Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-8.
Lecture-20.4.
Time-frequency plane.

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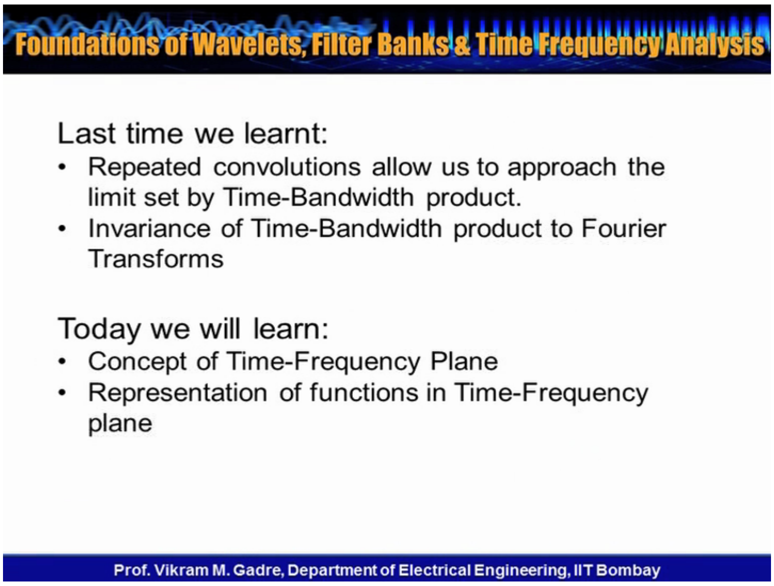

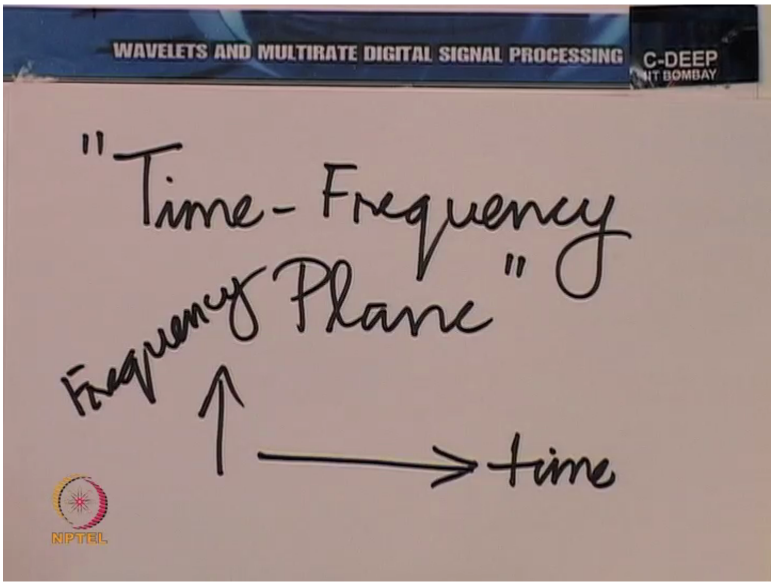

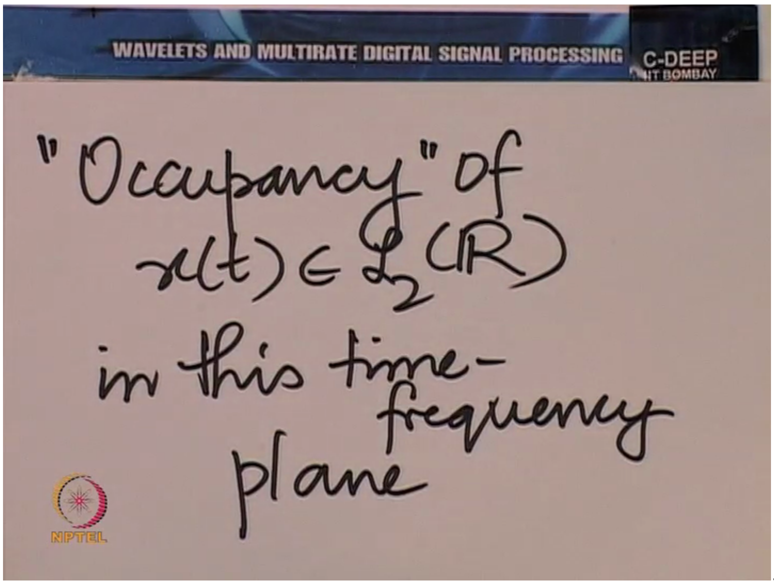

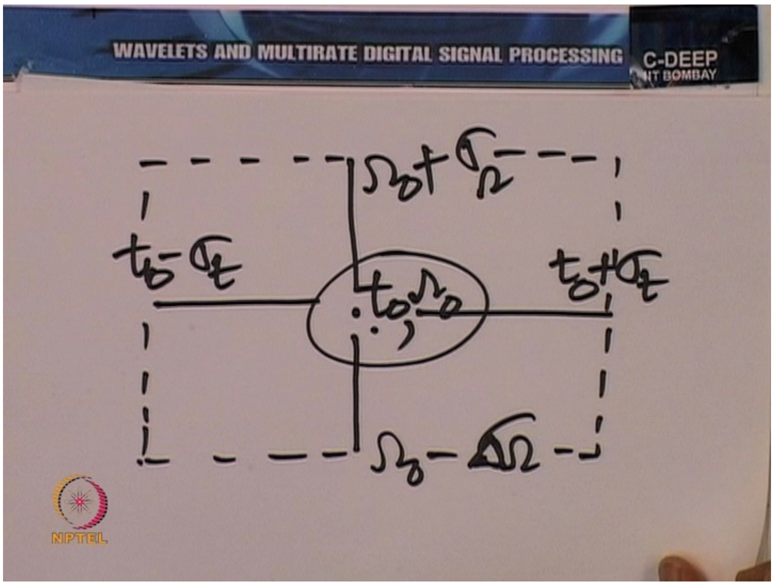

Now, with this remark we would like to take the idea of time bandwidth product further. Now that we have identified 2 domains, let us put the domains together and bring out a new domain, a 2 variable domain. So we shall henceforth talk of what is called a time-frequency plane. Essentially a plane in which one axis, say a horizontal axis represents time and the other axis, see the vertical axis represents frequency.

And therefore what the uncertainty principle says is that if you wish to describe the occupancy of a function display in, you know you can think of each function in L2R, we can think of the occupancy of xt in this time-frequency plane, occupancy is notional. And this occupancy can be thought as being from t0, Centre in time to t0 plus Sigma t on one side and t0 minus Sigma t on the other. this is the horizontal axis and the vertical axis we could centre it at capital Omega 0, namely the frequency Centre.

And we could spread it to Omega 0 minus Delta Omega 0 or Sigma omega if you please and above because take it to Omega 0 + Sigma omega. So this is in some sense notionally the spread, so you could think of that function xt as located in a rectangle which is centred at t0 Omega 0 here which has a width or horizontal spread of 2 times Sigma t and a vertical spread of 2 times Sigma omega. So let me show the whole time-frequency plane in some sense.

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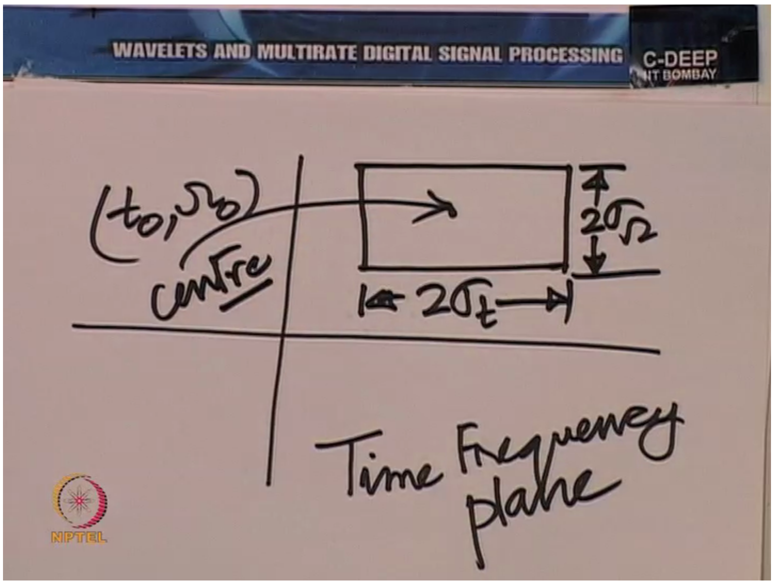

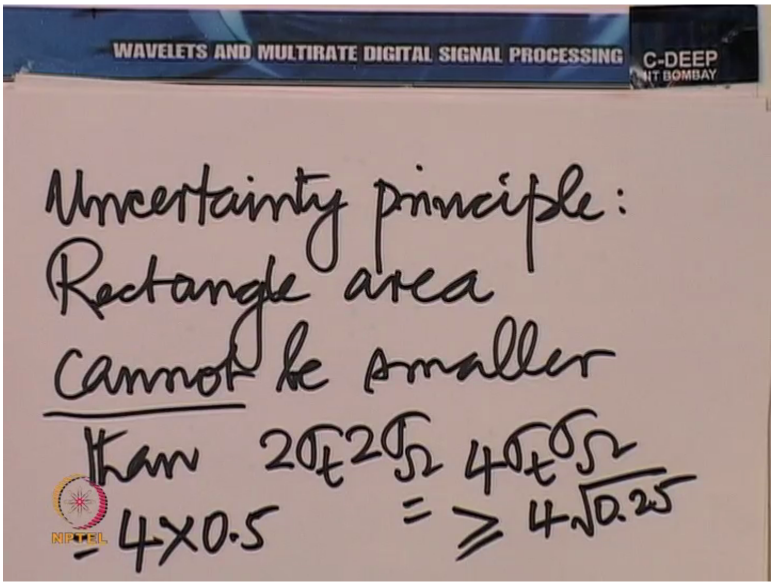

What we are saying is here you have the time-frequency plane, so let us mark it, this is the time-frequency plane notionally. And a function xt occupies a rectangle in this time-frequency plane, Centre t0, Omega 0 spread 2 Sigma t horizontally, 2 Sigma omega vertically, a very interesting concept. A function in L2R lies in a certain region of the time-frequency plane, it occupies a certain area in the time frequency plane and what the uncertainty principle says is that this rectangle cannot have an area smaller than a certain number.

Uncertainty says the rectangle area cannot be smaller than, well how much, 2 Sigma t into 2 Sigma omega, that is 4 Sigma t Sigma omega greater than equal to 4 times square root of 0.25, which is 0.5. So that is 4 into 0.5 that is the smallest area that it can have, 2, 2 units. the area of the rectangle cannot be smaller than 2 units. Now within that limitation you can change the width and the height that is also the positive side of the uncertainty principle. And in fact if you wish to cover the time frequency plane with functions, what do you mean by covering time-frequency plane with functions? It means using functions which occupy different such rectangles in such a way that it gives you different information in the time and frequency domain about another function.