Foundations of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-1.
Lecture -1.3.
Haar wavelet.

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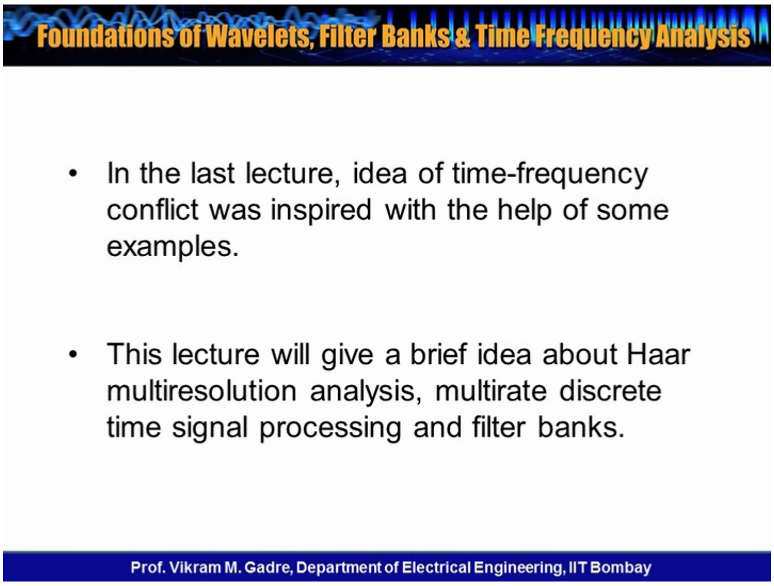

So we are going to build up the whole idea of wavelets by starting from what is called the Haar multiresolution analysis. Haar incidentally is the name of a mathematician, call him a mathematician, call him a scientist, what you will but one of the beautiful things that this gentleman proposed was that what is called the dual of the idea of Fourier analysis. What do we do in Fourier analysis, we allow even discontinuous waveforms, we allow non-smooth waveforms and we convert them into sum or a linear combination of extremely smooth functions, namely the sine waves.

Haar said can be do exactly the dual? Can we in principle take smooth functions and convert them into a linear combination of effectively jagged or discontinuous functions? Why on earth would one like to do something like that? Again, let us reflect for a minute. A few years before this might have seemed silly to do but today it is not. What are you doing when you are doing digital communication? You are transmitting audio, you are transmitting pictures and you are doing all this actually with a large number of discontinuity.

How does one record digital audio? 1 Firstly samples, so one takes values of the audio signal at different points in time, one digitises them and one then records those digital values as a stream of bits, all of these are highly discontinuous operations. You are forcibly introducing discontinuity in time and on top of that you are introducing discontinuity in amplitude by quantisation. So wanting to represent the beautiful smooth audio in terms of very discontinuous Bitstreams is very very beneficial to digital communication.

And in fact none of us complain, when we have a good digital audio recordings, sometimes we even say that a digital audio recording is better than analog recording as we had in the past. So going from smooth to nonsmooth has its place in modern communication and signal processing. And when Haar proposed that one should look at the whole philosophy and the whole principle of being able to go from smooth to nonsmooth, perhaps he was looking into the future, when this would be absolutely essential.

What we are going to do in the very 1st few lectures immediately following this is to look at one whole angle of wavelets and multirate digital signal processing based on the principles that Haar propounded. So we are going to look at what is called the Haar multiresolution analysis. And in fact if we understand the Haar multiresolution analysis in depth, we actually land up understanding many principles of wavelets, many of the essentials of multirate processing, specifically what is called to band processing very well.

So we shall draw upon the Haar multiresolution analysis to understand some of the basic concepts that underlie this course. And of course build upon them further later on. From the Haar we shall progress to slightly better multiresolution analysis, better in what sense we will understand. And there are many different families of such better multiresolution analysis, one of them being what is called the Dobash family. Dobash is again the name of a mathematician scientist who proposes that family of multiresolution analysis.

As I said at a certain point in the course immediately following this, we shall then look at the uncertainty principle, fundamentally and in terms of its implications. From there we shall move to the continuous wavelet transforms. So in the Haar multiresolution analysis, we have a certain discretization in the variables associated with the wavelet transform. Later on we shall go to what is called the continuous wavelet transform where the variables, the independent variables that are associated with the wavelet transform, all become continuous.

Following that we shall look at some of the generalisations of the ideas that we build up earlier in this course and towards the last phase of the course we shall look in-depth at some of the important applications to which wavelets and multirate digital signal processing provide great advantages. Now I would like to spend a little while in this lecture on building up in parallel some of the developments that took place to introduce the subject of multirate digital signal processing. What is multirate, what rate are we talking about here?

And why do we need to talk about multirate, why is it connected with wavelets? Let us go back to the audio example or maybe let us 1st go to the biomedical example. In the biomedical example we said we would have quicker parts in the response and slower parts in the response. The slower parts of the response are likely to last for a longer region in time, the quicker parts of the response are likely to last for smaller regions in time.

So here other than the concept of being able to localise on a certain region of time and of course correspondingly on frequency, there is also the distinction between what kind of localisation is required for higher frequencies and lower frequencies. If we spend a little bit of thought and time in understanding these 2 kinds of components, we will realise that most of the time when we talking about slower parts of the response or lower frequencies we are talking about compromising on what is called the time resolution.

So I bring in the idea of resolution here. Resolution means the ability to resolve, the ability to be able to identify specific components. So for example frequency resolution relates to being able to identify specific frequency components. And going further and being a little more pinpointed, when I am talking about frequency resolution, what I am saying in effect is, suppose I have 2 sine waves whose frequencies start coming closer and closer together. Over what region of time do I need to observe them so that I can actually identify 2 frequencies separately.

How can I resolve the 2 frequencies? How much can I narrow down on the frequency axis? Now what we are talking about is not so much how much we can narrow down but how much we need to. When we talk about higher frequency content or things that vary quickly, it is often, though not always the case that we are willing there to compromise on frequency resolution but we want time resolution. So things that take place quickly and are transient, short lived, demand time resolution and things that occupy the lower frequency ranges which last for a long time demand frequency resolution.

So, very often it is true that were one goes down on the frequency axis, one demands more frequency resolution, the ability to resolve frequencies more accurately as opposed to time resolution, the ability to resolve which time segment it occurs. And when one goes to higher frequencies, one normally demands more time resolution and lesser frequency resolution. One is asking for how closely one can identify 2 segments or 2 parts of the waveform which vary quickly, that means one is trying to narrow down on the time axis and ongoing so, one must compromise on the frequency axis.

So this is what brings us to the idea of multirate processing. You see, it means that when I talk about bands of higher frequencies, I must use smaller sampling rates in a discrete time sampling system. When I am talking about lower frequency ranges, I must use larger time sampling points or sampling intervals. Why must I do so, to be most efficient in the processing operation. When I am talking about lower frequencies, so in an evoke potential waveform if I am trying to look at the slower components, I should not unnecessarily Sample too frequently.

It only increases my data burden and does not offer me anything special. On the other hand, when I am analysing the quicker components, it is inadequate to use a low sampling rate. I would be doing injustice to the components. For those of us who might be exposed to the concepts of sampling and aliasing, if I am not faithful in my sampling rate of the quicker components, I would introduce aliases, I would introduce spurious effects which I do not want. So all in all we recognise that it is not a good idea to be using the same sampling rate for all frequency components.

So unlike a basic course on discrete time signal processing where we assume all sequences are at the same sampling rate, here we need to deal with sequences that are effectively at different sampling rates in the same system. That means we also need to deal with systems that operate with different sampling rates and that is why we talk about multirate discrete time signal processing. Now at a conceptual level we understand very well why there is a close relationship between multirate discrete time processing, the idea of uncertainty, requirement of resolution.

And if we go further, then, when we do multirate discrete time signal processing, we also bring in a new concept of filter banks versus filters. As I said, if we go back to the biomedical example, there is the effect or there is the desire to separate components. So when I wish to separate components, naturally I wish to have many different operators all at once. So I need a system of filters which not only have certain individual characteristics but which also have collective characteristics.

So I need to be able to analyse and then synthesise and all this with localisation included. This is what we mean by a filter bank. So a bank of filters as opposed to a single filter in discrete time signal processing refers to a set of filters which either have a common input or a common point of output or summation output. This concept of a bank of filters and in fact 2 banks of filters, an analysis filter bank and a synthesis filter bank taken together is very central to multirate discrete time signal processing.

We shall be looking at that concept in great depth, so we shall be building up the idea of a 2 band filter bank in reasonably great depth in this course. The concept of a 2 band filter bank is of great importance in being able to construct wavelets. In fact, we shall see even from the Haar multiresolution analysis example that there is an intimate relationship between the wavelet or the Haar wavelet and the 2 band hall filter bank. So much so that if I construct a properly designed 2 band filter bank, I also construct a multiresolution analysis that goes with it, so to speak.

All this is very exciting and what we intend to do in the lectures that follow from here is to take these concepts one by one. So in the next few lectures we intend to talk about the Haar multiresolution analysis to build up certain basic ideas from it, in the lectures that follow, as I said before, we intend to talk about the uncertainty principle and its implications. Following that we move from discrete to the continuous wavelet transform and then generalise both the continuous and discrete wavelet transforms to a broader class of transforms.

With that then we come to the end of this 1st introductory lecture on the subject of wavelets and multirate digital signal processing. And proceed therewith in the next lecture to talk about the Haar multiresolution analysis. Thank you.