Microwave Theory and Techniques

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Module - 03
Lecture – 11
Smith Chart and Impedance Matching - I: using Quarter Wave Transformer

Hello everyone. In the last few lectures, we have talked about transmission lines and in the transmission lines, we started with coaxial cable, then we talked about strip line, micro strip line as well as wave kind. Then we have taken a case where a transmission line is loaded with the load impedance which is equal to Z L. We took a few cases of that; for example, we saw that a small transmission line which is shorted; that means, Z L equal to 0 behaves as an inductance as long as the length of the line is less than lambda by 4. And if the line is open circuited, it behaves like a capacitance. Case 3 was where we had terminated the line with load impedance Z 0 and in that case, we saw that the input impedance always remains at 0 irrespective of the length of the line.

The fourth case we had considered where the length of the line was lambda by 4 and we had seen that any arbitrary load impedance can be transformed to any other impedance. Now, this transformation happens only at single frequency. So, what happens at other frequencies? So, today we will look into different things like smith chart, impedance matching because if the load is not match with the source impedance, then maximum power transfer does not happen.

So, let see one by one, all these things, so first of all how do we define input impedance?

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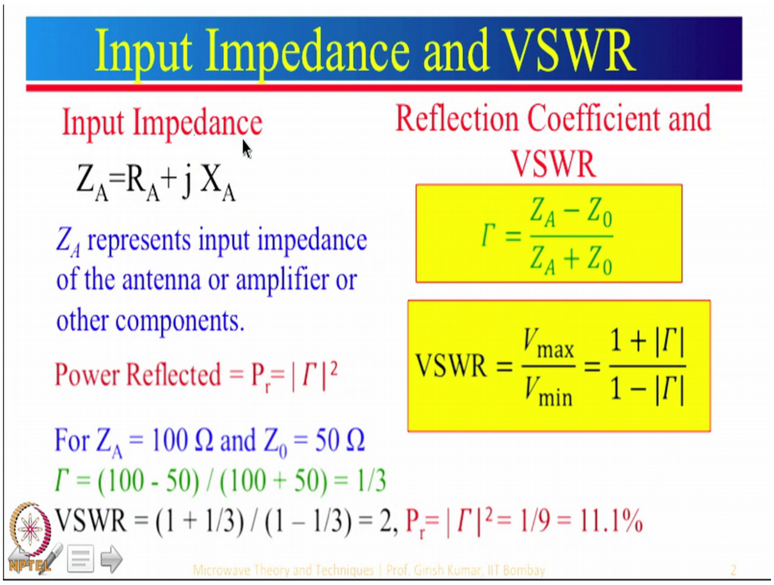

So, input impedance of any particular line or it can be of an antenna or an amplifier or other components can be defined in terms of real and imaginary terms R A and j X A. So, from this input impedance, we can find out the reflection coefficient. The reflection coefficient is given by Z A minus Z 0 divided by Z A plus Z 0.

So, just let us take a very simple case of if Z A is real value; that means, X A is equal to 0. We will take other cases also one by one. So, if Z A is equal to let us say 100 Ohm and if we say that the characteristic impedance of the line is 50 Ohm, then from this equation we can calculate the reflection coefficient, so which will be 100 minus 50 divided by 100 plus 50. So, that is 50 divided by 150 is equal to 1 by 3.

Let us also define another quantity which is VSWR. It stands for voltage standing wave ratio. This is defined as V max divided by V min and the expression for VSWR is given by this equation. You can see here that this one here is actually taking magnitude of Reflection Coefficient. Reflection Coefficient here is a complexed number. Of course, I took example of Z A is equal to 100 Ohm hence it was a real quantity, but Z A in many cases will be a complex number. So, then reflection coefficient will also be complex number but however, over here we take the magnitude of the reflection coefficient.

So, from here, we can calculate the value of VSWR, but just you look at the limiting cases, so if reflection coefficient gamma is 0 and this is 0, VSWR will be equal to 1 and if reflection coefficient magnitude is 1, then 1 plus 1 will be 2, but 1 minus 1 is 0. So, that goes to infinity; so that means, VSWR can vary from 1 to infinity. So, let us see what happens in this particular case. So, reflection coefficient is 1 by 3. We substitute the value. So, from here we can say 1 plus 1 by 3 divided by 1 minus 1 by 3. So, this will be 4 by 3 divided by 2 by 3 which comes out to be 2. So, for reflection coefficient 1 by 3, VSWR comes out to be 2.

Let us also define one more quantity which is a power reflected. So, what is power reflected? Let us say if we are giving an input and that one is going to some load, the maximum power transfers theorem says that the maximum power will get transfer if the load impedance is equal to source impeder. But if load impedance is not equal to source impedance, then part of the power will get reflected back.

So, let us see what is the power reflected, but power reflected that is given by P r which is equal to gamma square. Again you can see, it is magnitude of the gamma. So, if we not take this particular case, so here P r is nothing but gamma square which is equal to 1 by 3 square is 1 by 9 and if you see the percentage wise, it is actually 11.1 percent.

Now, many cases this particular VSWR equal to 2 is acceptable. So that means, really speaking we are accepting 11.1 percent reflected power. The many a times, they do talk about this is approximately equal to 10. So, just remember it is 11.1, but many times VSWR equal to 2 is approximately considered as reflected power less than 10 percent which is not really the case.

So that means, you can see here if load impedance is 100 Ohm, we are going to have this much of a reflected power. Suppose, if the load impedance was 50 Ohm, so what would have happen? So, 50 minus 50 would have been equal to 0, then reflected power would be equal to 0. So, let us see now, we had seen in the previous case that a quarter wave transformer can be used to transform the load impedance to 50 Ohm. So, let us see what happens now.

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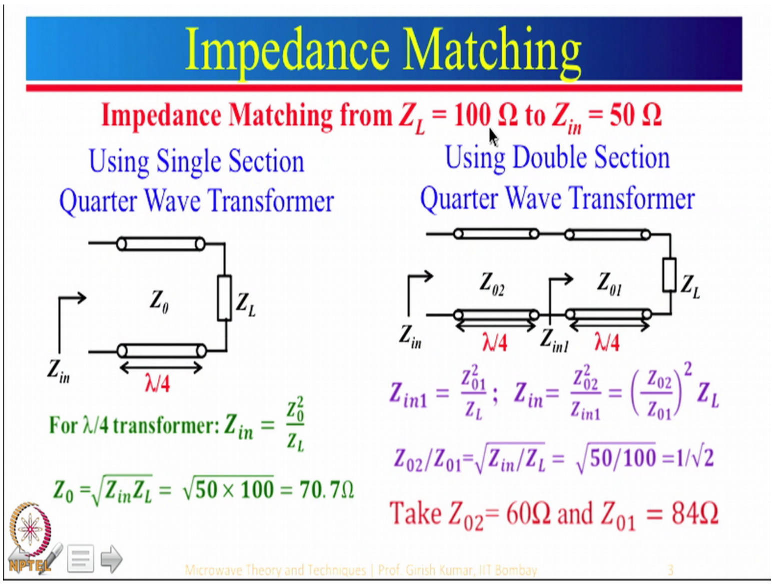

So, we will take a case where we want to do impedance matching from 100 Ohm to 50 Ohm.

Now, one of the simple things you may thing that if the load impedance is 100 Ohm, I can put 100 Ohm resistor in parallel and I would get Z input equal to 50 Ohm. Now, that is a bad solution because if I put a 100 Ohm resistor in parallel with load impedance of 100 Ohm, then 50 percent power will get dissipated in that 100 Ohm resistor. So, that is not a good solution at all.

So, let us see if you use single section which we had done in the previous lecture of a quarter wave transformer. So, let just go through the step. So, let us say this is the load impedance, that is a transmission line of characteristic impedance Z 0, length is lambda by 4 and this is where we want to know what is Z input. So, for lambda by 4 transformer, we had seen that Z in is given by Z 0 square divided by Z L. So, here Z L is given to be 100 Ohm, Z in desired is 50 Ohm. So, we can calculate the value of Z 0. So, this is Z 0 which is square root of you can see from here Z in multiplied by Z L. So, 50 multiplied by 100, we get 70.7 Ohm.

So, if we use a transmission line of length lambda by 4 at a given frequency, then we can transform this 100 Ohm impedance to 50 Ohm impedance. Now, remember one thing; this particular thing will happen only at a single frequency where this length is equal to lambda by 4, ok. So, for example, assuming that this is everything is in the air and we are designing let us say quarter wave transformer at 1 gigahertz. So, at 1 gigahertz wave length will be 30 centimeter. So, 30 divided by 4 will be 7.5 centimeter.

So, let us say we have chosen this length to be 7.5 centimeter. So, now, this 7.5 centimeter will be lambda by 4 only at 1 gigahertz. So, what will happen at point a 9 gigahertz or point 8 gigahertz or 1.1 gigahertz or 1.2 gigahertz? We will see that in the next slide what happens as the frequency changer ; however, let us look into one more thing here which is a using double section quarter wave transformer. So, what is being done over here compare to this instead of using a one lambda by 4 section, now we have use two lambda by 4 section.

So, here the characteristic impedance of this particular transmission line is Z 0 1 and the characteristic impedance of this line is Z 0 2. So, we want to find out what is Z input here and we want to this Z input to be equal to 50 Ohm. And we know that Z L is equal to 100 Ohm in this particular case. So, let us see first what is Z in 1 at this point. So, Z in 1 will be nothing but Z 0 1 square divided by Z L which is given over here.

So, now from here, we can go to the next step and that is Z input. What will be Z input? Now, Z in is nothing but Z 0 2 square which is over here divided by whatever is the load impedance and what is the load impedance here that is equal to Z in 1. So, we substitute Z in 1 over here and now we substitute the value of Z in 1 from here. So, if we put over here, this will become now Z 0 2 and this term will come here which is Z 0 1 and this term will go to the numerator. So, the term which is now equivalent to Z in is Z O 2 divided by Z O 1 square Z L.

So, for this particular case now, we want to find out what are the values of Z O 2 and Z O 1. We can see from here Z L is 100, Z in 1 is 50. So, from here you can say that Z O 2 divided by Z O 1 will be equal to square root of Z in 1 divided by Z L. So, if we substitute the value now, this is equal to 1 by square root 2. Now, you can see over here that there is only one equation and there are two unknowns. So that means, now we have to choose one of the value. So, you have all the freedom to choose different values of Z O 1.

So, let just take some cases here. So, if I take Z O 2 as 50, then Z O 1 will be 70.7 Ohm. We can take Z O 2 as 100. In that case, this will be 100 multiplied by square root 2 that will be 141. We can take Z O 2 as 10 and then this will be 40 known. So, now, among all these choices which one is the best choice ? So, you really have to understand some of these things. So, many a times analysis problem is relatively simple, but design problem is where there at certain challenges and you have to really think what value is the perfect value.

So, here I just want to mention you here. So, this is one of the best option and will show you later on why these are the best option. So, this one here we have taken as 60 Ohm and that gives Z O 1 will be equal to 60 multiplied by square root 2 and that comes out to be approximately equal to 84 Ohm. So, let see here. So, this was 100 Ohm. So, from 100 we went to 84, then we went to 60 and then we went to 50. So that means, we are moving from 100 to 50 in two steps from 100, we use 84 and then from 84, we use 60 and then we went to 50 Ohm.

Now, suppose if we had taken this as 100 only, so this would be 100 then this will be 100 and now this is going to be calculate from this particular case here. So, this will be then 70.7. So, in reality if I use 100 here we are not really using advantages of two section quarter wave; 100 here, 100 here; that means, this particular thing has not done anything. So, we actually speaking are getting only one quarter wave doing it is effective job.

Now, suppose if you take this as let say 10 Ohm over here and 14 Ohm over here, so let say then from 100 we are going to 14, then coming to 10 and then getting 50. Now, that is again a not a very good idea because let us say thing about us, we want to go from let say 100 to 50 ok. So, that is our step. So, it is always better from 100 you take step in between and you go between; think this way that we want to go from point a to point b, what do you generally do? We will like to use the shortest path. What you like to go from here 100, go to may be 120, then may be some 150 and then come back to 50 not a good idea. Similarly from 100, it is not a good idea to go to 10 and then you come to 14 or then come to 50.

So, try to go step by step process. So in fact, you can actually just you tell you also there are other things also to be done; something like we can also use a tapered line which will have an impedance of 100 Ohm over here and the impedance of 50 Ohm here. So, you can simply use a tapered line. So, as we discuss in the transmission line case for 50 Ohm, width will be more and for 100 Ohm, width will be less. So, all you do width is you take a larger width here and taper rate down to this and taper rate down to this here. So, that will be 100 and this will be 50.

Now, you thing the same thing now, this is 100 Ohm tapered to 50 here. So, if you try to take approximate average of this that may come around this here and then from here to here, you try to take approximate average it will come out to be roughly 60 Ohm. So, now, let me show you if you use multiple quarter wave sections what can happen?

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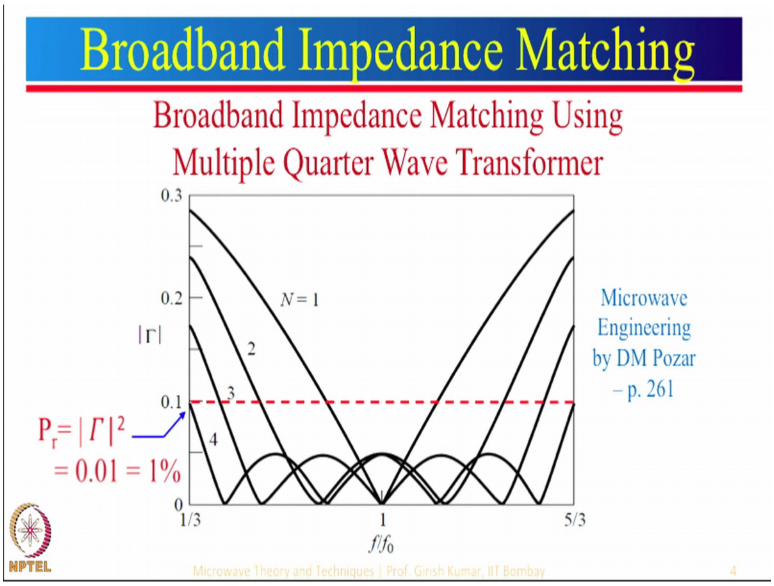

So, these results I have taken from this particular book here. I have even given the reference page number also. So, if we use one quarter wave transformer, so this is what is being used over here. So, if you use one quarter wave transformer, so let us see what we have shown. This is the frequency ratio f by f 0, this is 1 by 3, this is 5 by 3.

So, if you really take the ratio of this to this so; that means, if I can achieve this particular bandwidth this can be 5 is to 1 bandwidth and the advantage of this normalized thing is you can design this particular quarter wave transformer at 1 gigahertz or 100 megahertz or 500 megahertz or 3 gigahertz; whatever is your desired frequency, this concept will be valid.

So, what we have here let see first the response of a single quarter wave transformer. Now, I have drawn this dotted line over here. This dotted line corresponds to reflection coefficient equal to 0.1. So that means, reflected power will be only 1 percent ; so that means, here in this particular case, we are allowing that 1 percent power can reflect but 99 percent power will get transmitted to the load.

So, you can see that in this particular case for n equal to 1, this will be the bandwidth over which reflected power will be less than 1 percent. So, you can draw a line from here to here and you can see that this bandwidth will be relatively small. If we took n equal to 2, this is what I was I had shown you in the previous slide that two quarter wave transformer. So, if you use two quarter wave transformer, this will be the response and you can see now the bandwidth is definitely much larger.

Now, instead of using 2, if you use 3, so this will be the response, if you use 4 quarter wave transformer, then you can see that the bandwidth obtain will be all most 5 by 3 divided by 1 by 3 which will be 1 is to 5 ratio. So, this bandwidth can be from 1 gigahertz to 5 gigahertz or it can be let us say, 100 megahertz to 500 megahertz. In fact, at one time, we had a one very broad band requirement. So, I had design even n equal to 10 quarter wave transformer.

Now, actually people start thinking then if we use too many quarter wave transformers, my overall length increases. That is not really the case actually speaking because these quarter wave transformers when you keep adding one the thing. Your center frequency is also changing. So, it is not really that at that particular frequency, it will be very very large. As I mention, for a tapered line itself if you just take a length about lambda by 2, it will do a fairly good job of impedance matching over very large bandwidth.

However, I want to also tell you the best possible solution. The best possible solution is that from this part which is let us say 50 Ohm to 100 Ohm, instead of using tapered line, if you use exponential line like this and exponential line. So, if this is a exponential variation that will give us the largest bandwidth, the only thing you have to do it is the length of the exponential line should be greater than lambda by 2 at the lowest frequency ok.

So, remember it is at the lowest frequency not at the highest frequency. If you do that, you can actually design a very broad band impedance matching. Now of course, this matching is only good for real impedance as we mention but impedance can be a complex number and for complex number, calculations become little bit more involved. So, there is a graphical way of doing the whole thing and let us see what is that.

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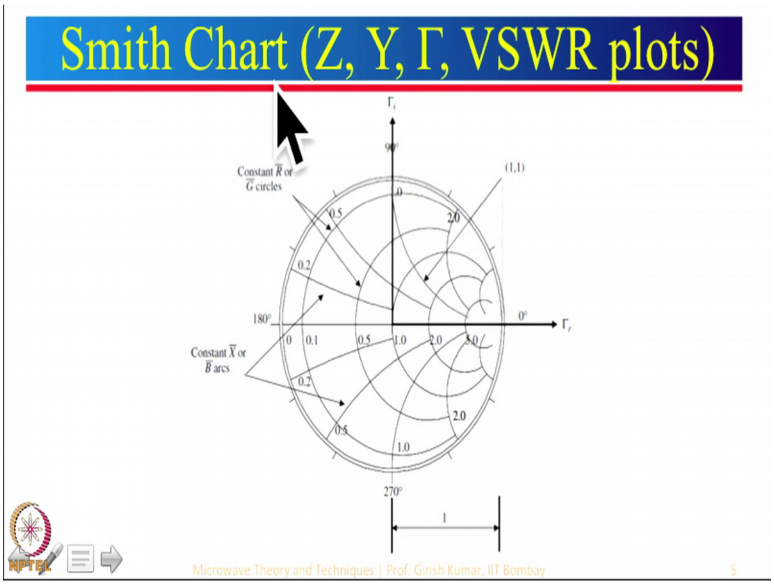

So, one can actually use Smith Chart. Now, smith chart can be use to plot impedance, admittance, reflection coefficient as well as VSWR. I am sure many of you would have studied smith chart in the electromagnetic waves or some other course, but since we are going to use smith chart in this particular course very heavily.

So, let me just give little bit of a brief overview of smith chart. So, let us say this is the smith chart and on this smith chart, you can see various these lines are there. So, let us see first of all this horizontal line what is written over here. So, it is 0, 0.1, 0.5, 1, 2, 5 and this becomes infinity. So, you can actually think about this as a real axis. So, let me first talk about only impedance and then we will talk about other thing.

So, when we look at this part here, think about this as a real axis. Now, most of you are familiar with let us say if we want to plot real and imaginary, generally what you do? You draw a real line and you draw vertical line which is a imaginary axis. So, here real axis is very similar to that horizontal line which you do except that do not now thing about a negative r. I will tell you how this Smith chart can be use for negative r value also, but here it is only for the positive r value. But you can also see here this is from 0 to 1, what actually this is? This entire smith chart is a normalized plot.

So, for example, let say if the load impedance is suppose 100, then you normalize with the respect to characteristic impedance let say that is 50. Then the load impedance divided by the characteristic impedance 100 by 50 will be equal to 2. Suppose, if the load impedance is 10 Ohm, so 10 divided by 50 will be 0.2. So, 0.2 will be somewhere here.

So, now this is the plot you can say for real impedance. So, what happens if the impedance has inductive or capacitive part? So, the above portion is basically for positive portion, the lower portion is for the negative value. So, same thing if you recall again this is the real and imaginary. So, even imaginary we generally put positive numbers above and negative numbers down below. The only difference over here in the normal Cartesian or x, y coordinator all the imaginary numbers go vertically up or vertically down. Here, these things go in circular fashion.

So, let see now what we have here. So, we will just go with this 0, 0.1, 1.5, 1 and so on and then you see this number here 0.2. Actually this is a part of the big circle which is not shown; it is only shown up to here. So, you can call at a arc of a big circle. So, along this entire line the imaginary part remains 0.2; again these are normalized value, along this entire thing this is 0.5, along this it is 1. So, this is the plus part, here is the negative part. So, along this particular thing, this is minus 0.2, this will be minus 0.5, this is minus 1, this is minus 2. You see also these other circles here. So, these are known as constant r circle; for Z, it can be constant g circle for y, but let us first focus only on z. So, you can see that these are the constant r circle. What is that mean? So that means, if you look at this point or this point or this one or here everywhere, the value of the r will remain 0.1. If you look at this here 0.5, if you move along this or along this, the real value is always going to be 0.5 and these are the constant x as you can see here or here. So, for impedance, it will be constant x, of course here x is positive, here x is negative.

So, this is the general smith chart. So now, you might to under where is a reflection coefficient. So, this one here just to repeat one more time, so all of these things will show the real positive value, all these are imaginary positive value, all these are imaginary negative value and for Z, this is 0. So, 0 Z equal to 0 implies short circuit, Z equal to infinity implies infinity.

Now, if we thing about again the Z parts. So, all these positive numbers will actually imply inductive values and all these negative values will imply positive values. So, will take examples of these things one by one; suppose, we want to now plot y, so again these numbers remain as it is, the significance only changes, that is y is equals to 0 but y equal to 0 now will represent open circuit, not short circuit as Z equal to 0 and this is y equals to infinity. So, y equal to infinity will represent short circuit. So, remember that the numbers which are written here will remain same; interpretation you have to do.

So, same thing here now y is positive. Positive y means it is capacitive and negative y would mean inductive ok. So, please remember this particular thing. Now, let us see where is reflection coefficient. Now, one of the interesting thing you can see that below this there is a line shown over here. In fact, this particular horizontal line which is shown that actually is a reflection coefficient equal to 0 point and this is a reflection coefficient equal to 1. And anything along this reflection coefficient well vary. So, suppose this is 0, it will be 0.5, this will be say 0.5, 0.7, 0.8, 0.9 and so on.

So, the reflection coefficient equal to 0 over here and you can even correlate. What is the reflection coefficient takes place? Z in minus Z 0 divided by Z in plus Z 0. What is Z in is equal to? Z in equal to Z 0 here, so that means, reflection coefficient is 0 and this when here is reflection coefficient 1 and this angular thing will give the angle of the reflection coefficient. Suppose if the point is here which is then reflection coefficient 1, suppose if it is somewhere here, then the reflection coefficient value can be correspondingly measure from here. But now suppose if the reflection coefficient is somewhere here, so what you do? You actually draw a line like this and measure this particular angle and that will give the angle of the reflection coefficient.

Now, where is VSWR? In fact, VSWR whatever are these numbers here, so suppose it is 1, then the VSWR will be 1, at this point VSWR will be 2; over here, VSWR equal to 5. Let us take an example where we can look at all these things one by one.

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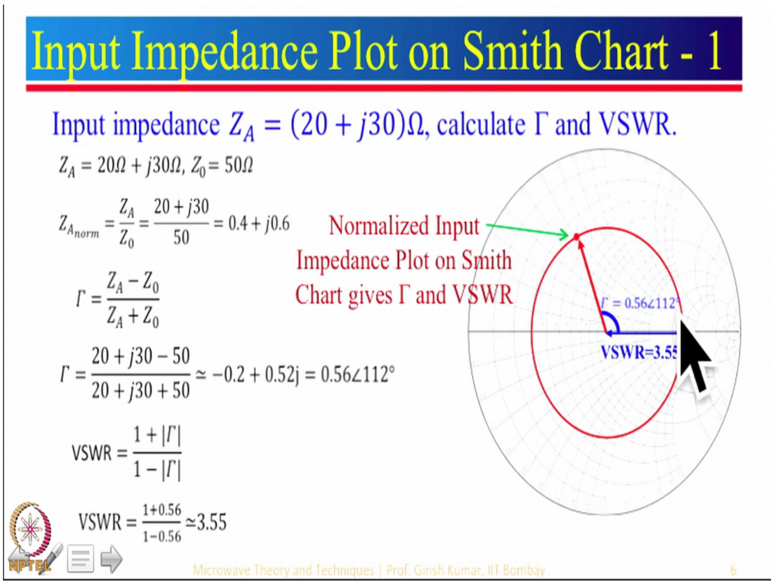

So, here is an example where input impedance is given by Z A equal to 20 plus j 30 Ohm and the required parameters are that we want to calculate reflection coefficient and VSWR. Of course, we can use the conventional thing and that is we can use this particular formula for reflection coefficient. So, Z A is 20 plus j 30 Ohm and Z 0 is 50 Ohm. We substitute this value over here, solve these complex numbers and from here we can calculate what is the reflection coefficient and that comes out to be 0.56 and then 112 degree.

And from here we can calculate the value of VSWR which is given by this particular expression and so, what we need? We only need magnitude of the reflection coefficient which is 0.56. So, you put 0.56 and here and that would give us the value of VSWR equal to 3.55. So, this is the way you can do the calculation; now, will tell you, how to do the calculation using the smith chart or using the graphical way.

So, here what you do? You take this particular impedance; the first step would be to normalized. So, we normalized this impedance. So, 20 plus j 30 divided by 50. So, 20 by 50 is 0.430 by 50 is 0.6. Now, we need to plot this particular thing on the smith chart. So, what you do step by step thing, first look at the real part which is 0.4.

So, along this smith chart, you see from here 0 then you know it is 1 and you know it is infinity. So, you look it the point which is 0.4, then you look at the other part. Other part is plus which is imaginary plus. So, for plus we need to go up. If it was minus, we have to move downward and then what you do? You look at plus j 0.6. So, from here you actually move and then from 0.4, you move and you stop at a point where you can see that these are the constant reactive circle. So, you stop at a point which is 0.6.

Now, this point here what you do? Now, you draw a circle. So, when you draw the circle here, what you can see whatever is this value which is crossing the real axis that value here will directly give you the VSWR value which is 3.55. Now, from here you can draw the line and you can draw the line so that axis is from 0 to 1. Whatever is this value, you can note down that value and that value will be 0.56 and you can now measure this particular angle and that will be 112 degree and you can see that this angle is exactly same as this particular angle here.

So, there are two ways to calculate reflection coefficient and VSWR; one is you can calculate using these expression; another one is you can use smith chart and plot these values and use graphical way to calculate reflection coefficient and VSWR. We will take more cases in the next lecture. So,

Thank you very much. We will see you next time, bye.