Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Tools in –D data Analysis (STFT and CWT).

Good afternoon everyone, I am Shivam Bharadwaj of the Department of electrical engineering, IIt Bombay, I am the TA for the course foundation of wavelets, filter banks and time frequency analysis. Today I am here with you to discuss some of the important time frequency data analysis tools like STFT or widowed Fourier transform and CWT or continuous wavelet transform.

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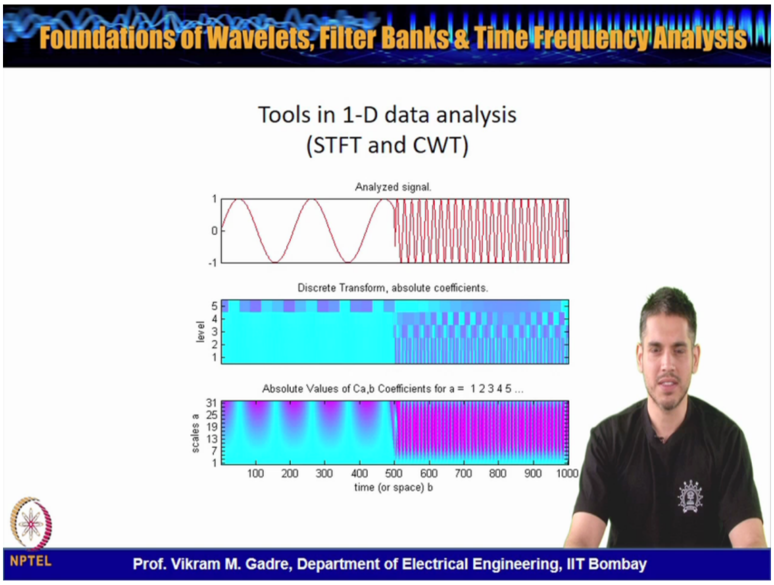

We will simultaneously discuss the advantages and limitations of these 2 techniques over the conventional signal processing techniques like Fourier transform. So let us begin with, now consider the following 2 signals x1t and x2t and take 10 to 15 seconds to look at these 2 signals closely. Now as one can expect these 2 signals are completely different in their behaviour as far as time domain is concerned. In the 1^{st} single or x1t, there are 4 different sinusoids of different frequencies which are present throughout the time interval, whereas in the 2^{nd} signal the sinusoidal components occur at different points in time.

So, one can easily noticed these few points, both the signals have the same frequency components, the occurrence of these different frequency components is different for both the signals. Now what do we want to know is precise occurrence of these frequency components that are present in the signals, okay. Let us begin with in fact Fourier analysis and see if Fourier analysis can achieve this goal or not.

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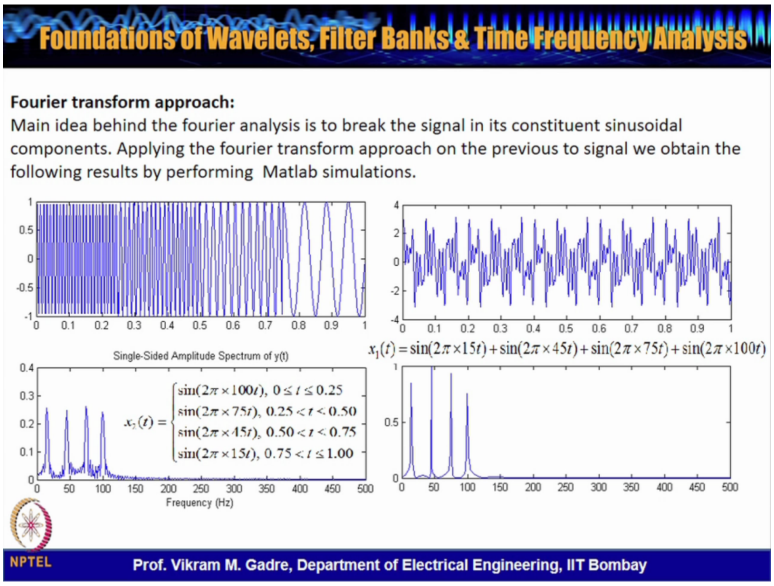

So the main idea behind the Fourier analysis is to break the underlining signal in its consequent sinusoidal components. So as we go ahead and apply the Fourier analysis algorithm which is readily inbuilt in Matlab, we obtain the following 2 results. So, as we can see on the left-hand side you can see the signal x2t and on the right-hand side you can see the single x1t. Now as we have seen in the equation, underlining equation of these 2 signals, the sinusoidal components in x1t are present throughout the time, and whereas in the 2nd signal x2t, these 4 sinusoidal components occur at different instants in time.

But, now if we go ahead and look at the magnitude plot or the amplitude spectrum of both these signals, what we will see is the thing which is represented before. This figure Fourier transform clearly indicates information about the different frequency components which are present in the signal. But the exact time, when these components are occurring is not brought out by the Fourier analysis. So this is one of the main limitations of the Fourier analysis. I would like you all to think as to why this problem arises with Fourier analysis.

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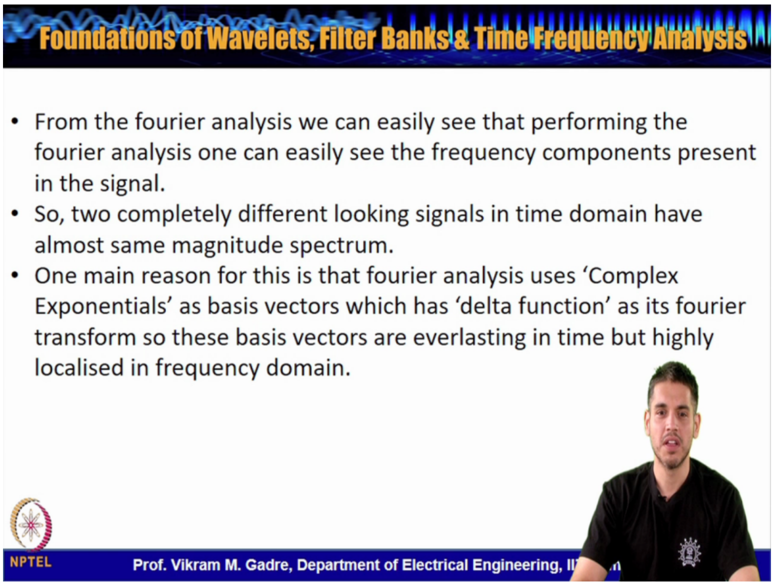

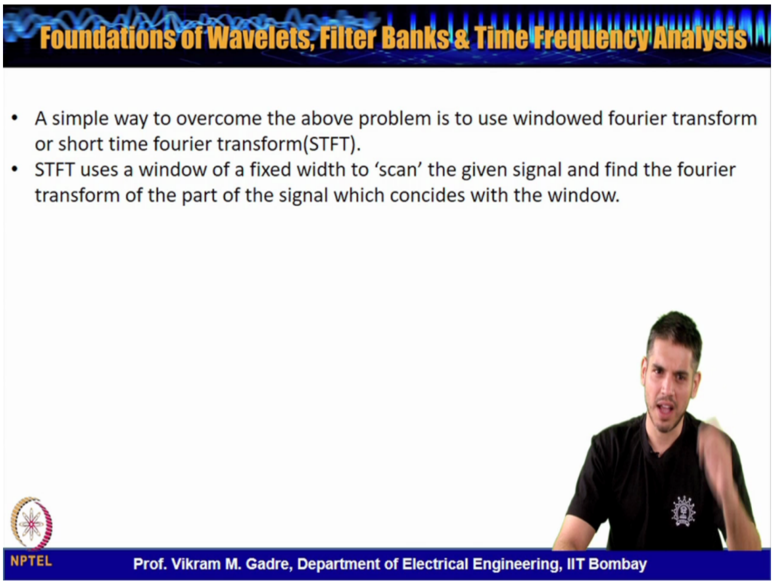

One hint is that if we look closely at the Fourier analysis formula, than the basis that we are using in Fourier analysis are the complex exponentials. And complex exponentials are made of sine and cosine waves which are everlasting waves in time domain but if we look those at those waves in frequency domain, then we have 2 impulse for cosine and similarly 2 impulses for sign. This means that the frequency localisation of sine and cosine wave is better as compared to their time localisation.

And this is the primary reason why the Fourier analysis tells you the precise frequency components but is not able to bring into the light the precise occurrence of these frequency components. Now, can we overcome this problem? The answer is sure, yes, we can of course overcome this problem by using a technique which is popularly known as widowed Fourier transform or short time Fourier transform.

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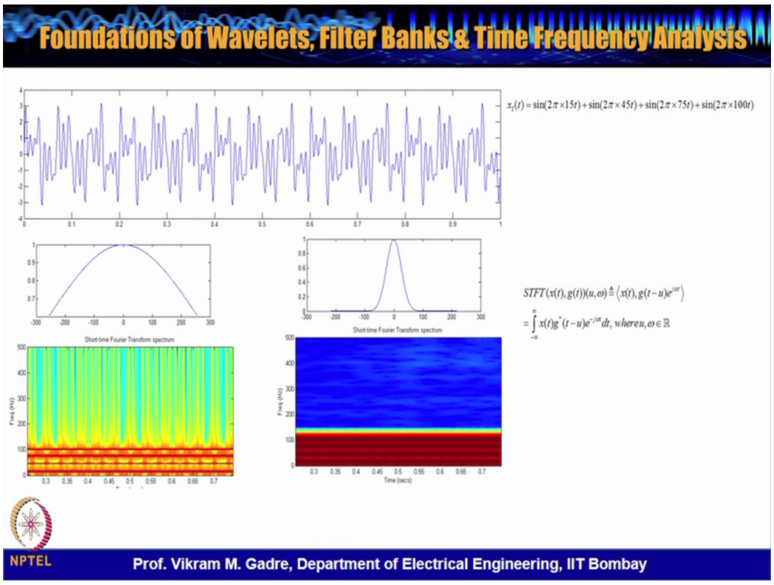

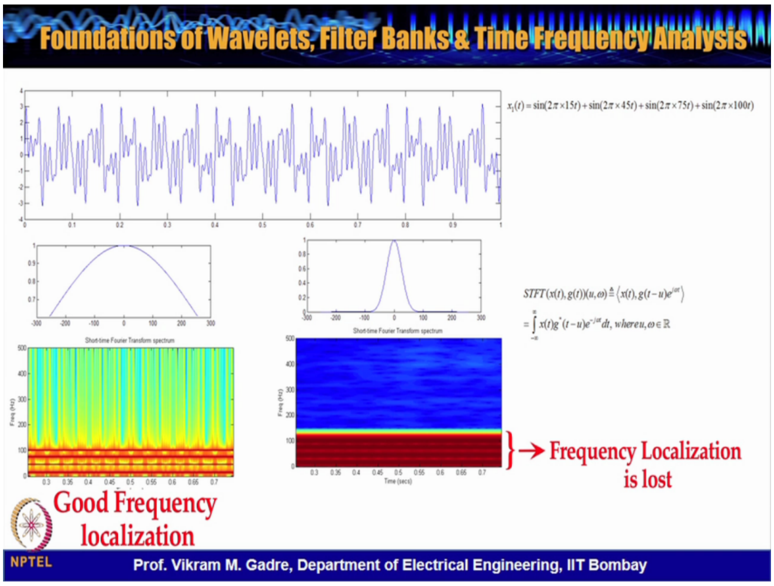

So as you can see now that our and aligning signal which is x1t, as you can see all the sinusoidal components of 15 hertz, 45 words, 75 hertz and 100 hertz are present throughout the time from 0 to 1. So, now if we go ahead and look at the STFT analysis equation which was taught by Sir in the class, from this analysis equation we can easily see that in STFT analysis, we have to 1st choose a window with respect to which we want to do the short time or widowed Fourier transform. This window we have represented by Gt and xt in this equation represents a underlining signal x1t.

So now we are considering 2 different windows for our analysis. For the 1st window which is on the left-hand side, you can see that it has a wider time duration, whereas the 2nd window which is shown on the right-hand side, you can see that it is of lesser time duration. Now, from our Fourier analysis concepts, we know that if we use a window of wider length, wider time duration, then a frequency localisation is improved. On the other hand if we use the window of shorter time duration, then our frequency localisation is lost, which is clearly visible in these figures, which is clearly visible in the figures as you can see that on the left-hand side the spectrogram with respect to a wider window, you can clearly identify on the Y axis which reads of the frequency, the different frequency components which are present in the signal.

While on the right-hand side, the analysis which is done with the help of a shorter time length window, the frequency localisation is lost. Since in this signal the difference sinusoidal components were present for all time duration, we might not as well care for time, good time localisation, our aim should have been to identify the different frequency components and that can be easily be done by Fourier transform also and by short time Fourier transform if you are using, then it is highly advisable to use a larger window length.

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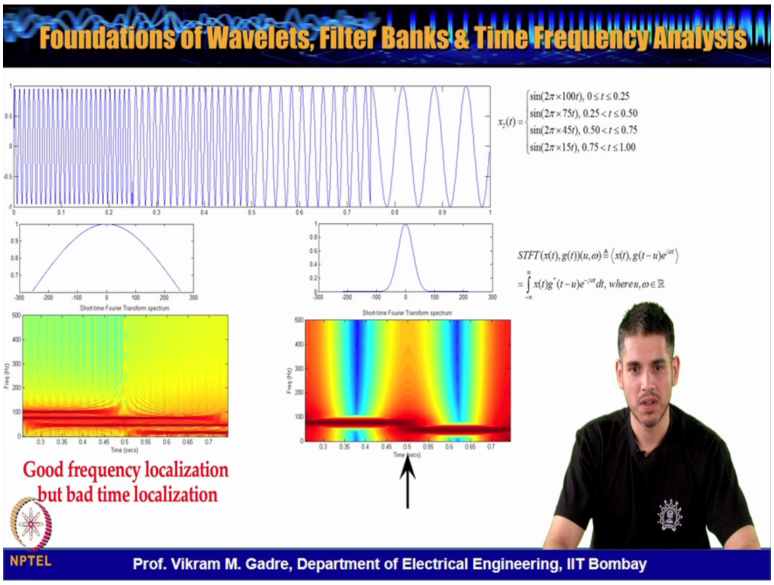

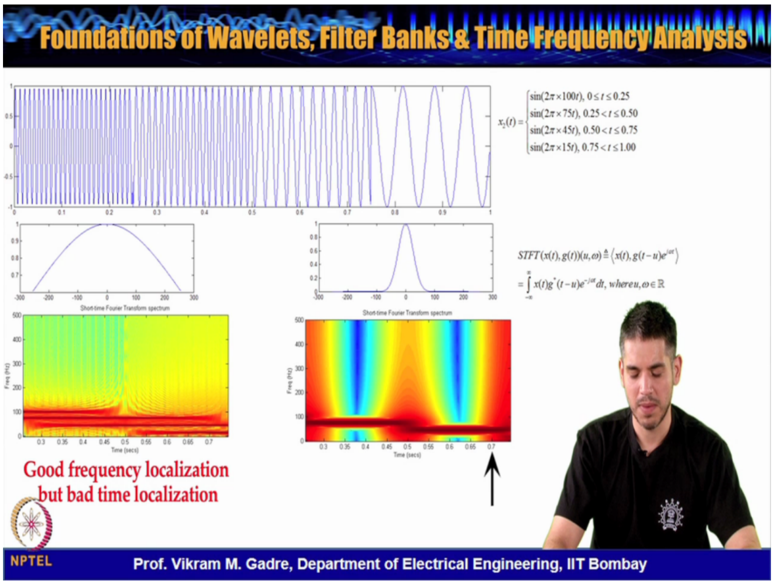

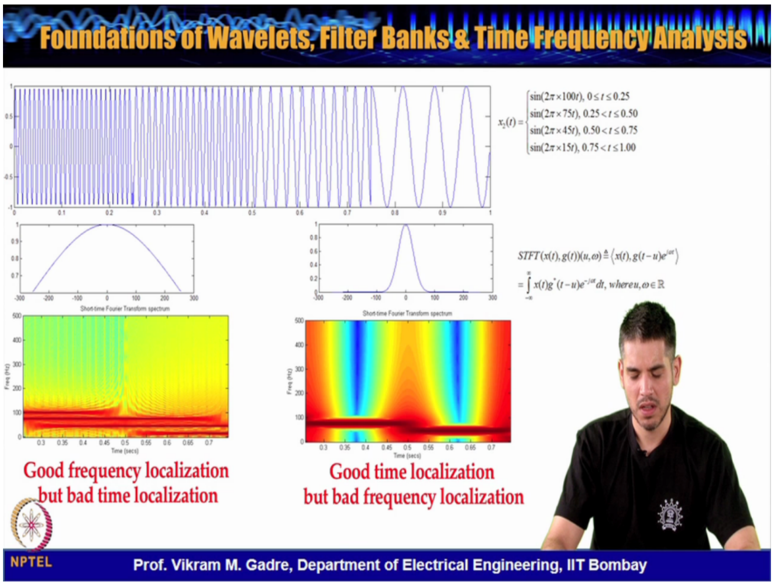

Now, consider this signal, in this signal x2t, different different sinusoidal components occur at different instants of time. For analysis again, we have chosen 2 types of windows, type I window is the window which is shown on the left-hand side and again it has a wider time duration, so because of its wider time duration, doing STFT analysis with this window will more precisely tell as the different frequency components but the occurrence of time information is lost.

So as you can see the time information is not clearly brought out by our STFT analysis but the frequency information which is represented by these black arrows is brought out exactly by this analysis. Whereas if we look on the spectrogram which is on the right-hand side below the window which is of lesser time duration, we can exactly point out the occurrence of the time of these frequency components, which is being done by the use of these black arrows.

Now again, what is the limitation of this, this seems to be a very appropriate and advanced technique as compared to Fourier transform but again there are certain limitations. Once you fix the window size, any transient which is lesser than that window size, that analysis will not be able to capture the underlining frequency visiting those transients. So what is the next, what the next step can be?

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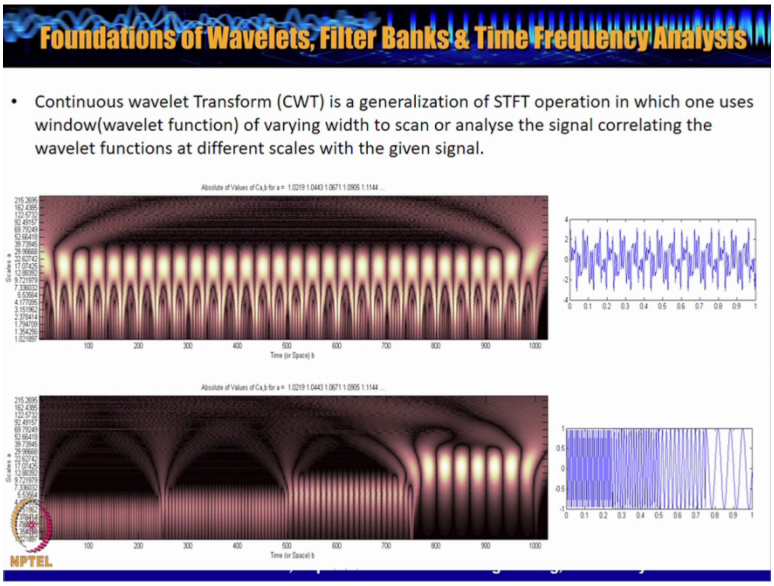

If we are able to change the window length in the runtime, in the scanned runtime, then we can somewhat obtain a better analysis and this is what is brought out by CWT or continuous wavelet transform. If we do the CWT analysis in Simply Matlab, then the following kind of spectrogram will be provided to you by the wavelet toolbox of Matlab. Now this figure if you look closely, it has 2 axis, one is the time-space and another is the scale. Now one can ask the following question that where is the frequency information and where is the time information.

So time information can be read out directly from the x-axis. Here the time spaces written from 0 to 1000, if we go ahead and normalise and divide every point on the x-axis by 1000, we can exactly obtain the time axis as 0 to 1 and you can read out the time information, whereas Y axis is marked as scale. Now, the scale, as in windowed Fourier transform are here again the indication of the frequency components present in the signal or the frequency components which are brought out by the CWT analysis.

And by slight tweaking of commands in Matlab, you can actually convert this scale or the y-axis readings into their frequency, respective frequency reading, there is a slight trick in Matlab, I encourage you all to go ahead and try that trick and convert instead of scales on the Y axis, try to have frequency readings on the Y axis. Now for the 1st single again, as we can see, that some part of the scale axis always occur throughout the x-axis.

Means some parts from scale 1 to scale as you can see to approximately 30, scale 30 are occupied throughout the time axis. So, if you convert these scales into frequency by using that Matlab trick, you can find that from what frequency to what frequency are present, means you can find the frequency range of the signals that are present in the signal, underlining signal to be analysed. Now, in the 2nd figure which is shown below, you can precisely see that this figure brings out the precise occurrence of different frequencies at different instances in timescale.

You can see that for one 4th of the time, starting one 4th of the period in the signal, there is one frequency present and that frequency can be easily read out from their respective scales. Similarly for another quarter, there is another scales, another level of scales or another frequencies that are present and similarly for the 3rd quarter and fourth-quarter. So what have we seen till now?

We have seen that CWT analysis because of its advantages that you can vary the window length in the runtime of the computation, CWT analysis is preferred when you want precise time-frequency analysis, precise time frequency analysis. Again, neither STFT is bad, nor CWT is bad, there are some applications where there is favourable to you CWT and there are certain information, use of STFT is highly recommended in literature. Thank you.