Foundations of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-1.
Lecture -3.3.
Scaling Function for Haar Wavelet.

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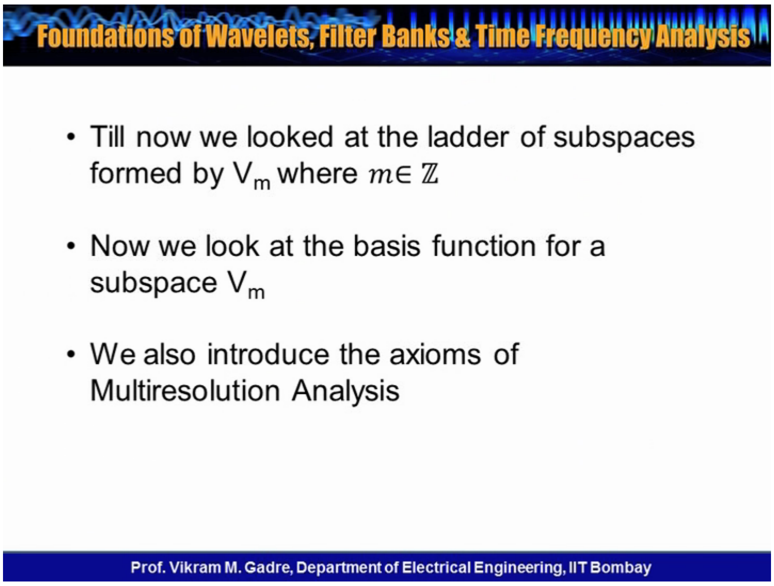

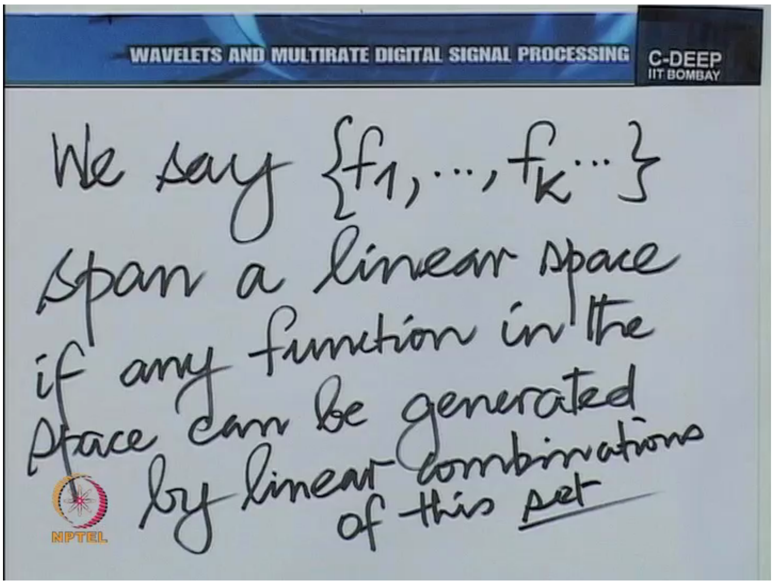

You know, yesterday I told you that there is this beautiful idea about just one function psi t, its dilates and translates going all over to capture incremental information. now we need to state that formulae too but in order to move in that direction, we 1st need to bring in as I said another function which will span V0. So we need to bring in this idea of spanning. We say a set of functions, we say a set of functions, let us say f1 to let us say fk and so on, span a linear space, if any function in that linear space can be generated by the linear combinations of this set.

Now again there is this subtle distinction between finite linear combinations and infinite mere combinations, I do not wish to dwell on those distinctions at the moment. But what we are saying, what we mean by span, so when we talk about the span of a set of functions, we are talking about all linear combinations of those functions and therefore the set or the space, in fact the space of functions generated by all linear combinations of that set. So, now we ask a question which should make our life easy.

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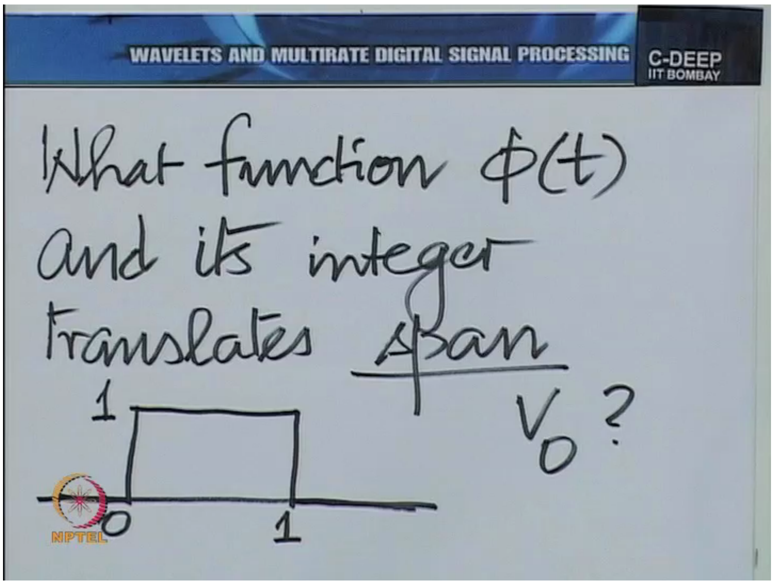

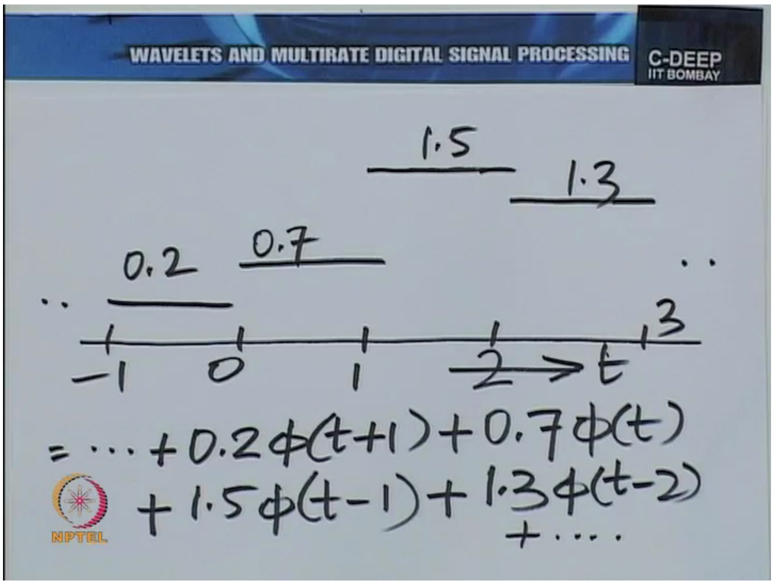

What function, we asked this question before but now we answer it, what function, suppose we call it Phi t and its integer translates span V0? And the answer is very easy, in fact if you were to visualize a function which is 1 in the interval from 0 to 1 and 0 else, lo and behold, you have the answer. So what we are seeing is any function in V0 can be written like this, summation n over all the integers CN Phi t - n, so essentially integer translates of Phi.

And these are the piecewise constants here. Just to fix our ideas, let us take an example here. So what we are saying is, for example, suppose I have this example of a function in V0, alright, so I have 0, 1, 2 and 3 and so on. The values here is let us say 0.7, the value here is 1.5, the value here between 2 and 3 is 1.3, the value between -1 and 0, let us say is 0.2 and so on, then and this could continue, this function can be written as, well dot dot dot +0.2 times T +1+0.7 times Phi t +1.5 times Phi t -1+1.3 times Phi t -2 and + dot dot dot and so on. Simple enough, not at all difficult to understand.

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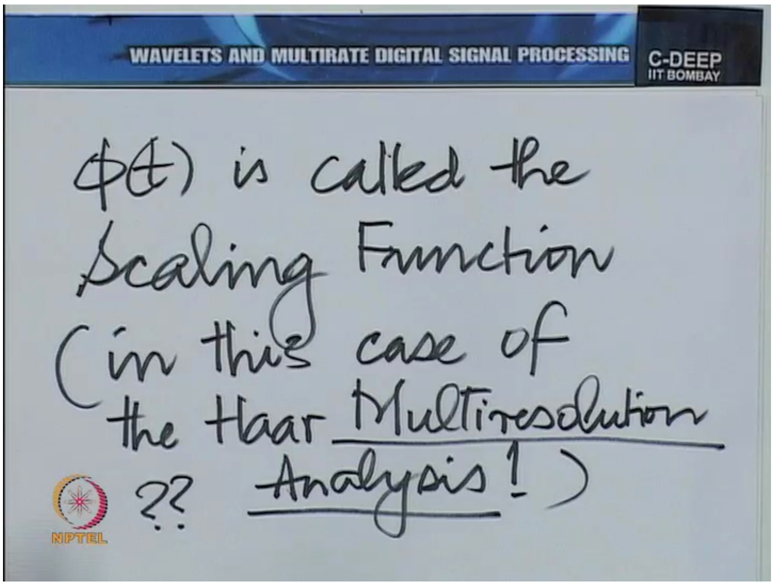

So we have this single function Phi t whose integer translates span V0. now the subtle point is that if you were to go to any space Vm, so the same thing would carry forth. So it is very easy to see that any space V can be similarly constructed. In fact they can be more precise, we can write down Vm is essentially the span overall n belonging to Z of Phi 2 raised to the power m t - n. So just as we looked at the wavelet yesterday and said it is a single function which can allow details to be captured, we now have this function Phi t which captures representations at a resolution.

It is a very powerful idea if you think about it. If you want to capture information at a resolution, at a certain level of detail that is all the information upto that resolution, you have the function Phi. If you wish to capture the additional information in going from one subspace to the next, you have the function psi, the wavelet. Now we need to give this function Phi T a name, we shall call it a scaling function. Now of course here, the Phi T that I had drawn in this context is the scaling function of the Haar wavelet for the Haar multiresolution analysis.

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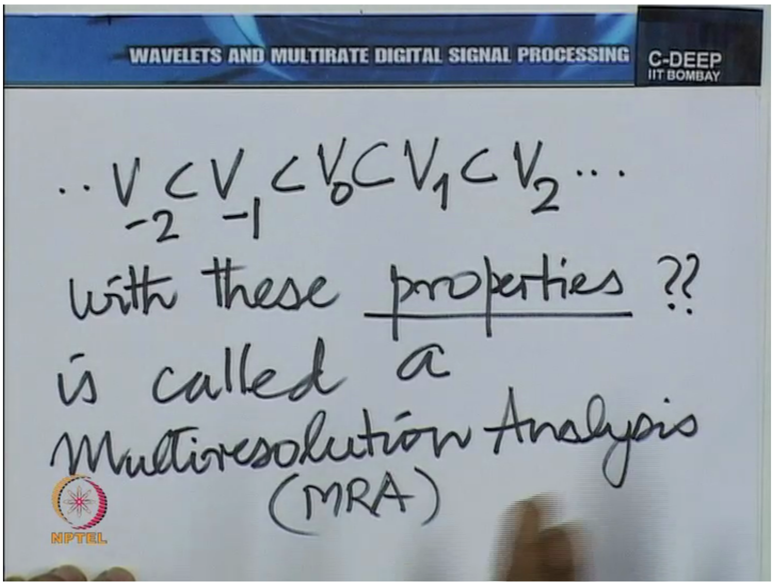

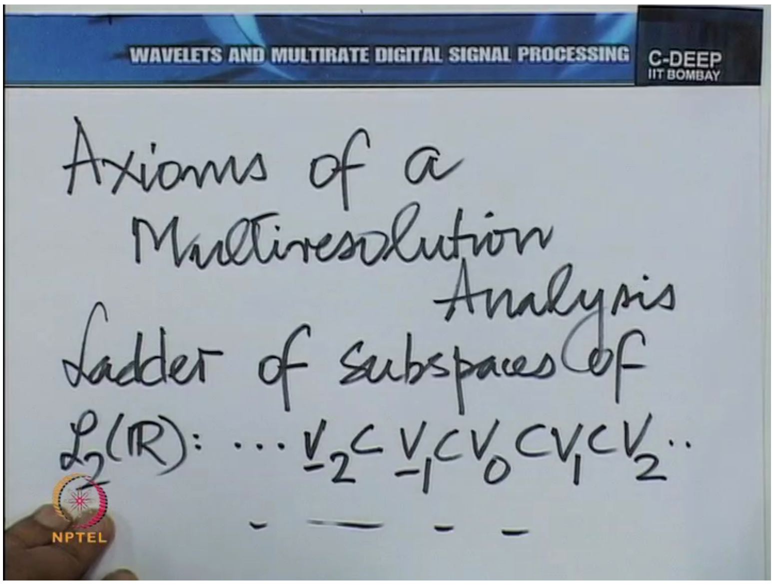

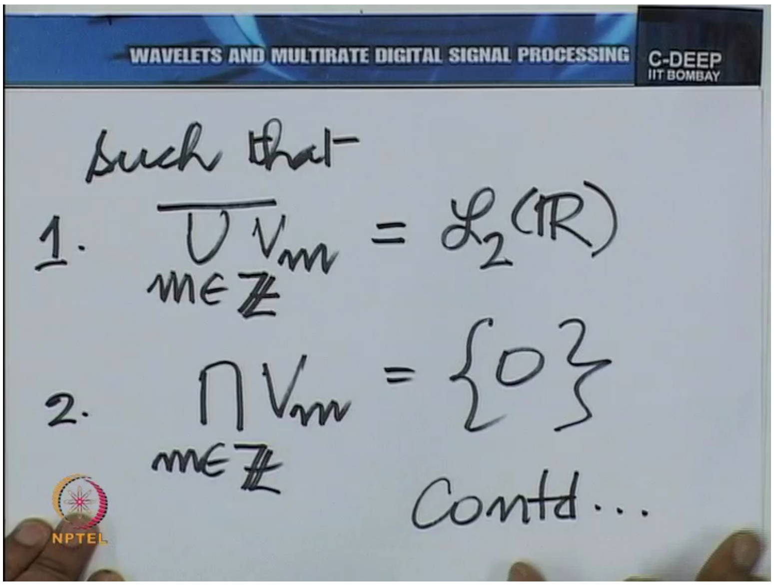

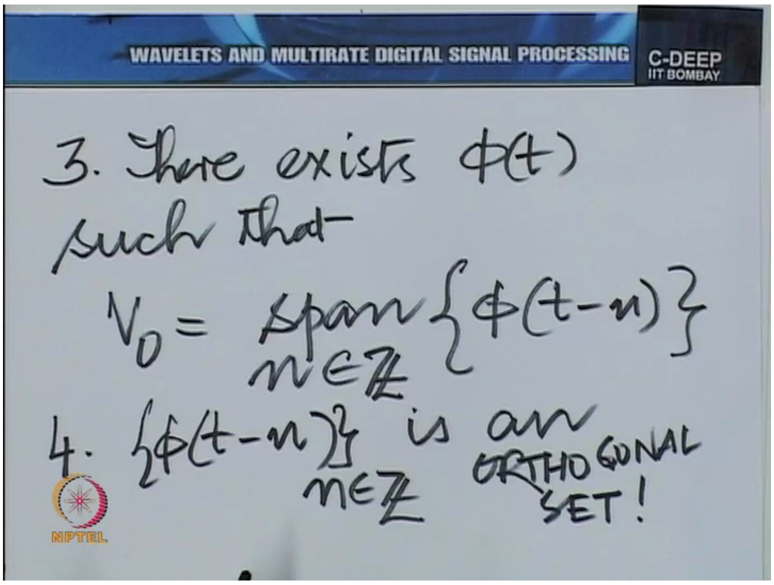

Now what is this multiresolution analysis, I have suddenly brought in this word. So what is this? Well, this ladder of subspaces that we are talking about here with these properties is called a multiresolution analysis or an MRA for brief, of course in this case the Haar multiresolution analysis. Now what properties, we need to put them down formally once again. We have introduced 2 of them and one more subtly, now we need to put down axioms very clearly.

So let us put down what are called the axioms of a multiresolution analysis. So the 1st axiom is of course that is a ladder of subspaces of L2R and we know what the ladder looks like, such that, axiom number-one union over all integers closed Vm is equal to L2R. Intersection overall integers Vm is the trivial subspace with only the 0 element. These are not all, further… There exists a Phi T such that V0 is the span over all integer n of Phi T - n. Point 4, in fact you know it is not just span, there is something more.

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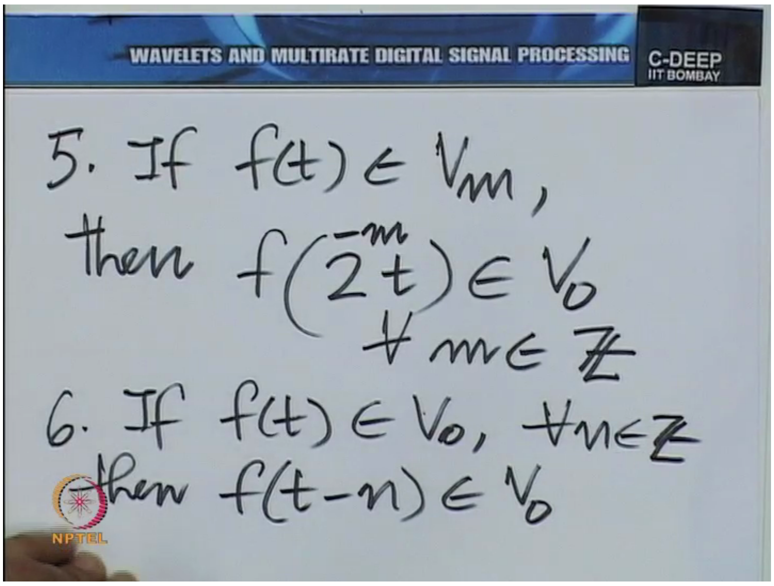

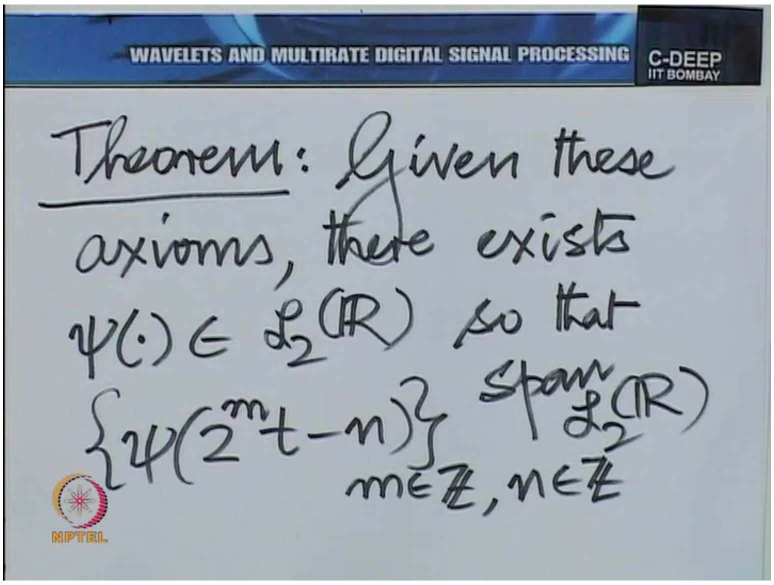

This Phi T - n over all n is an orthogonal set, this is a deeper issue here. Now we will explain in more detail the notion of orthogonality in the next lecture but for the moment let us be content to put this down as an axiom. Next, if ft belongs to Vm, then f 2 raised to the power MT, 2 raised to the power - MT belongs to V0. So for example if ft belongs to V1, then ft Y2, or f 2 raised to the power - 1 T belong to V0 for all M belonging to Z.

And if ft belongs to V0, then ft - n also belongs to V0 for all integer n. So these are the axioms of multiresolution analysis, that means this is what constitutes the multiresolution analysis. And here we have taken the Haar multiresolution analysis to build the idea up but the whole abstraction is that we can have several different phis and then to end this lecture, the corresponding psis. So where does the psi come in, it comes in what is called the theorem of multiresolution analysis.

Given these axioms, there exists a psi belonging to L2R so that psi 2 raised to the power mt - n for all integer m and all integer n span L2R. This is a very very significant idea, in other words this is exactly what we said yesterday. Take dyadic dilates and translates of a function psi and you can cover all functions or go arbitrarily close to any function in L2R as you desire. We have built this idea up from the Haar example but in the next lecture we shall try and build a little more abstraction into what we have done and proceed further from there. Thank you.