Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-8.
Lecture-21.3.
STFT: Duality in the interpretations.

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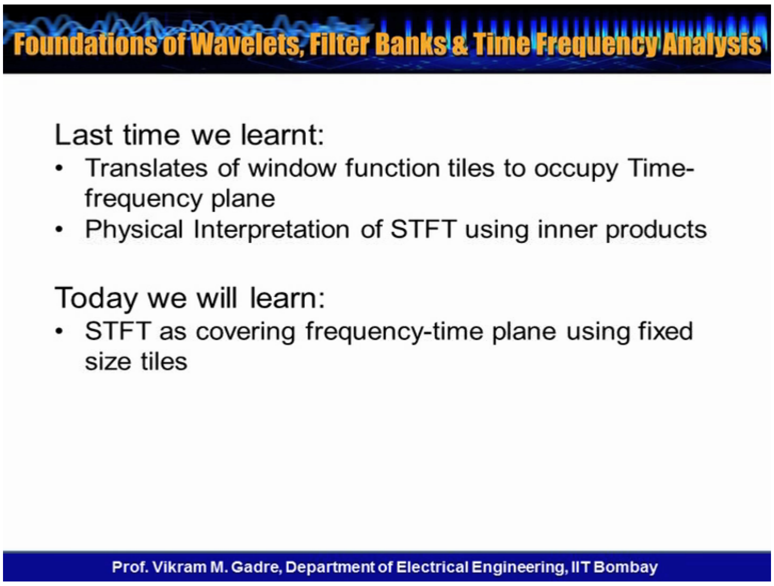

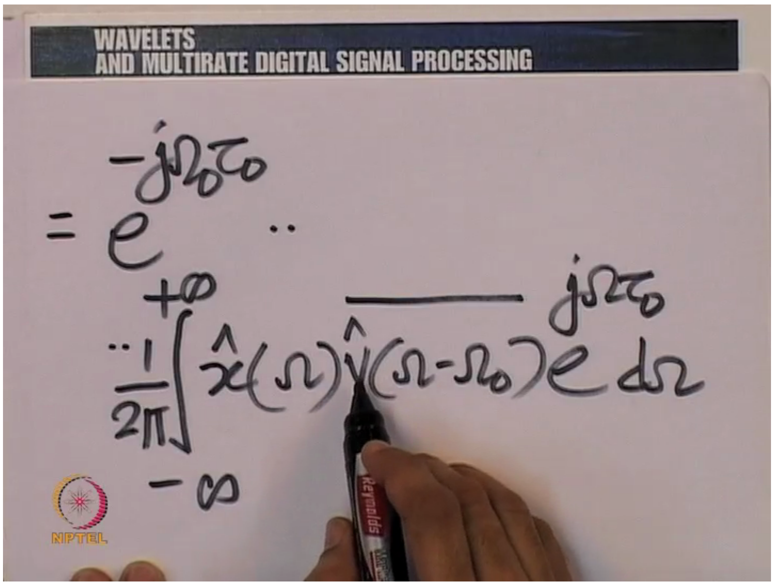

Let me put back before you the frequency interpretation 1st and then we will see how is it similar to the time interpretation that we had a few minutes ago. So the frequency interpretation is like this. You know, if you look at it, this is essentially a constant term, its magnitude is 1, this is again a constant, 1 by 2 pie, so one need not pay too much attention to these 2 terms. Essentially it is this integral which is important.

In this integral what are we doing, we are multiplying the Fourier transform of the function which is being analysed that is x, the Fourier transform is x cap of capital Omega by the Fourier transform of the window shifted to lie around R Omega 0. So we are trying to analyse the content the Fourier transform around Omega equal to Omega 0. And we are doing so at the point t equal to tau 0, if you please, because we are taking the inverse Fourier transform here.

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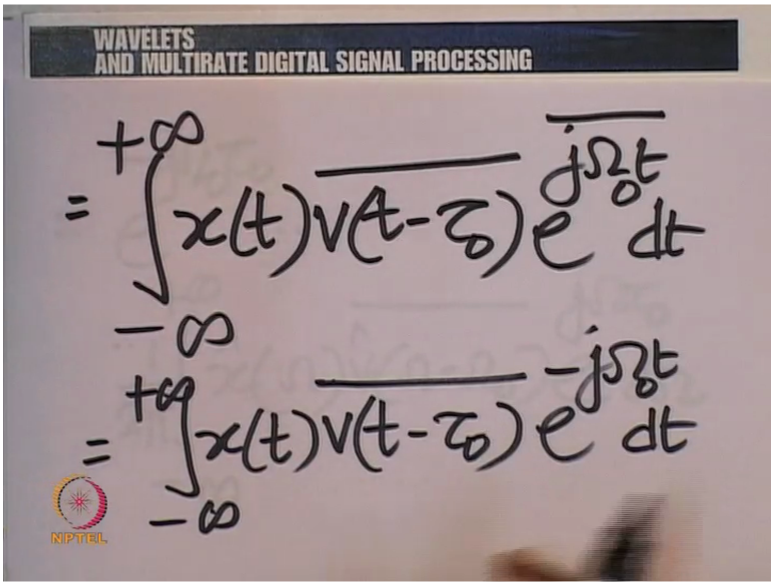

If you will just go back a few steps, we had the expression for the short time Fourier transform in time, let me put back that expression for you, that expression is here. What did we do there, we did exactly a dual thing in time. So we had this, you see if you, before we calculated, we had this essentially as a time expression and with this time expression what we did was the following. We multiply xt by a translate, so we extracted information of xt around the point t equal to tau 0 and then took the Fourier transforms at the point omega equal to Omega 0. This completes the beauty of duality in the interpretation.

We are doing exactly died do aloft what we have done in time, in frequency, that is what we have said. The operation is very similar in time and frequency. It extracts a region of time and it also simultaneously extracts a region of frequency. So in a certain sense the short time Fourier transform was a very good idea when it came. Although when they used the rectangular window, it was a bad idea. And the rectangular window was not sufficient for that reason, you had to taper off the window at both ends to make the function continues.

In fact all this is also the basis of the idea of windowed F I R filter design discrete time signal processing. We will talk about designing finite impulse response filters using Windows. Now the ideas that we have talked about here are also replicated there in a slightly different context, but let me not go into that for the moment. Come back to this point. So short time Fourier transform is one way of tiling the time frequency plane. What are the tile is doing in the short time Fourier transform, we must try and visualize them.

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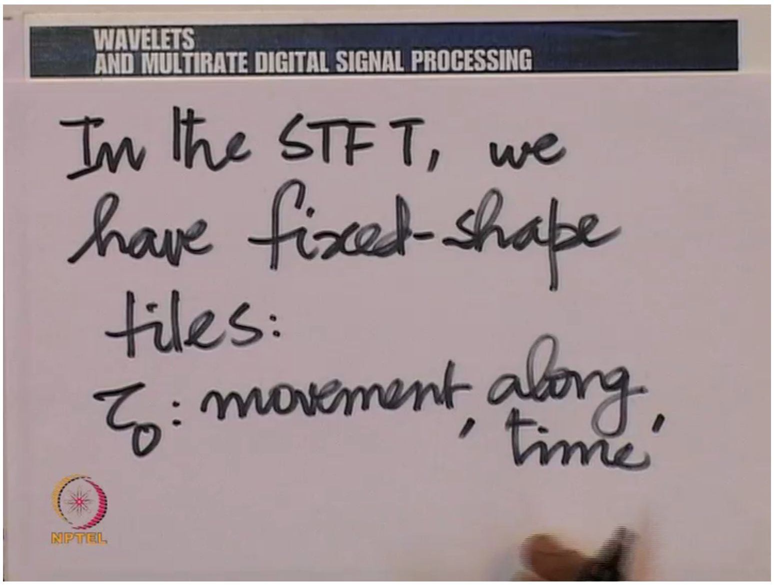

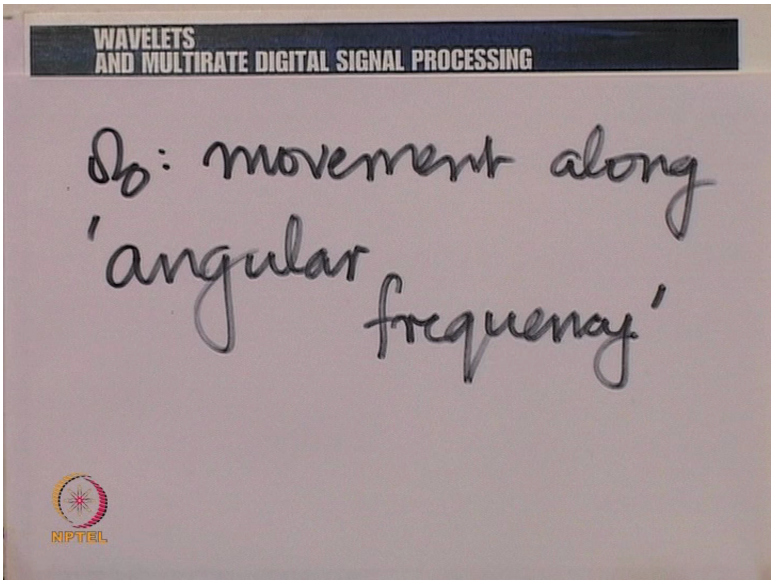

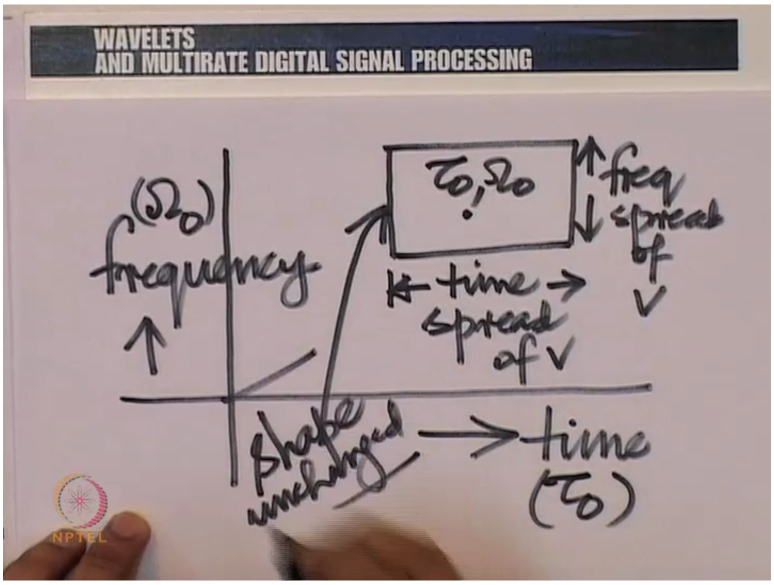

So in the STFT, we have fixed shape tiles, we call them fixed shaped tiles, tao 0 is the movement along time and simultaneously Omega 0 is the movement along frequency or angular frequency if you please. So in fact, I think it will be best to understand this graphically. What we are saying is we have this time frequency plane as we constructed the last time. And in fact the time frame is indexed by tao 0 and the frequency frame, the frequency region is indexed by capital omega 0, different values.

So at a particular town 0 and Omega 0, what the short-term Fourier transform does is to extract information in a region of the time frequency plane spread around tao 0 and omega 0 when this is indicative of the time spread or twice the time spread of V. And this is indicative of twice the frequency spread of V. And what we are saying is we can visualize this, think of this as a tile now and visualize this style as being moved along this two-dimensional plane, you know.

So forget about these axes for the moment, just visualize the time, just visualize the plane underneath as the time-frequency plane and time moving around. So different tao 0 and omega 0, that is what we are doing. The shape is unchanged, that is important, we should mark it here, shape unchanged.