Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-8.
Lecture-21.1.
STFT: Conditions for valid windows.

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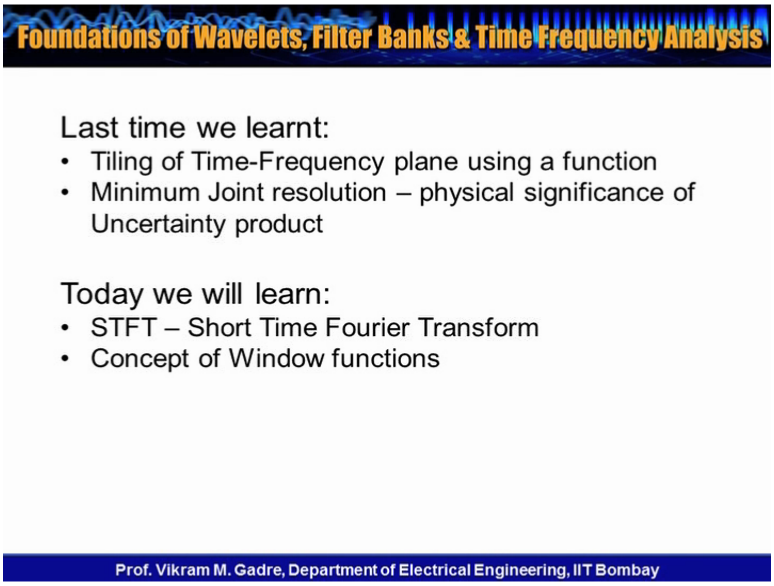

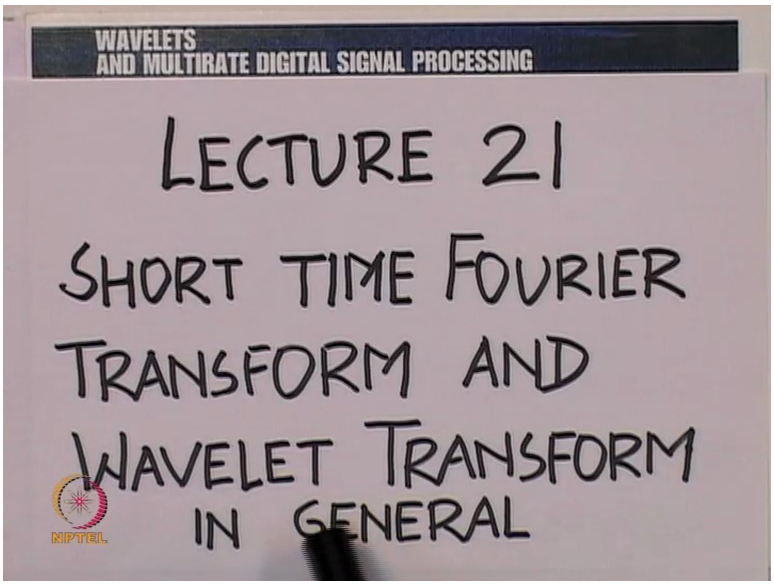

A warm welcome to the 21st lecture on the subject of wavelets and multirate digital signal processing. Let us recall in a few words what we discussed in the previous lecture and put the lecture today in perspective. In the previous lecture we had brought in the idea of time frequency plane. We could think as we saw of the time frequency plane as a floor if you please, a two-dimensional surface. And you used functions to put tiles on that surface, analysing tiles or synthesising tiles.

What the uncertainty principle told you was the smallest size that a tile could have. When I say size, I mean the smallest area that a tile could have. Of course, depending on the units that one uses, one would have different values for that smallest area. And all those limits ultimately come from the fact that the product of the time variance and the frequency variance must be greater than or equal to 0.25 or one fourth. Now what we intend to do today and that is captured in the title that I have given in the lecture today is to build a 2 kinds of transforms in a more generalised way based on this idea of tiling.

Namely the short time Fourier transform and the wavelet transform in general. So we wish to look at both of these from the perspective of tiling the time frequency plane. With that background let us go straight away to the 1st of these 2 transforms, namely the short time Fourier transform. In fact in some sense to replicate how things proceeded historically.

You know when people realised in analysing signals or in dealing with 2 domains simultaneously as we are trying to do, that there is a basic uncertainty that hits you when you try to do something like that, the 1st thing that they thought of was the simplest. Namely, if you cannot find out frequency components at a particular time, you could probably find them out over an interval of time. And the obvious way to do that is to chop, to break the signal into parts, into pieces, take the Fourier transform of each piece.

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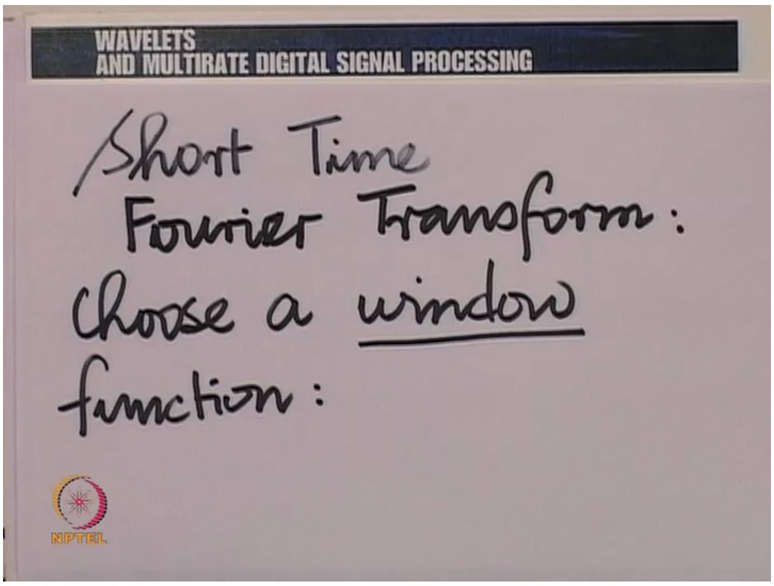

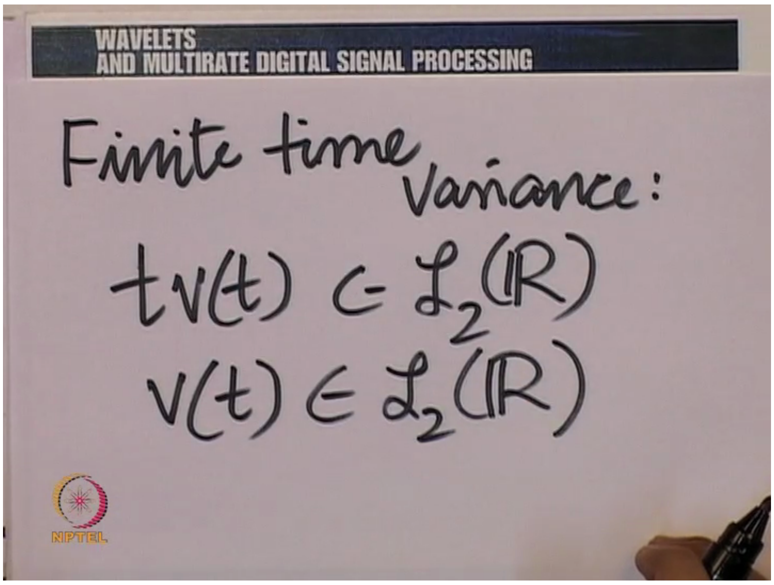

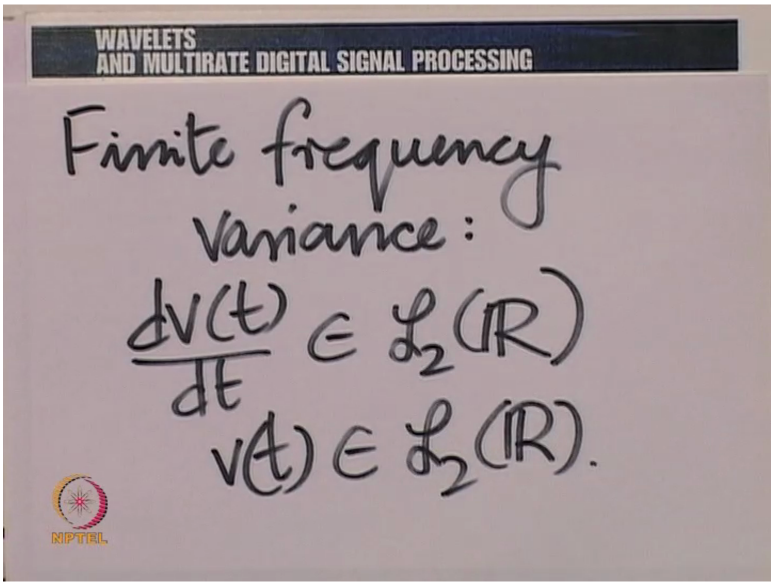

That is exactly what the short time Fourier transform proposed, let us explain this mathematically. So when people talked about the short time Fourier transform, they said, they said, well, choose 1st a window function. Let us call that window functions vt and we shall define that characteristics that we desire out of the window function. So essentially what we would want out of the window function is a finite time variance and a finite frequency variance. So let us write that down in mathematical terms.

Finite time variance, which essentially means you want t times vt to belong to L2R. Needless to say vt belongs to L2R, that is taken anyway. And we also want finite frequency variance, that means we want the derivative of vt to belong to L2R, given of course that vt belongs to L2R, let me just write that down for completeness. Now if we use this definition of a window function, the most common window or the simplest window that we have encountered so far disqualifies.

So the so-called rectangular window, the rectangular pulse in place of vt which we had been using as the scaling function in the Haar multiresolution analysis is disqualified. However we shall take it as an extreme case. An extreme case where one of these is not satisfied, the other is. So in the Haar multiresolution analysis we are using a so-called window function for the scaling function or for that matter even for the wavelet function which has an infinite frequency variance.

So it disqualifies from the point of view of frequency variance but it definitely qualifies from the point of view of time variance. So with that little background we could also take other windows, let us take a couple of examples just to give an idea. So last time we looked at the time frequency product of the triangular function, that could be a possible window.

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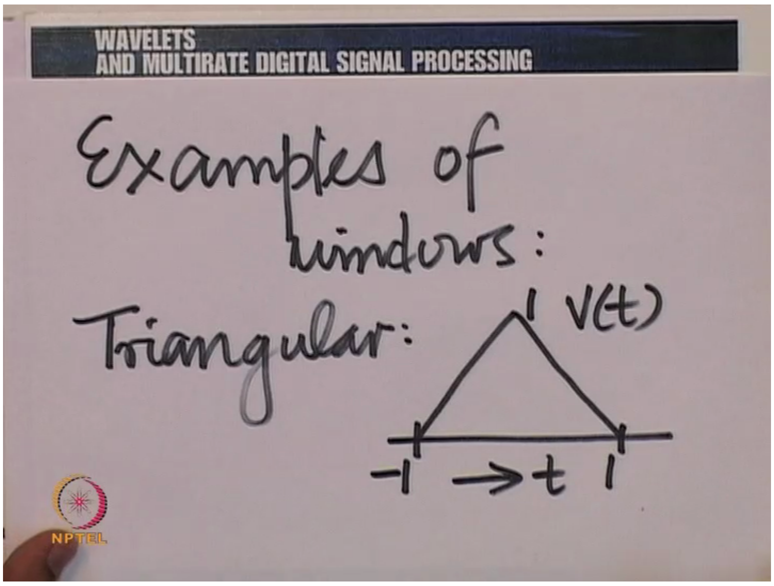

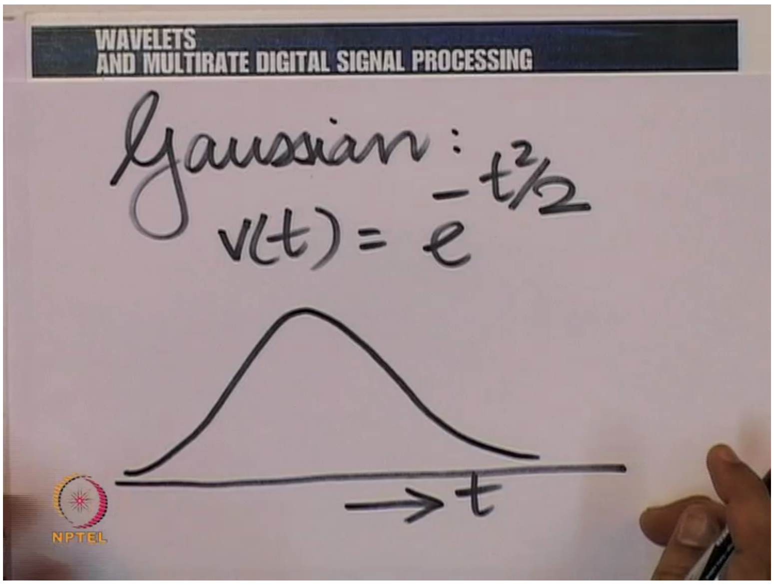

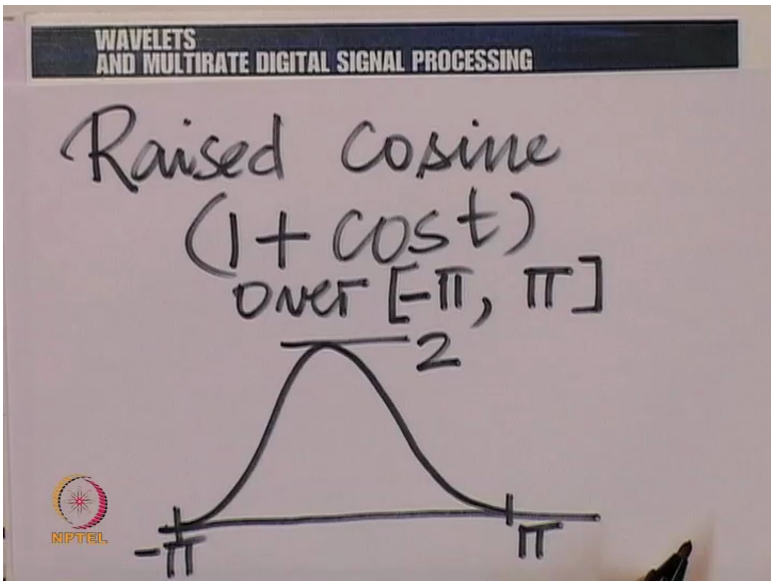

So, examples of Windows could be the triangular window, I will just sketch it. The Gaussian window, again let me sketch it. A so-called raised cosine window, which looks something like 1+ cos t over a limited interval, that would have an appearance like this, raised cosine mind you. At the 2 ends it would be 0 and it would have a maximum of 2 in between. Anyway, what you do notice commonly among all these windows is that there is a limit in time in spread and a limit in frequency in spread.

Now for the raised cosine case, I have not proved explicitly about this limit in frequency but in fact I leave it to you as an exercise to calculate the Sigma t squared, Sigma omega squared product for the raised cosine window. So with that we have chosen a window and we as I said allow that extreme poor case of the rectangular window with which we built our concept of multiresolution analysis as well.