Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-8.
Lecture-20.3.
More insights about Time-bandwidth product.

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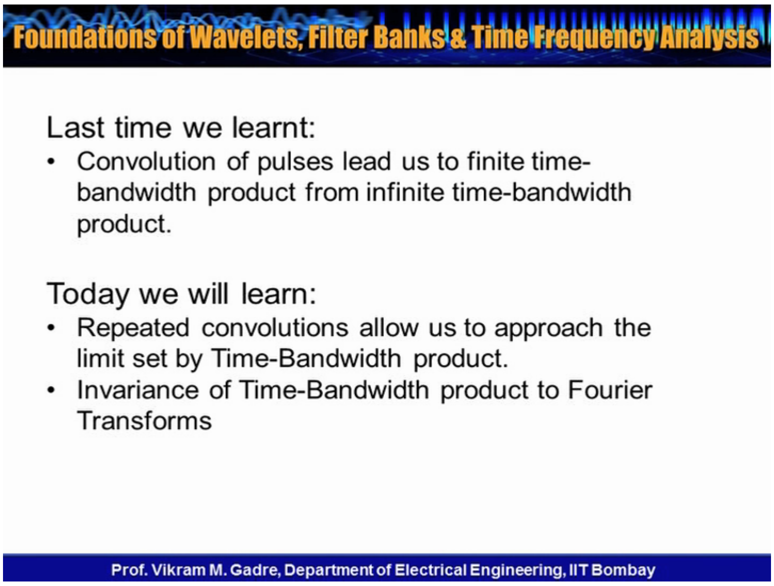

Now we seem to have an alternation of bad news and good news and yes that alternation takes one more step. the bad news is that now when you want to go from 0.3 to 0.5, you are going to have to work really really really hard. That is what nature does most of the time, I mentioned this before. Nature brings you tantalisingly close to an ideal and makes you work very hard to go anywhere close.

If you look at many fields, whether it is filter design, whether differential system design, you know to get an acceptable level of system performance, you may have to work just a little bit. To get that finesse in system performance, one has to really work hard in the difference between acceptable and fine performance may not be all that much all the time, that is true here as well. To go close to the uncertainty principle is not too difficult, to go any closer is very difficult.

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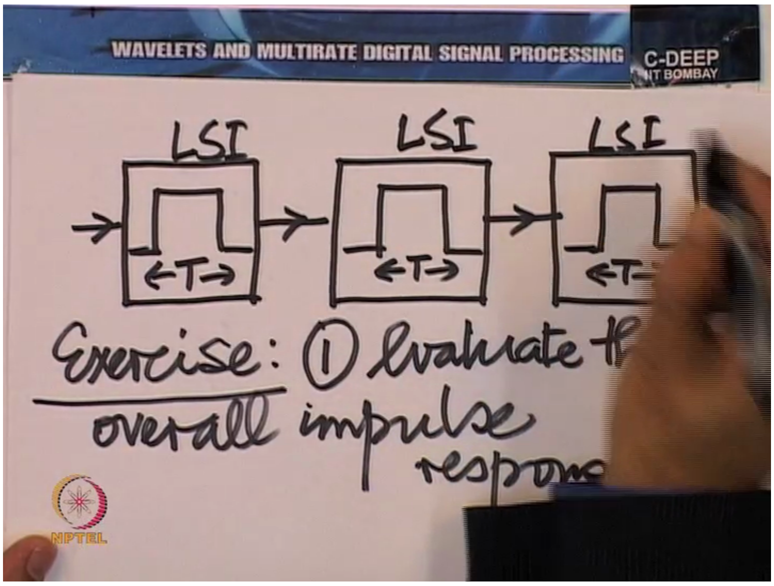

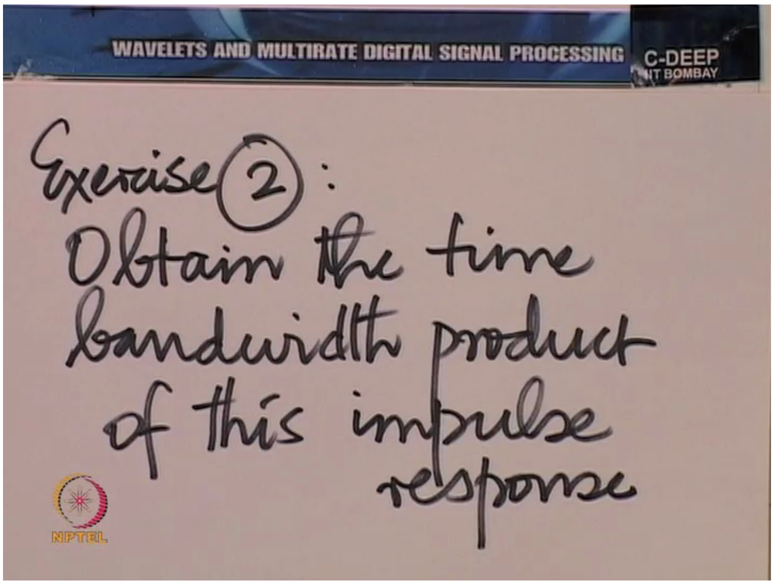

In fact one way of going closer is to repeatedly convolve the pulse with itself. So if you took this sample and hold system and if you make a cascade of one more such system with it, you get the triangular pulse. Now if you take cascade of 3 such systems, put one more in cascade, what I mean by that is, take this cascade, so I have a cascade of 3 LSI systems, each of whose impulse response is essentially a pulse. I shall leave a little exercise for you to do here.

The exercise is, number-one evaluate the overall impulse response. Exercise number 2, these are all LSI systems by the way, linear shift invariant system with the impulse response shown. Exercise 2, obtain the time bandwidth product of this impulse response. And naturally when you do that you would be inclined to compare it with the number 0.3, you hope that it should be less but I leave it to you to see what it actually is. In fact it will be interesting to do this further.

And it will be even more interesting to see if you can come up with an inductive argument, that is not really worth away. Each time you put one in the cascade, you are going to do better in terms of time bandwidth product, are you going to go closer to 0.25? I leave it to you to take a couple of steps, it should be an interesting thing to do. Anyway let us make one more remark. You know, it is not just a compactly supported function like this one which has a time bandwidth product of 0.3. Now I shall use very simple argument to show that the same 0.3 can come from and non-compactly supported function.

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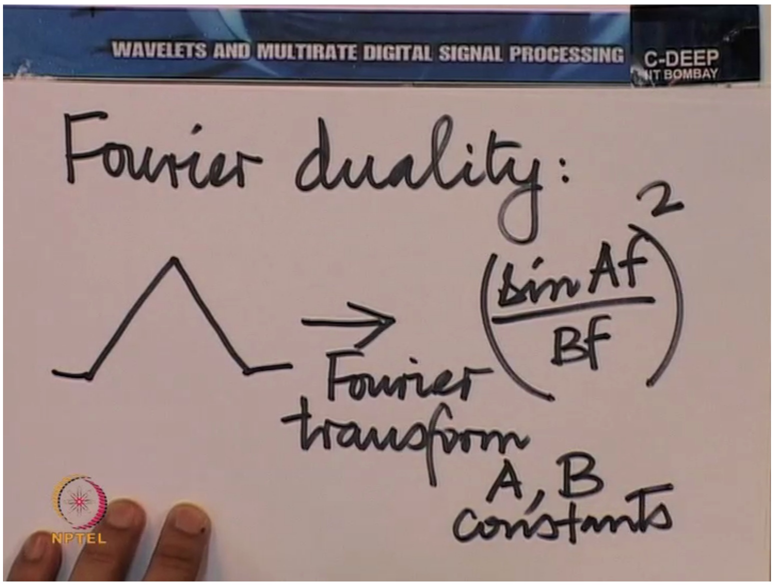

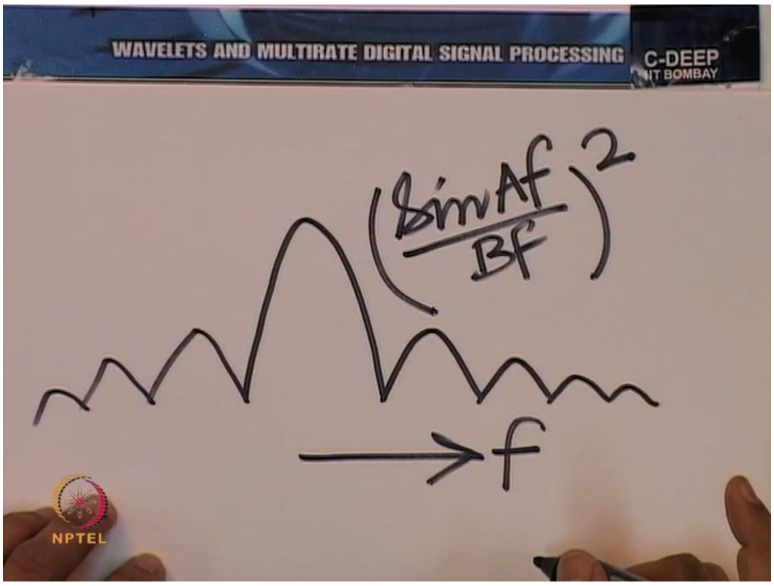

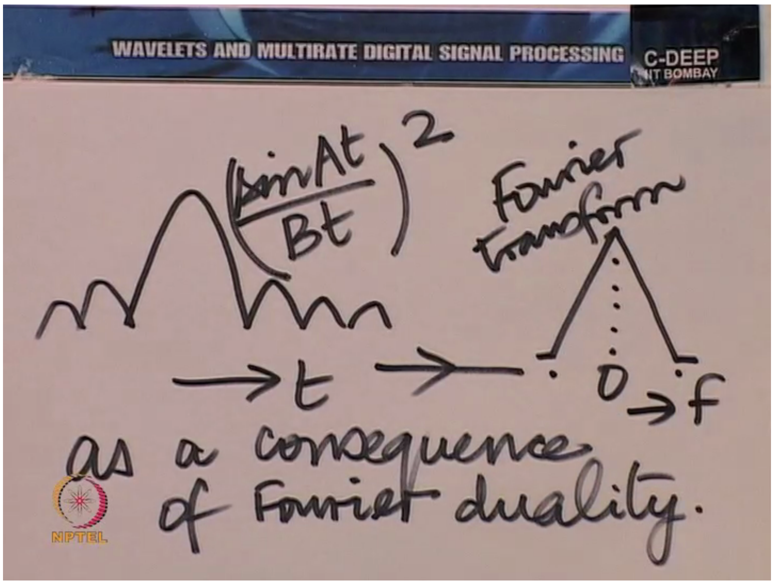

We use the principle of Fourier duality and we know that the Fourier transform of this triangular pulse in time is of the form sin AF by BF the whole squared, A and B are constants. You can suitably evaluate the A and B, that is not important, what is important is the form. So in fact we can even sketch it, the Fourier transform will look something like this as a function of F. Now, the important question to ask is what is the Fourier transform, if this is treated as a time function?

And that is what Fourier duality would give us. It would tell us that if we considered a time function like this, something like sin At by Bt the whole squared, its Fourier transform is going to look something like this, I do not need to mark the limits here, this is 0, as a consequence of duality. This is the Fourier transform, function of F here. So what we are calling the time variance for the triangular pulse becomes the frequency variance for this, for the functions the sin At by Bt kind of function here and what we are calling the frequency variance for the triangular pulse becomes the time variance for this.

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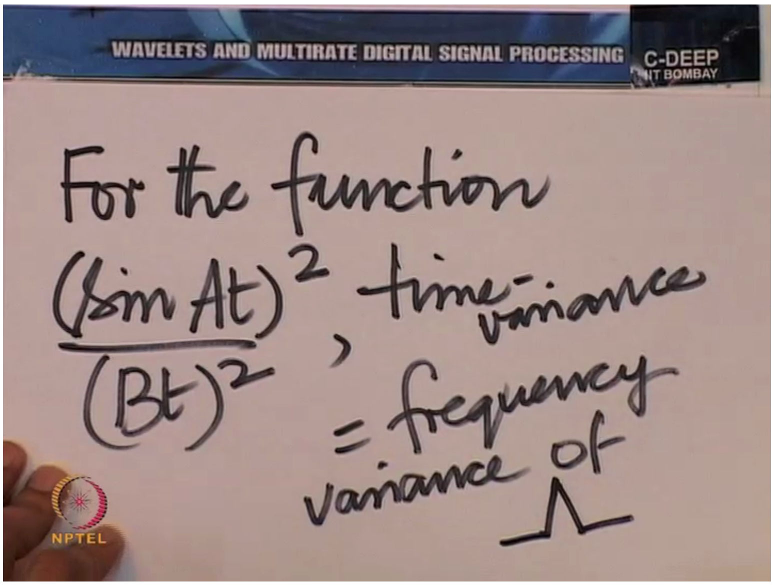

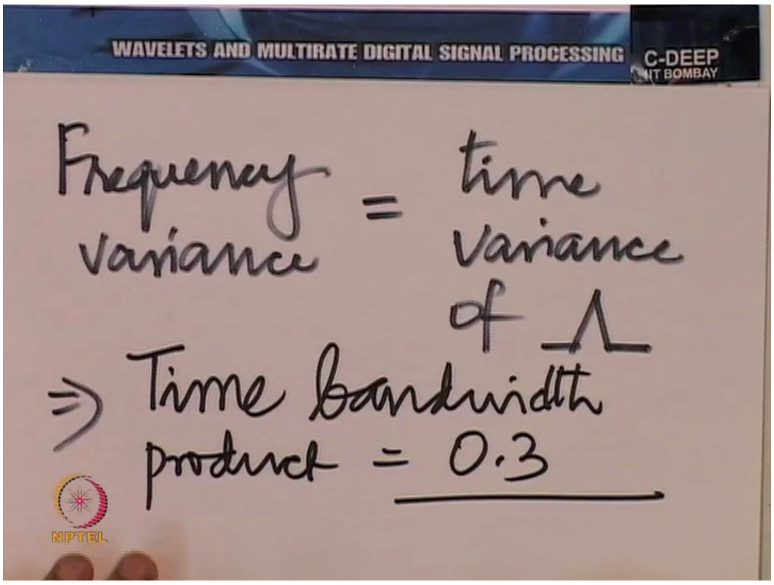

So in other words, when you take the Fourier transform of a function and ask what is its time bandwidth product, the time bandwidth product is the same. So for this function, for the function sin At by Bt the whole squared or this, time variance is equal to the frequency variance of the triangular pulse and the frequency variance is equal to the time variance of the same triangular pulse. And therefore the time bandwidth product is very easy to calculate, in fact the time bandwidth product would simply be 0.3.

Now we have a partial answer to the question that we raised the last time. Can you change the shape and maintain the same time bandwidth product? Yes, you can. In fact what we have just done answers many questions or brings up many different conclusions. One is, we have discovered one more kind of invariance of the time bandwidth product and let us write that down. The time bandwidth product is invariant to Fourier transformation. And this is a very deep kind of invariance.

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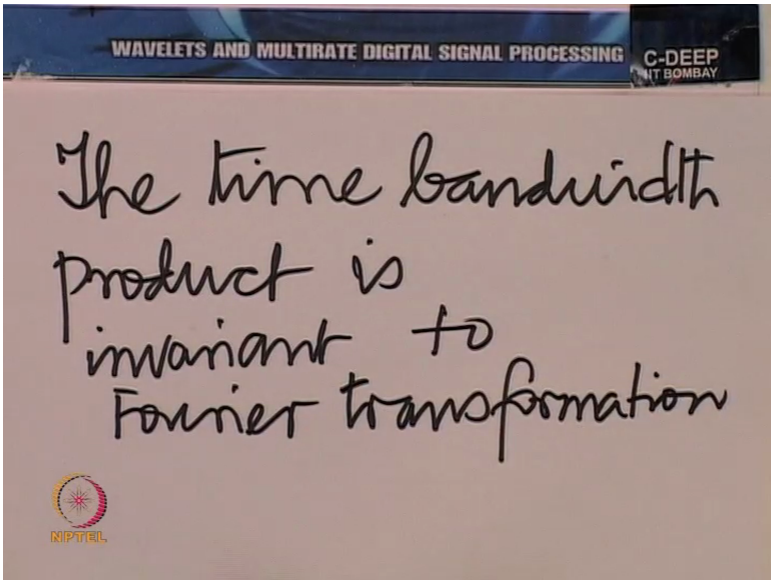

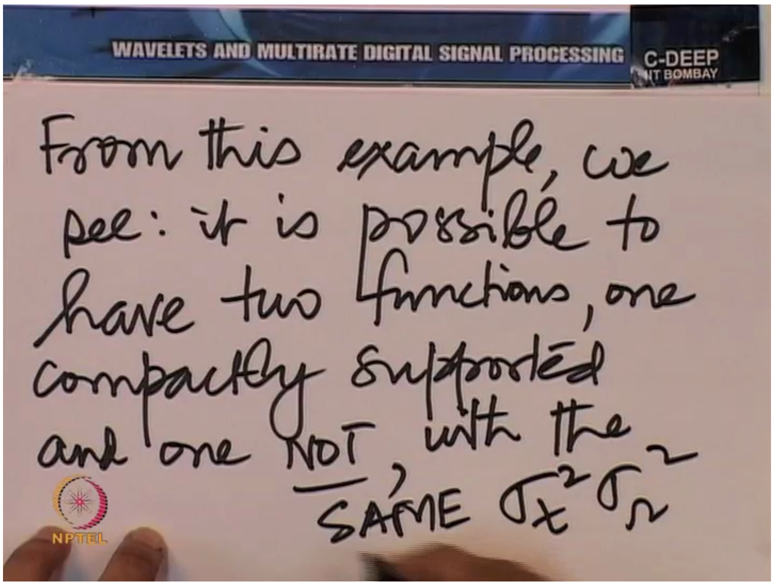

One more conclusion that we have drawn from here is that we can have both compactly and non-compactly supported functions, namely functions which are nonzero on a finite interval and functions that are nonzero over an infinite interval with the same time bandwidth product. So it is possible, from this example, we see it is possible to have 2 functions, one compactly supported and one not, with the same time bandwidth product.