way that is look at the port 1 here this is S 11. What is b 1? Reflected wave. What is a 1? Incident wave.

So, what we are basically saying S 11 is nothing but reflected wave at port 1 divided by incident wave at port 1. Let us see what is S 22. S 22 we have to look at the port 2 here what it shows here this is a incident wave, this is the reflected wave provided this is terminated with the match load to get a 1 equal to 0 so that means, S 22 is nothing but a reflected wave which is b 2 divided by a 2. So, this is same thing as reflection coefficient at port 2. This is the same thing as reflection coefficient at port 1.

Now, let us see what is S 21. S 21 is defined as b 2 by a 1. What is b 2? Wave going over here. What is a 1? Incident wave over here; provided a 2 is equal to 0 so that means, if we terminate this thing into a match load, then what is the transmitted wave to this particular value divided by the incident. So, hence this you can say will actually give the gain of the amplifier or loss of any given circuit. So, that is why many a times b 2 is mentioned as transmitted wave when a 1 is the input here ok. But if you look at S 12, what is S 12? That is b 1 divided by a 2 provided a 1 is equal to 0. So that means, if input is given at port 2 which is a 2 then what is the wave going towards this. So, in this particular case if the input is at port 2, then b 1 becomes transmitted wave. So, b 1 divided by a 2, ok.

So, in the next lecture we are going to talk in more detail about S parameters, how these reflection coefficients and transmission coefficients are define. So, just you summarize today’s lecture. So, we actually started with the very simple ABCD parameters which are define in terms of input voltages and current and output voltages and current. And the benefit of that is that for one network if this input voltage and current is defined in terms of output voltage and current then that output voltage current becomes input voltage and current for the next port, and then we can find the next to next port output voltages and currents. So, you can keep cascading these things and simply by multiplying ABCD of first to second to third or fourth or nth different components can be there and you can find ABCD parameters of the entire network simply by doing the matrix multiplication.

And then we did look at a few simple examples, one was series impedance, then another one was shunt impedance, then we looked at the transmission line, and then we also looked at one case with consisted of series shunt series, ok. And after ABCD parameters we looked at S parameters. Now, ABCD parameters are defined in terms of voltages and currents, S parameters are defined in terms of incoming wave and outgoing wave. And in the next lecture we will see that how ABCD parameters are related with S parameters. How we can convert from ABCD parameters to S parameters and also we will look at what are the different properties of S parameters, and what happens when you go from 2 port to n ports. So, what are the different things we will look into that and we will also see how to calculate S parameter for a given network.

So, with that thank you very much. See you next time. Bye.