Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-8.
Lecture-20.5.
Tiling the Time-frequency plane.

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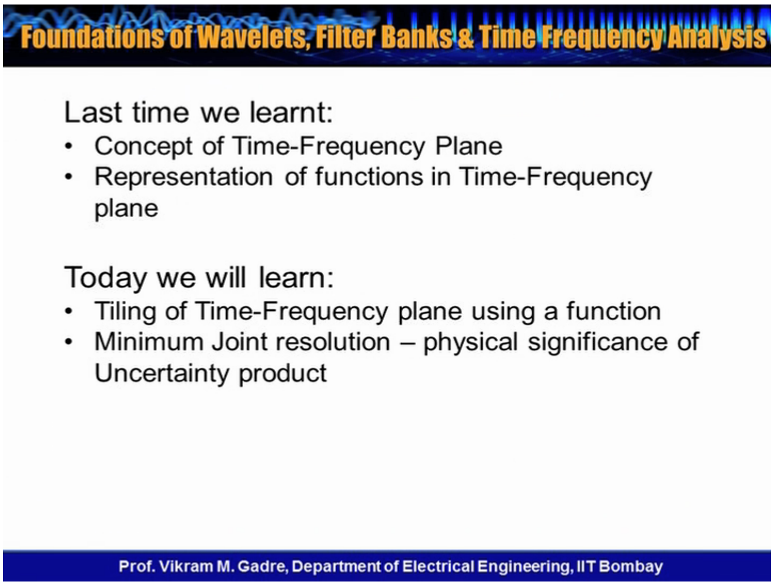

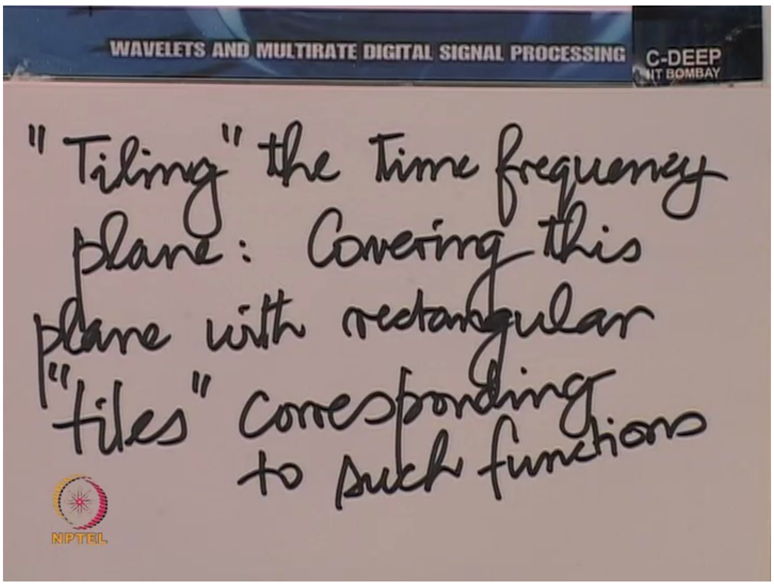

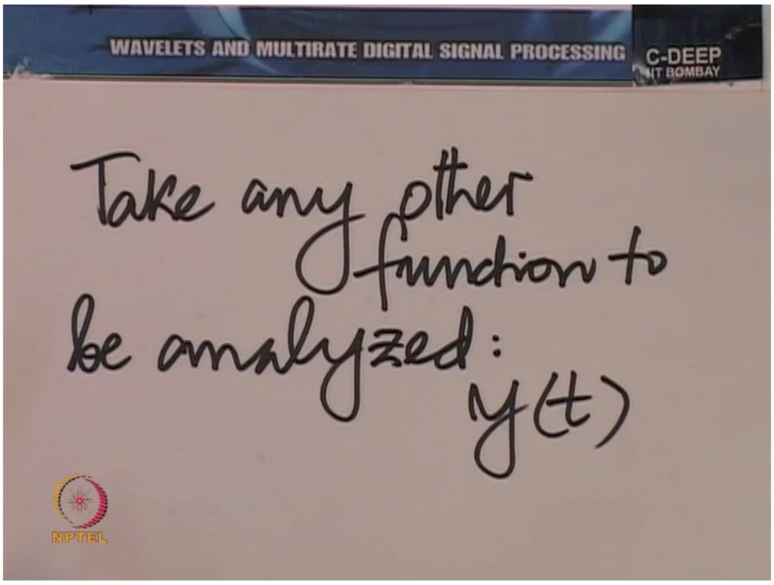

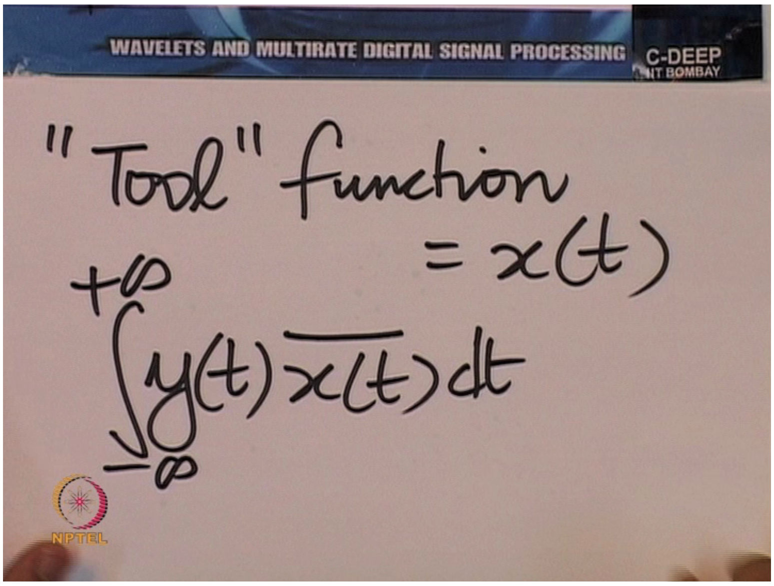

Now we can talk of what is called tiling the time-frequency plane. It essentially means covering this plane with such rectangles, with rectangles corresponding to function. In fact I will say rectangular tiles corresponding to such functions. And what we are essentially saying is that when we take the dot product of such a function, you know, let us, let us be specific. So when we take any other function, take any other function to be analysed, let us say y of t.

And let this tool function so to speak the x of t, then Parseval's theorem says that if I take the dot product of yt with xt, essentially yt x bar t dt, then the same thing happens in the frequency domain. Parseval's theorem says, this is equal to 1 by 2 pie, do not worry about the factor 1 by 2 pie, it is essentially just a normalising constant, the important thing is inside. Fourier transform of y, Fourier transform of x complex conjugated, integrated over omega.

So what this means physically is if I take the dot product, if I take the projection of yt on such a tool function xt in time, I am doing the same projection in frequency. So by projecting this yt on xt in time, I am essentially extracting information about yt in a time region between t0 minus Sigma t and t0 plus Sigma t. Parseval's theorem tells me simultaneously I am also exciting information of the Fourier transform of y in a region captured between capital Omega 0 minus Sigma omega and capital omega 0 + Sigma omega.

Simultaneously I am extracting information in the time frequency plane about yt in that rectangle using a tool function. So when I take the dot product of yt with such a tool function, I am immediately extracting information about yt in that rectangle of the time frequency plane. And now we have an interpretation, the rectangular region over which you want to extract information about a function yt cannot be smaller in area than 2 units.

Well, you know number 2 is not the point, I mean you can always change that number by changing the unit. We use a certain unit here, in fact we will use angular frequency as far as frequency goes, if you used hertz frequency we would get a different number that, that is not the point. The point is that there is a minimum rectangular area over which you can view yt. There is a minimum joint resolution, you know in the sense you cannot go finer than that resolution when you look at the 2 domains together.

But the good news is that there are many different ways in which you can look at a small domain when it is within the uncertainty limit. In fact now tiling has a different interpretation. If I wish to analyse a function, I think of the function in the time domain and in the frequency domain together. So essentially I am viewing the function in a joint domain and I wish to see how the function looks in a joint domain. Let me give an example, suppose I have what is called a chirp function.

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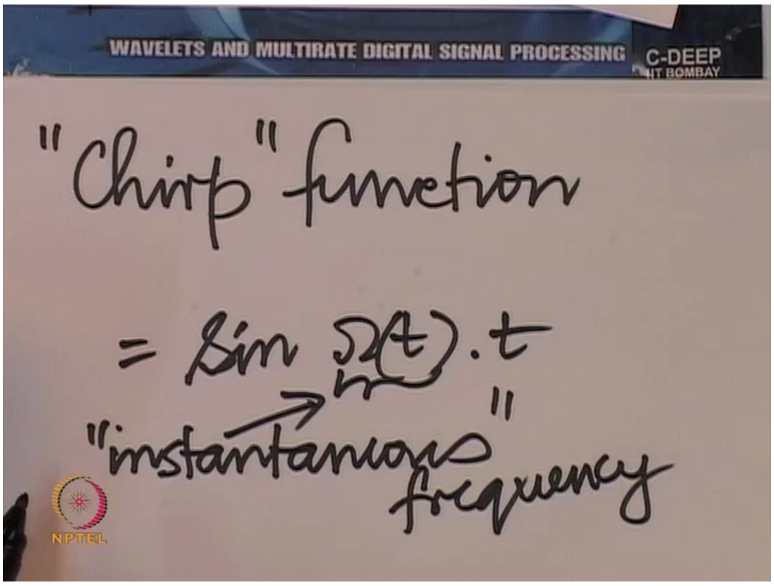

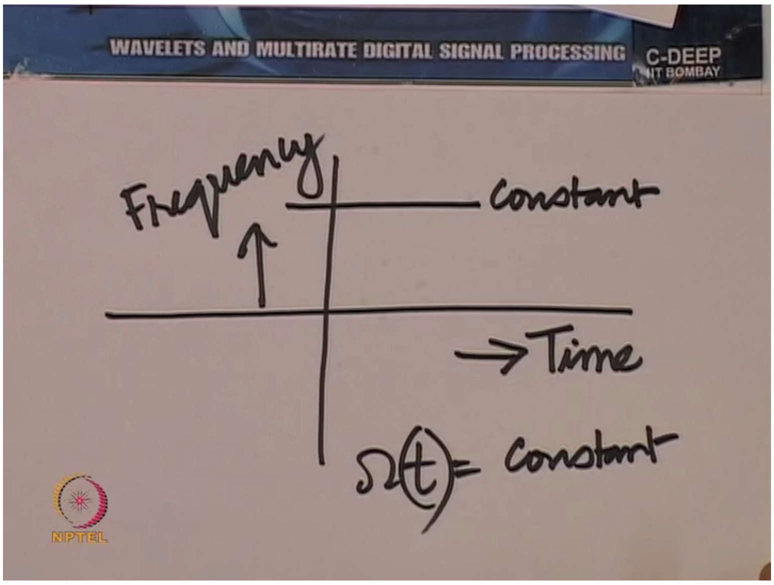

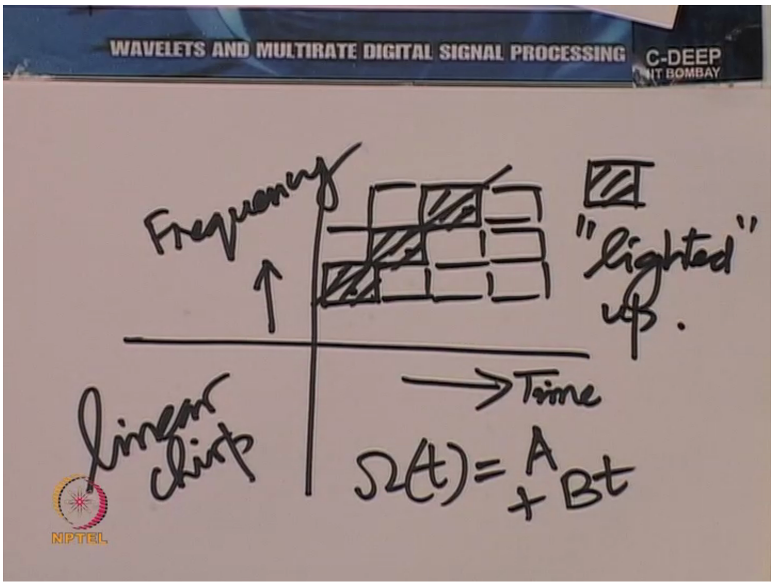

You know, a chirp function is named after the sound of birds. When birds chirp broadly or crudely, the chirp waveform has a pattern which is a continuously changing frequency in time, instantaneous frequency. So it is something of the form say sin Omega as a function of t t. So essentially this instantaneous frequency so to speak of the sine wave is a function of time. Now, you know an important question in analysing the function that one encounters sometime into Radar or Sonar is trace this variation of the instantaneous frequency in time.

And there the uncertainty principle hits hard. So what we are saying is, you know in the time frequency plane, suppose omega t is a constant function of time, is a constant function of time, then we are talking about a frequency independent of time. Suppose that is a linear function of time, which is often true, something like say this, omega as a function of t is some A plus Bt, this is called the linear chirp.

What would you try to do with the tool? You would try to trace this pattern and that is where the uncertainty principle hits you. It says that you can only put rectangles that looks something like this and you can never really trace what is happening within that rectangle. You could crudely say if the rectangles, you know, suppose you think of putting many rectangles on this time-frequency plane, so you have rectangles like this, put them all over and you would see that these rectangles are lighted up so to speak.

So, you know, I will shade these, the shaded rectangles are lighted up. In other words if I looked at the dot product of this function yt which has this linear chirp nature with this set of tiles, that means many of the functions which are on different styles in this time frequency plane. The tiles in which the frequency efficiently is prominent would be lighted up, that means the magnitude would be large there, the intensity would be large of the dot product.

Now all that would indicate is that, you know, it would show you the discrete points. So here for example if you look back in this time frequency plane, each of these rectangles would correspond to a single point here. It would show this point, this point and this point, the ones that lie on the line as lighted up. Alright. So these, these points would be lighted up here. But you cannot go closer than these points.

So of course these points would lie on what can be seem to be straight line but you would not know what has happened between the points. That is what the uncertainty principle says. You cannot get instantaneous frequency as a function of time exactly. But you can do it as closely as you desire by taking smaller rectangles and the smaller the area of the rectangles that you take, to within of course the uncertainty principle, the better you can make this estimate, one of the meanings of the time frequency plane and its tiling. We shall see more in the next lecture, thank you.