

**Microwave Theory and Techniques**  
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**Module - 02**

**Lecture - 10**

**Transmission Lines - II: Transmission Line Model, Open and Short Circuited  
Lossless Transmission Lines**

Hello everyone. Welcome to today's lecture on Transmission Lines which is continuation of the previous lectures. So, in the previous lecture we had seen coaxial cable we had actually seen that what are the capacitance and inductance values, and from that we found out what is the value of the  $Z_0$  which is characteristic impedance of the line.

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## Co-axial Cable

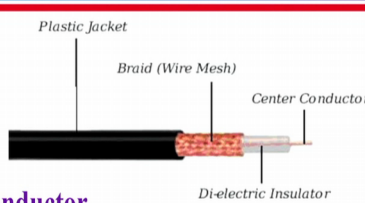
$$C = \frac{2\pi\epsilon_0\epsilon_r}{\ln \frac{D}{d}} \text{ (F/m)}$$

$$L = 0.2 \ln \frac{D}{d} \text{ (\mu H/m)}$$


**d = Outer diameter of inner conductor**  
**D = Inner diameter of outer conductor**  
 **$\epsilon_r$  = Dielectric constant**

$$Z_0 = \sqrt{L/C} \rightarrow Z_0 = \frac{60}{\sqrt{\epsilon_r}} \ln \frac{D}{d} \text{ (\Omega/m)}$$

For RG58C/U Co-axial cable:  $d = 0.91 \text{ mm}$ ,  
Dia. of dielectric =  $2.95 \text{ mm}$ ,  $\epsilon_r = 2.1 \rightarrow Z_0 = 48.7 \Omega$

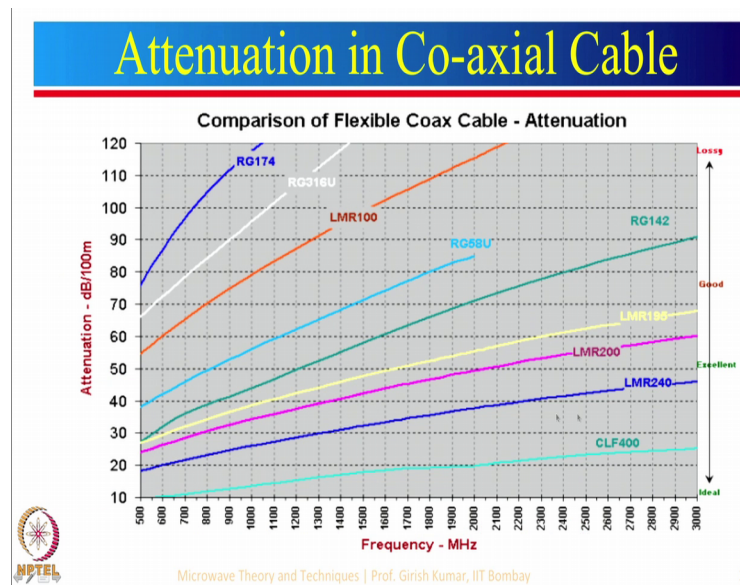


Plastic Jacket  
Braid (Wire Mesh)  
Center Conductor  
Di-electric Insulator


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Of course, this one assumes it is a lossless line.

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But then we did look into what are the losses. So, one can see that the losses can be very high or losses can be even very low. And of course, you pay more price for these kind of coaxial cable.

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### Specifications of RG58C/U

**Electrical Specifications**

Description	Minimum	Typical	Maximum	Units
Frequency Range	DC		5	GHz
Impedance		50		Ohms
Velocity of Propagation		65.9		%
Shielding Effectiveness	45			dB
Operating Voltage (AC)			1,900	Vrms
Nominal Capacitance		30.8 [101.05]		pF/ft [µF/m]

**Performance by Frequency Band**

Description	F1	F2	F3	F4	F5	Units
Frequency	0.01	0.1	1	5		GHz
Attenuation, Typ	1.4	4.9	20	60		dB/100ft dB/100m
Input Power (CW), Max	650	170	44	15		Watts

**Mechanical Specifications**

Diameter	0.195 in [4.95 mm]
Weight	0.025 lbs/ft [0.04 Kg/m]
Min. Bend Radius (Installation)	0.98 in [24.89 mm]
Min. Bend Radius (Repeated)	1.96 in [49.78 mm]

**Construction Specifications**

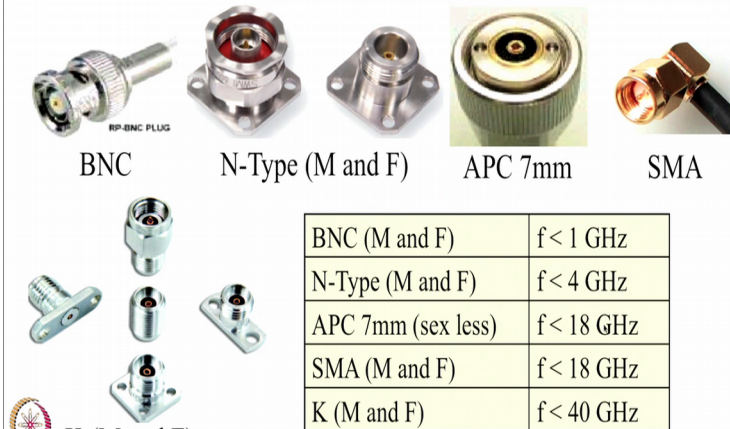
Description	Material and Plating	Diameter
Inner Conductor	Copper, Tin, 19 Strands	0.036 in [0.91 mm]
Dielectric	PE	0.116 in [2.95 mm]
First Shield	Tinned Copper Braid	

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Then we did look at the specification of one of the coaxial cable, followed by different types of connector.

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## RF Connectors



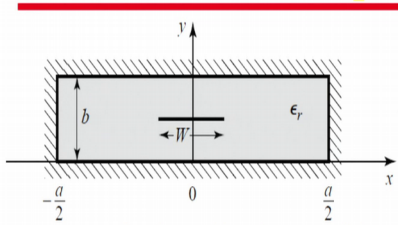
BNC (M and F)	$f < 1$ GHz
N-Type (M and F)	$f < 4$ GHz
APC 7mm (sex less)	$f < 18$ GHz
SMA (M and F)	$f < 18$ GHz
K (M and F)	$f < 40$ GHz

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So, I did mention to you about different types of connector, but just to tell you at RF these days the maximum number of connectors which are being used are N type or SMA conductor type these are the two most commonly used. Of course, at millimeter wave you have no option, but use these connectors.

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## Strip Line



$$Z_o = \frac{30\pi}{\sqrt{\epsilon_r}} \frac{b}{W_e + 0.441b} (\Omega/m)$$

$W_e$  is the effective width of central conductor.

$$\frac{W_e}{b} = \frac{W}{b} \begin{cases} \text{for } \frac{W}{b} > 0.35 \\ \left(0.35 - \frac{W}{b}\right)^2 \text{ for } \frac{W}{b} < 0.35 \end{cases}$$

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Then we talked about strip line. So, I did mention to you how coaxial line can be represented in terms of strip line, because there it is circular here it is rectangular.

Then from strip line we looked into different types of substrates what are the substrates used from low cost to very high cost. After that we talked about micro strip line we do not look into what are the fringing fields associated with these thing.

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## Microstrip Line

$$\epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left( \frac{1}{\sqrt{1 + 10 \frac{d}{W}}} \right)$$

$$Z_0 = \begin{cases} \frac{60}{\sqrt{\epsilon_e}} \ln \left( \frac{8d}{W} + \frac{W}{4d} \right) & \text{for } W/d \leq 1 \\ \frac{120\pi}{\sqrt{\epsilon_e} [W/d + 1.393 + 0.667 \ln(W/d + 1.444)]} & \text{for } W/d \geq 1 \end{cases}$$

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And because of the fringing field effective dielectric constant is present which is slightly less than dielectric constant of the substrate. Because now, wave is not completely confined inside the dielectric material part of the field is in the air.

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## Microstrip Line Design

For a given  $Z_0$ ,  $W/d$  can be found as:

$$\frac{W}{d} = \begin{cases} \frac{8e^A}{e^{2A} - 2} & \text{for } \frac{W}{d} < 2 \\ \frac{2}{\pi} \left[ B - 1 - \ln(2B - 1) + \frac{\epsilon_r + 1}{2\epsilon_r} \left\{ \ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right\} \right] & \text{for } \frac{W}{d} > 2 \end{cases}$$

Where,

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \left( 0.23 + \frac{0.11}{\epsilon_r} \right)}$$

$$B = \frac{377\pi}{2Z_0\sqrt{\epsilon_r}}$$

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And then we looked into for a given value of W one can find the characteristic impedance. So, this is an analysis equation, and then we looked at the design problem, where  $Z_0$  is given and we need to find what is W by d or W by h.

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## Microstrip Line Design Problem

For FR4 substrate ( $\epsilon_r = 4.4$ ) of height (h) = 1.6 mm, find the value of microstrip line width (W) for characteristic impedance ( $Z_0$ ) of 100  $\Omega$ .

**Design:**

$$A = \frac{Z_0}{60} \sqrt{\frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{\epsilon_r} \left(0.23 + \frac{0.11}{\epsilon_r}\right)} = \frac{100}{60} \sqrt{\frac{4.4 + 1}{2} + \frac{4.4 - 1}{4.4 + 1} \left(0.23 + \frac{0.11}{4.4}\right)}$$


$$= 2.899$$

$$\frac{w}{d} = \frac{8e^{2A}}{e^{2A} - 2} = 0.443 \Rightarrow w = 0.71 \text{ mm}; \quad \epsilon_e = \frac{\epsilon_r + 1}{2} + \frac{\epsilon_r - 1}{2} \left(1 + \frac{10h}{w}\right)^{-1/2} = 3.05$$

**Verification using analysis equation:**

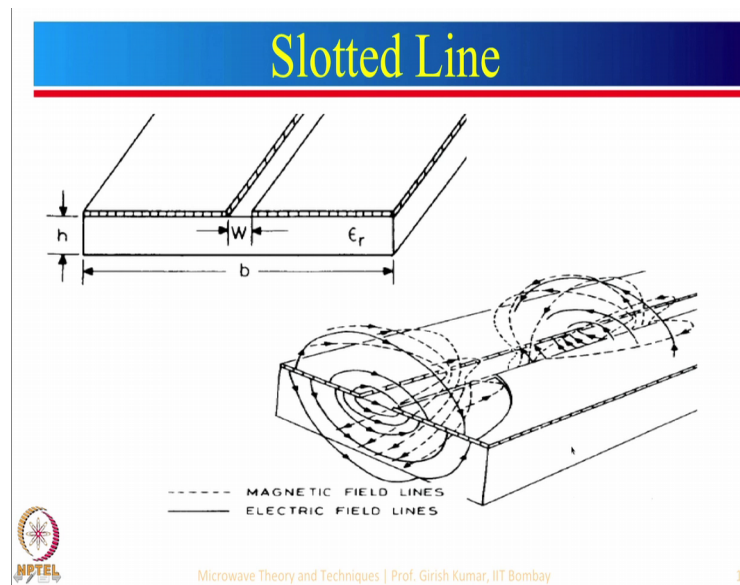
$$\frac{w}{d} < 1 \Rightarrow Z_0 = \frac{60}{\sqrt{\epsilon_e}} \ln\left(\frac{8d}{w} + \frac{w}{4d}\right) = 99.62 \Omega$$

$$\text{Percentage Error in } Z_0 = \frac{100 - 99.62}{100} \times 100 = 0.38\%$$


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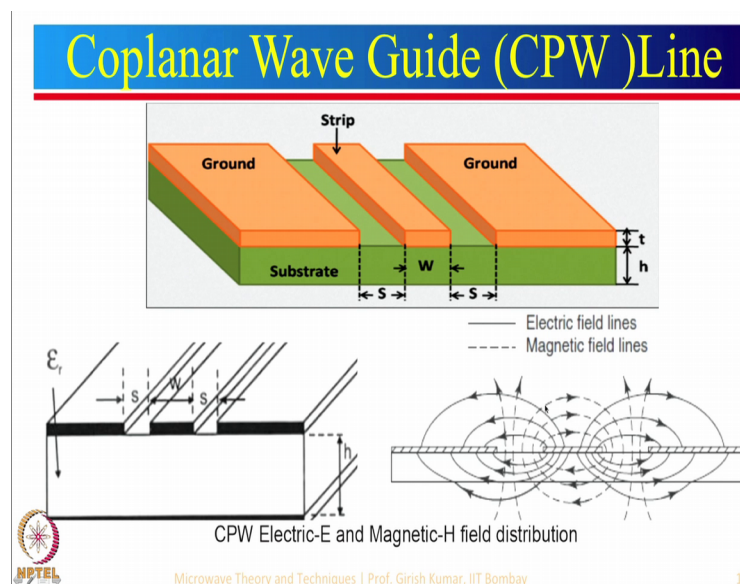
Then we took one design problem so where we have taken a one particular substrate and then for characteristic impedances of 100 ohm. We can find out the value of W which comes out to be 0.71 mm and epsilon effective was 3.05 which is less than 4.4. And we verified this designed equation using the analysis equation and we saw that the percentage error is relatively very small.

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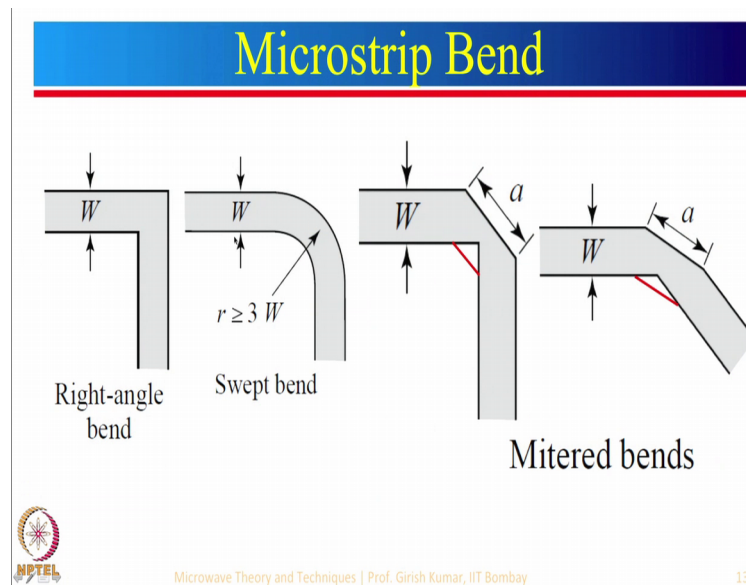
Then we also looked at some other variations like slot line, co-planer waveguide, but I did mention that these lines are relatively more lossy; in a sense that there will be more radiation losses.

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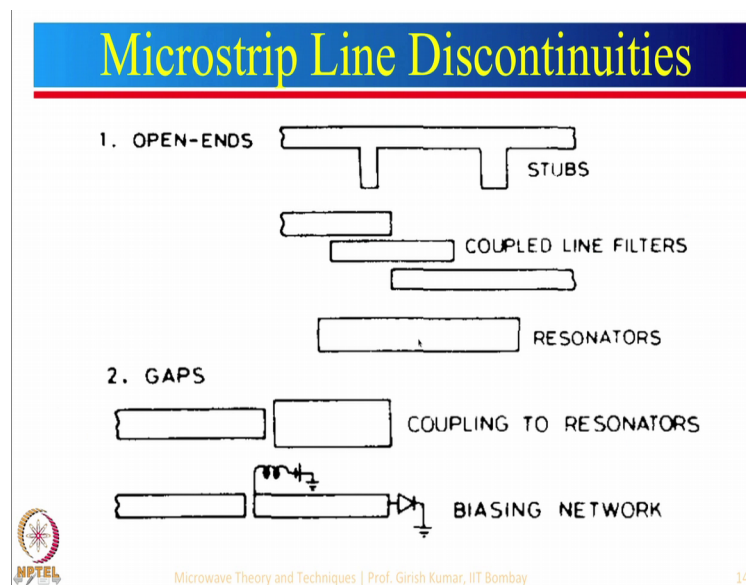
So, there many a times better option as an antenna then they are options as micro strip circuit. So, I prefer generally micro strip circuit. So, that is why in this course we will focus more on the micro strip circuit. Then I did mention to you about how to do a proper layout.

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You will see later on that bends are very very common in most of the different configuration. So, instead of using a bend like this use this bend which is most preferable one if not possible use a bend which has something like this over here. So, right angle bend you use to mitered bends over here. And if the angle is not 90 degree still I recommend that you use this kind of a configuration then of using only this type.

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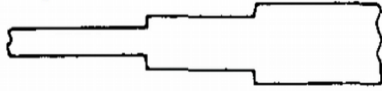


Then we also looked at different types of micro strip discontinuities which are open and gap.

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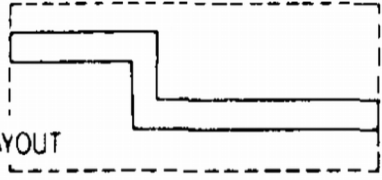
### Microstrip Line Discontinuities (contd.)


3. STEPS IN WIDTH Impedance Transformer



4. RIGHT-ANGLED BENDS

CIRCUIT LAYOUT




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Then step in width, right angle bend, T junction, cross junction.

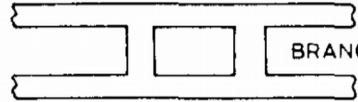
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### Microstrip Line Discontinuities (contd.)

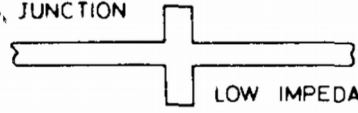
5. T-JUNCTIONS




BRANCH LINE CIRCUITS



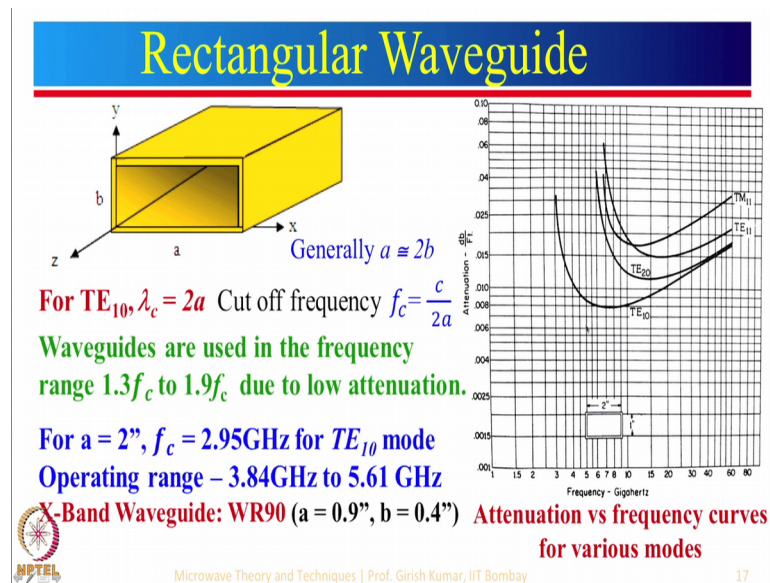
6. CROSS JUNCTION



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In fact, there are many other discontinuities are also there which we will discuss as we move along in the course. But these are the most important ones or more commonly appearing in different circuit configuration.

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Now, let us move to the next one rectangular waveguide. You have already been introduced to rectangular waveguide in the previous lectures, but I will just have a little bit of a repetition but add a few additional practical concepts about this over here. So, a rectangular waveguide is defined by its two dimensions  $a$  and  $b$  of course, then there will be a length of that. Now, this  $a$  here basically defines the cutoff frequency. So, if we are going to operate at fundamental mode  $TE_{10}$ . What is 10? 1 basically means that there is a one half wavelength variation over here.

So, what is half wavelength? Well, suppose if I take a wave. So, wave will be let us say 0 goes to maximum comes to 0 and goes back to 0. So, that will be a complete wave form. Here what happens? The wave is going from 0 it goes to maximum and then it comes to 0 here. Now, this is the distribution for voltage. Why voltage? Because this is a short circuit, so for short circuit voltage will be 0 and it will be 0 here in between it will be maximum. But if we look at the current distribution then the current will be a maximum over here. So, the current will be let us say maximum then it will go to 0 and then it will go to minimum.

So, these waveguides are defined by at the cutoff frequency and the cutoff frequency first we need to find out the cutoff wavelength  $\lambda_c$  is given by 2 times  $a$ . And once we know  $\lambda_c$  we know that cutoff frequency is nothing but  $f_c$  is  $c$  divided by  $\lambda_c$  and  $\lambda_c$  is equal to  $2a$ .

Now, generally speaking waveguides are not used just immediately above the cutoff frequency. So, for example, I will take an example and then I will explain you. So, suppose a waveguide which is shown here as an example whereas,  $a$  is 2 inches and  $b$  is 1 inches. So if  $a$ , is equal to 2 inches you can actually do the calculation, so  $\lambda_c$  will be  $2 \times 2$ . So,  $2 \times 2$  will be 4 inches you multiply that inches into 2.54 to get centimeter and then from that you use the concept  $f_c$  is  $c$  by  $\lambda_c$  and then the cutoff frequency comes out to be 2.95 gigahertz for TE<sub>10</sub> mode.

However, you cannot use waveguide at 3 gigahertz or so, the reason for that this is here is the attenuation curve versus frequency and these curves are given for different mode here TE<sub>10</sub> mode and other mode. Now, TE<sub>10</sub> mode is known as dominant mode, majority of the applications use TE<sub>10</sub> mode dominant mode.

So, let just look into that here. Now, see this is frequencies 2.59 close to 3 gigahertz. So, let us see here the frequency this is 3 gigahertz. So, if I go vertically up here at 3 gigahertz you can see that the attenuation is relatively very high. And this attenuation now is decreasing, so you can see that in this particular region relatively attenuation is small. So, you can see that these numbers are very small compare to even coaxial line so in fact, in general rectangular wave guides have much lower losses compare to a coaxial line or even a micro strip line.

So, here in general just to mention so this is 2.95 and generally this particular wave guide is used in this particular frequency range which rarely corresponds to  $1.3 f_c$  to about  $1.9 f_c$ . Now, there are two things are there one is of course, in this range there is a low attenuation but if you see low attenuations stretches all the way to up to this point then why this limit. The reason for this limit is that if, let us say  $b$  is equal to  $a$  by 2. Now, in that particular case if it is exactly  $b$  is exactly  $a$  by 2 then what happens at double the frequency this particular mode will start getting excited. So, this mode is TE<sub>10</sub> but the other mode which can come over here that can be now, TE<sub>01</sub> mode.

Also at double the frequency this particular thing here we will not be TE<sub>10</sub> any more at TE<sub>20</sub> will be something like this maxima it will go to 0 and then it will come like this here. So that means, higher order modes will start propagating which we really do not want hence there is a limit of  $1.3 f_c$  to  $1.9 f_c$ .



I have also want to mention that my waveguide are also mentioned defined by its numbers. So, for example, standard available is WR 90 now, actually in this number itself the dimension of a, is hidden actually. So, over here a is actually equal to 0.9 inches. So, you have just 90 you put 0.9 inches

Now, just you tell you in this particular case b is equal to 0.4 inches, so which is not really half of a but half of a would have been 0.45 inches it is close to that. Just to give you some other numbers also for example, at one time we did work on WR 2300. So, 2300 actually means 23 inches. So, that is about this large waveguide, and that one had a b which is half of that a. But, because this size was so large there is a reduced height waveguide is also available where b was equal to a by 4. So, again when you are choosing a waveguide it is very important what is your frequency operation.

For example, you can do the calculation for WR 90 you will see that the cutoff frequency is actually less than 7 gigahertz. However, the operating range for this waveguide is generally given more as 8 to 12 gigahertz which is what is x band. So, be careful about this attenuation curved even the cutoff frequency is 3 gigahertz you do not really use the waveguide at 3 gigahertz, because attenuation will be very large at this particular frequency.

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Comparison of Transmission lines				
Characteristics	Coaxial Cable	Waveguide	Stripline	Microstrip Line
Modes: Preferred Other	TEM TM,TE	TE <sub>10</sub> TM,TE	TEM TM,TE	Quasi-TEM Hybrid TM,TE
Dispersion	None	Medium	None	Low
Bandwidth	High	Low	High	Low
Loss	Medium	Low	High	High
Power Capacity	Medium	High	Low	Low
Physical Size	Large	Large	Medium	Small
Ease of Fabrication	Medium	Medium	Easy	Easy
Integration with other Components	Hard	Hard	Fair	Easy



So, let us do a comparison of different transmission lines. So, for coaxial cable there can be different modes which has been discuss in the earlier lecture. So now, let just look at

some of the important thing. For example, coaxial cable has a bandwidth high, but I just want to mention here little bit be careful about it a coaxial cable can be used right from dc up to about 1 gigahertz or 10 gigahertz or even 40 gigahertz, but you have to pay lot of price for that.

Now, waveguide bandwidth as I mention it is generally low the reason for that is waveguide is defined by its mode and you can say that the bandwidth will be generally limited to about  $1.3 f_c$  to about  $1.9 f_c$ . So, strip line you can see that the bandwidth is relatively high, here it is slightly low again because of the dispersion or because of the radiation which is over there, ok. So of course, loss coaxial cable has medium loss, waveguide has very low loss and these have relatively higher losses.

Similarly now, the power capacity coaxial cable can handle medium power again in this particular as I want to mention there are several variations of coaxial cables are available. There may be a thin coaxial cable or there may be a, this thick coaxial cable also which can handle several kilowatts of power also. But waveguide of course, can handle very high power the reason for that is first of all there is a no center conductor. So, center conductor being thin in the other cases will have a high resistance. So,  $I^2 R$  losses will be more here there is a no center conductor and of course, the outer 4 walls are there which are made a 4 metallic wall so they can handle higher power.

Of course, the disadvantage with this is that the physical size of this is relatively large, ok. You can see the advantage over here integration with components. You can see that with the micro strip line it is very easy to integrate with strip line fair the reason why the fair is that let us say if you have a one substrate then we have a line. If you want to put components on that then putting another thing will be very challenging because the reason if you put the layer this components or lamped elements will have certain thickness. So, there will be an air gap. So, you really speaking have to be very careful when you want to mount the components. So, that is why micro strip line is very very popular and these are really speaking used now, almost you can say in all your laptops mobile phone and other cases.

Let us now look at the basic concept of lossy transmission line, ok. You might have studied this particular thing in electromagnetic waves, but still we will look into some of the things as we are going to use this transmission line for several applications later on.

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## Lossy Transmission Line Model

For low-loss lines (R and G are very small):

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = j\omega\sqrt{LC} + R\sqrt{\frac{L}{C}}$$

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}$$

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So, little bit of a quick review. So, here what we have a very small coaxial line represented by the discrete values over here. So, what we have? A resistance and the length of the coaxial line is  $dz$ . So, that is  $R dz$ , then the inductor is let us say  $L dz$  the capacitance. What is the capacitance? You can say that capacitance is between the center conductor to the outer conductor or the capacitance can be between the line and the ground. And this particular thing will generally be there, because of the lossy substrate. So, the substrate is lossless then this term will generally be not there.

So now let just look at what are the different thing. So, this expression is a general expression which is a propagation constant and that is given by  $R$  plus  $j\omega L$  multiplied by  $G$  plus  $j\omega C$ , where  $R$  is the you can say the series resistance of the line, this is the inductor of the line, that is the conductance which is between the line and the ground and the capacitance.

So, if you assume that  $R$  and  $G$  are very small. So, in that particular case we can make an approximation over here. So, by making this approximation you can say that this term get simplified to this over here and if we now, assume that if  $R$  is tending to 0 then propagation constant is really equal to this particular term over here.

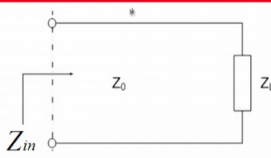
Now, similarly we have characteristic impedance expression which is now,  $R$  plus  $j\omega L$  divided by  $G$  plus  $j\omega C$ . If you actually look at it these two things look

kind of similar here it is a product here it is a ratio of the two terms. So, if R is 0, G is 0 this will not be there j omega j omega will get cancelled and we get square root L by C.

So, you might be again familiar with electromagnetic waves concept, so where we have a transmission line of let us say length L and characteristic impedance of the line is Z<sub>0</sub> which is terminated in a load impedance.

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### Loaded Lossless Transmission Line




$$Z_{in} = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

**CASE 1:** For a short circuited line,  $Z_L = 0$   
 $Z_{in} = jZ_0 \tan(\beta l) = j\omega L$  for  $l < \lambda/4$

**CASE 2:** For an open circuited line,  $Z_L = \infty$   
 $Z_{in} = Z_0 / j \tan(\beta l) = 1/j\omega C$  for  $l < \lambda/4$

**CASE 3:** For a matched load,  $Z_L = Z_0$   
 $Z_{in} = Z_0$  for all values of  $l$



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By the way all these formulation which we are looking at they are valid for all types of transmission line. So, this concept is valid for all of these, particular thing so you do not have to worry about whether it is good for coaxial or micro strip or strip line it is valid for all those cases.

So now, for a loaded lossless transmission line what is generally done I will just give you a typical thing you defined a transmission line in terms of let us say voltage at this port, voltage at this port and that voltage you might have seen that is given in terms of cos hyperbolic term and sin hyperbolic terms in terms of voltages and current. And similarly current is written in the similar form. So, we take the ratio of v and I which gives us the value of Z input.

And the cos hyperbolic term and sin hyperbolic term which has let us say cos hyperbolic gamma L and we just saw that the gamma can be written as alpha plus j beta and alpha is 0 for lossless line which is j beta and that is what the terms are gamma is written in terms

of beta and that cos hyperbolic term and sin hyperbolic term get simplified to cos term and sin term. And by taking the ratio of that we get this particular expression.

This expression if you have a little doubt you can refresh your old notes or you can look at any one of these books which I have mention earlier you can read Pozar book or Collins book or any other even Jorden Polymen book and so on which you might have studied.

Now, what we are interested today in is to use this particular transmission line to realize different types of component. And as we will see at microwave we will see later on that we require very small inductances or capacitances which are very difficult to purchase from the market they are not available as of the shelf, so many a times we realize these inductors and capacitors using these transmission line. So, let just look into it how we can realize inductor or capacitor using these transmission lines.

So, let just look at this expression again. So,  $Z_{in}$  is given by  $Z_0$  which is characteristic impedance of the line you can see here there is a load impedance and then  $Z_0 \tan \beta L$   $\beta$  is nothing but propagation constant. Just to tell you. So,  $\beta$  here is equal to  $2\pi$  by  $\lambda$ , where  $\lambda$  will be equal to  $\lambda_0$  by square root  $\epsilon_r$  in case of coaxial cable or in case of strip line, but for micro strip line  $\lambda$  will be equal to  $\lambda_0$  divided by square root of  $\epsilon_{eff}$ .

So, now let just look at different cases. So, the case number one for a short circuited line so that means, there is a transmission line over here which is short circuited. So that means,  $Z_L$  is equal to 0 and now, if you put  $Z_L$  equal to 0 over here this is 0, this is 0. So, what happens now? Simplify  $Z_0/Z_0$  gets cancelled we get  $j Z_0 \tan \beta L$ .

Now, you can see that this particular term is  $j$  here and this term can be represented as  $j\omega L$ , but there is a condition over here I will explain one by one what are these things. So, remember if there is an inductance what is the impedance of an inductance is equal to  $j\omega L$ . What is this impedance?  $j Z_0$  which is somewhat similar to there, but this will happen only if  $L$  is less than  $\lambda/4$ . Why this condition? So, let just look into this over here. So,  $\beta$  is equal to  $2\pi$  by  $\lambda$ . If  $L$  is less than  $\lambda/4$  then this term will be less than 90 degree and tan of anything which is less than 90 degree will be positive. So, this will be a positive number  $j Z_0$  some positive number which is equivalent to there.

So, we can say that is small transmission line which has to be smaller than  $\lambda/4$  which is short circuited can realize an inductance. Now, you might wonder well if there is a short circuit over here current will be flowing through here voltage is equal to 0 here how does it realize inductor. Well, the reason let just look into here because we are working at a very high frequency. And at high frequency what happen? If this is a short circuit this point may not be short circuit because there will be a voltage wave form.

As I mention suppose if there is a short here, then from short it will go to maxima it will go to 0, then it will go to minima it will go to 0. So, think about from 0 to  $\lambda/4$  length it goes from 0 to maximum. So, a short circuit here really will become maxima over here if the length is precisely equal to  $\lambda/4$ .

Now, let us see what happens if length is greater than  $\lambda/4$ , but less than  $\lambda/2$ . If it is greater than  $\lambda/4$  and the less than  $\lambda/2$  then  $\beta L$  will be between 90 degree and 180 degree for that case this quantity will be negative and if this is negative then this quantity will not be valid so in fact, a negative over here will realize a capacitance. So that means, a length which is between  $\lambda/4$  to  $\lambda/2$  will actually realize a capacitance. However, generally we do not do that as we will be obvious from the next one. So, we generally use a smaller line which is less than  $\lambda/4$  and if it is short circuited it can realize a inductance.

Let us see the case number 2, case 2 is open circuit. So, for open circuit  $Z_L$  will be equal to infinity. So, let us see what happens in this case.  $Z_L$  equal to infinity and  $Z_L$  is infinity. Now, please do not try to cancel this infinity infinity directly. First of all you have to see this is infinity. So, any term which is added to infinity will be negligible. So, it goes away. Now, this is infinity any term which is added to infinity will be negligible. So, this is negligible, this is negligible. So now, you can cancel  $Z_L$  with  $Z_L$  over here. So, what we will get?  $Z_0$  divided by  $j \tan \beta L$ .

Now, this is equivalent to  $1 / j \omega C$  for length less than  $\lambda/4$ . Again the discussion is similar to what I had mentioned over here. So, if length is less than  $\lambda/4$  this particular thing will be less than 90 degree. So, tan of less than 90 degree will be positive so that will be equivalent to this.

So that means, I can realize a capacitance using a small transmission line which is an open circuit. So, again if you just think from circuit theory point of view these things



may not make a much sense. Open circuit you always think current is not going to flow how it can realize inductance again you have to think this way. So, if this particular thing is open circuit and if the length is close to  $\lambda/4$  then open circuit will become short circuit over here and anything in between. So, what will happen? So, let us say if this is an open circuit. So, for open circuit what will be the current, current will be 0 voltage will be maximum

So, this current which is 0 here which try to go to maxima if it is  $\lambda/4$ ; suppose if this length is  $\lambda/2$  but still it will go from 0 may be half the way of maxima maximum would have been here but you will have a some finite value of the current. So, that is the difference in a normal circuit theory and in at microwave because at microwave frequency is very high wavelength is very small. And just to give you a little bit of an idea also see if you have working at let us say 1 gigahertz, then what is  $\lambda$ ?  $\lambda$  will be 30 centimeter. What will be  $\lambda/4$ ? 7.5 centimeter. Now, these things can be put on a micro strip line or you can use a coaxial cable.

But if you try to realize inductor using this particular concept at let us say 1 megahertz. Now, add to 1 megahertz wave length is going to be 300 meter and  $300/4$  is 75 meter. So, I do not think anybody would like to use up to 75 meter length just to realize a inductor one would rather by a inductor from the market or of the shelf.

Then let just look at the third case here for a matched load which means  $Z_L$  equal to  $Z_0$ . So, if  $Z_L$  is equal to  $Z_0$ . So, what will happen? If  $Z_L$  is  $Z_0$  here  $Z_L$  is  $Z_0$  this will get cancelled over here. So, we can say  $Z_{in}$  is equal to  $Z_0$  for all values of  $l$ . So, it is independent of  $l$ . In fact, it is independent of frequency also.

So, what is the significance of this? Now, the significance of this is that if  $Z_{in}$  is equal to  $Z_0$  that means, the source over here is equivalent to 50 ohm and this one here is terminated. So, what happens? If this is 50 ohm here and if this is 50 ohm if you recall there is a maximum power transfer theorem. What maximum power transfer theorem says, in order to transmit the maximum power from the source to the load, load impedance should be equal to source impedance. So, now, you can see that  $Z_{in}$  is equal to  $Z_0$ , even though there is a transmission line in between but the power transfer will be now maximum, so whatever we give at the input side all the power will go to this particular thing and that is the significance of this here.

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## Loaded Transmission Line (Contd.)

**CASE 4:** For  $l = \lambda/4$ ,  $\tan(\beta l) = \infty$


$$Z_{in} = Z_0^2 / Z_L$$

Known as Quarter Wave Transformer

If  $Z_L = 100 \Omega$  and  $Z_{in} = 50 \Omega$  required

$$Z_0 = \sqrt{(Z_{in} Z_L)} = \sqrt{(100 \times 50)} = 70.7 \Omega$$

**CASE 5:** For  $l = \lambda/2$ ,  $\tan(\beta l) = 0$

$$Z_{in} = Z_L$$


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Let us look at two more cases here. One case is where length is actually equal to lambda by 4. So, when length is lambda by 4 that will be tan beta L which is tan 90 degree this term will become infinity. And if you now, substitute this particular thing in the previous equation then Z in become Z 0 square by Z L and this particular thing is known as quarter wave transformer. Let just take an example.

Suppose if Z L is equal to hundred ohm and we need Z input equal to 50 ohm. Why we need Z input 50 ohm? Because majority of the sources have the output impedance which is equal to 50 ohm; so if I want to connect one particular source to this particular load 100 ohm then maximum power transfer will not take place.

However, if we use this concept of quarter wave transformer and let us see what happens now. So, we know Z L 100, desire is 50 ohm we use this particular equation. So, if I use a characteristic impedance Z 0 of impedance 77.7 ohm that means, a load impedance hundred ohm is transfer to source impedance or input impedance of 50 ohm which will be matched to this. And since the line is lossless there will be no losses in the line, so that means now whatever is the power coming from the input side will be deliver to the load because the transmission line is lossless.

I just want to mention one more case over here this particular case is useful many a times when you want to have a proper layout of the particular system. So, for example, if length is equal to lambda by 2. So, if you now put in tan beta L it becomes 0 and if this is

equal to 0 then  $Z_{in}$  become  $Z_L$  so that means, input impedance will be equal to load impedance if you choose length equal to  $\lambda/2$ , and this particular this is independent of the characteristic impedance of the line.

In fact, something like this we have to use specially in one of the application using series fed micro strip antenna array and in between we have to use the length which was  $\lambda/2$  to provide 180 degree phase shift. So, one can provide a 180 degree phase shift at the same time input impedance remain same as load impedance. So, if we use  $L$  equal to  $\lambda/2$  then input impedance becomes equal to load impedance.

So, today we have looked into coaxial line, micro strip line, strip line in brief and we saw little bit in detail about the waveguide. We looked into the comparison of different transmission line, and after that we talked about simple model of a transmission line, and then we looked at different cases case 1 2 3 4 5. And we actually saw that by using a small shorted transmission line we can realize a inductor or if we use small open circuit line we can actually realize capacitor. And we can use a  $\lambda/4$  line to transform any arbitrary impedance to some other impedance by using the characteristic impedance of the line. So, any load impedance can be transfer to the desired input impedance by using a properly design micro strip line or strip line or coaxial line of a given characteristic impedance.

So, in the next lecture we are going to look at various things related to quarter wave transformer, single section, multiple section; we will also look at smith chart, reflection coefficient, VSWR and so on.

Thank you very much. See you next time. Bye.