Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-8.
Lecture-20.1.
Discontent with the "optimal function".

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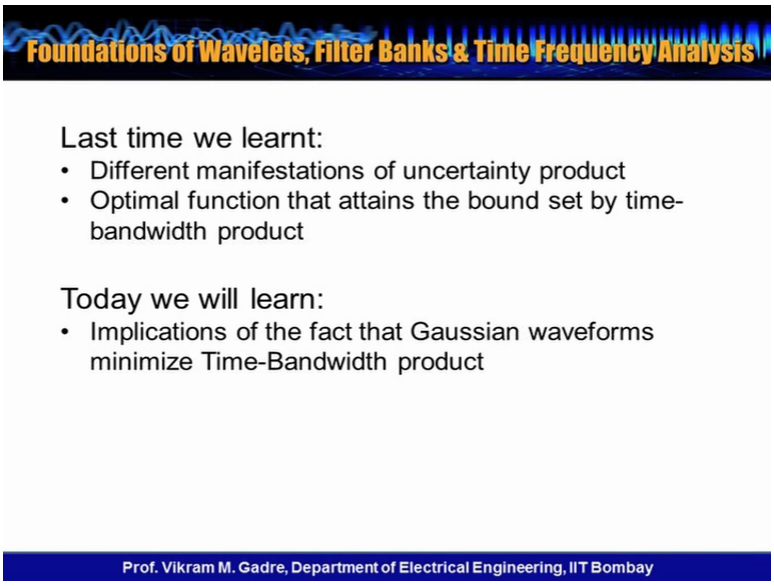

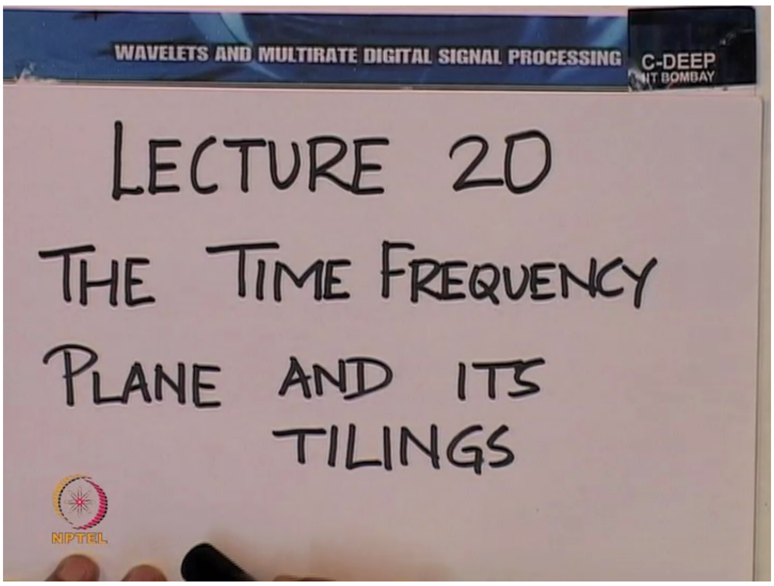

A warm welcome to the 20th lecture on the subject of wavelets and multirate digital signal processing. Recall what we had done in the previous lecture. We had built up the uncertainty principle to completion. We found that nature imposed a fundamental limit, if we look at the time bandwidth product, you cannot go below a certain number, that is what we finally inferred. In fact we also inferred which function could give us that minimum product. Let us therefore put the theme of the lecture today and some of the important conclusion that we had drawn in the previous lecture before ourselves to put our discussion in perspective.

So what we intend to do today is to talk about what is called the time frequency plane and the idea of tiling the time frequency plane. Just like you would tile a floor, you would tile a surface, we would tile the time frequency plane. And to recall what we have done in the previous lecture, we had drawn the following conclusions. Conclusion number-one, what is the minimum time bandwidth product that you can get?

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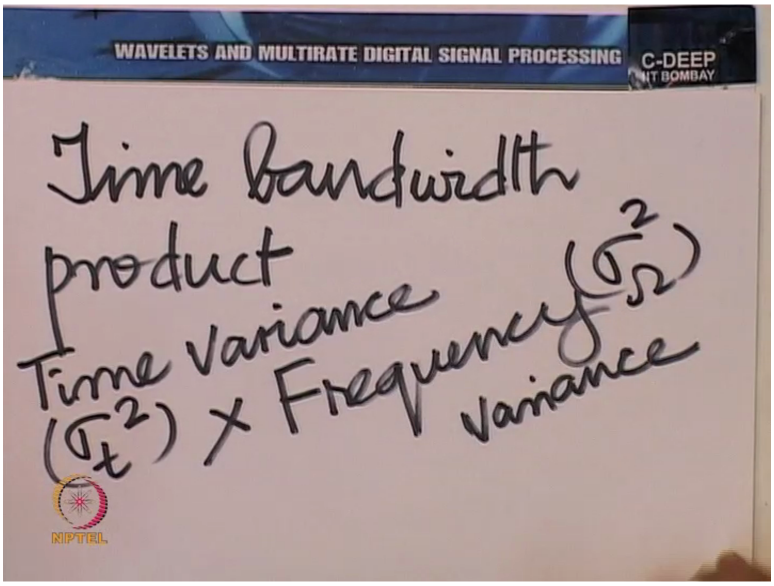

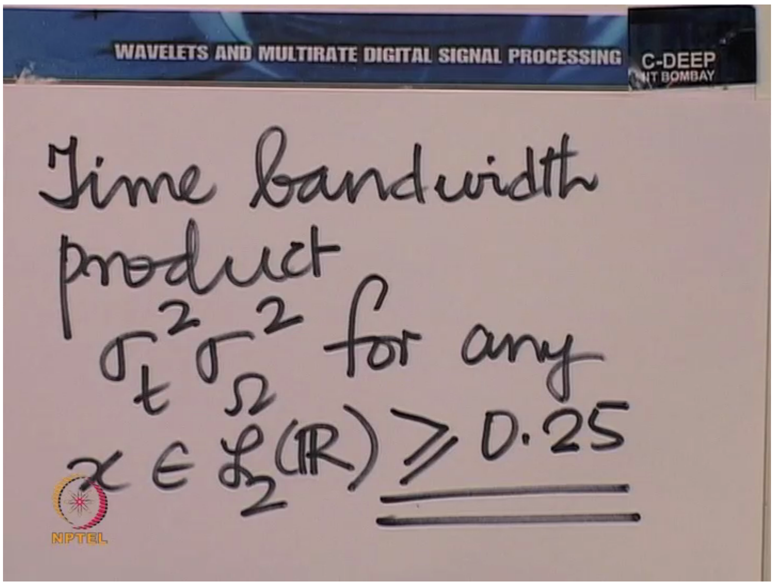

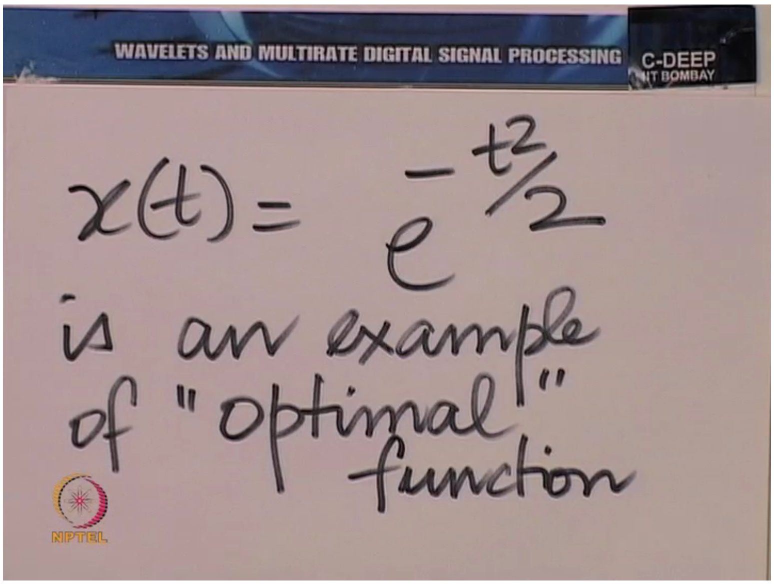

So recall that we had defined the time bandwidth product to be the time variance multiplied by the frequency variance. And we said that this quantity which we also described as Sigma t squared times Sigma omega squared cannot fall short of 0.25. So we said the time bandwidth product Sigma t squared Sigma omega squared for any function x belonging to L2R is always greater than or equal to 0.25. We had proved this the last time. And we also concluded which function xt in L2R gives us this time bandwidth product.

So we showed that xt is the Gaussian namely e raised to the power minus e square by 2 for example is an example of a so-called optimal function, optimal in the sense of time bandwidth product, all right. Now, other optimal functions can be obtained by modulating this with a term of the form e raised to the power J Alpha t square. So we saw the more general optimal function.

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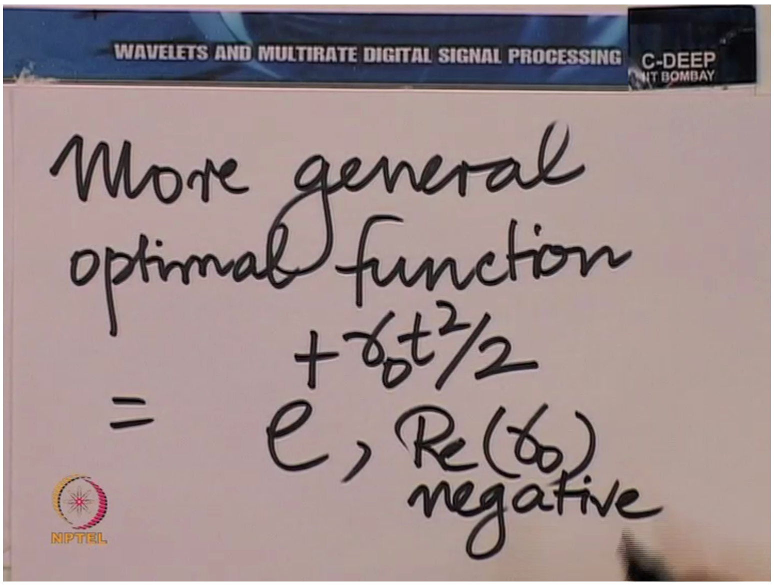

More general optimal function is of the form e raised to the power minus gamma 0 t square by 2 or you could say plus gamma 0 t square by 2 with the real part of gamma 0 negative. If you wrote minus gamma 0, of course you could say the real part is positive, either way, whatever you wish to write. So gamma 0 could be complex in general, that is what I mean. Anyway, essentially it is a Gaussian that is optimum. In a way that is good news, in a way it is bad news.

The good news is that we know what the optimal function is, we know that the Gaussian is optimum. The bad news is that the Gaussian is unrealisable in the exact sense in physical systems. You know, this may seem like a puzzling state, one of the favourite probability density functions of most scientists and engineers is the so-called normal or the Gaussian density. And in fact we go to the extent of saying that when we put together a number of independent identically distributed random variables, in most situations the some random variable goes towards the Gaussian.

So that is what is called the Central limit theorem. And therefore we also justify the use of the Gaussian density in most statistical situations. So much so that we are obsessed with the use of the normal distribution and we use the term variance to even denote the spread around the mean. We say, well, in a Gaussian the spread around the mean tells us as is indicated by the variance and tells us more or less in what range that variable lies, the variance, mean plus variance to mean minus variance.

So well, so that is the Gaussian for you. Then why are we saying this is physically unrealisable? I am talking about a Gaussian time waveform. Take for example the exponential time waveform or the exponential time waveform modulated by a sinusoid, these are easily realisable. Circuits which comprise of resistances, inductances and capacitances when excited, say with a step or even for that matter with a sinusoid give us either exponentially decaying sinusoids or exponentially decaying transients and therefore those are easy to generate with physical systems.

Unfortunately there is no meaningful physical system which can generate a Gaussian in the same way. So that is one of the fundamental reasons why Gaussian is good news in a statistical density but Gaussian is bad news as far as functions go. You know, I must mention that people talk about what is called Gaussian min shift keying or Gaussian minimum shift keying, GMSK in the context of digital communication. The word Gaussian there refers to the Gaussian pattern in the impulse response, whether it is in phase or in amplitude.

But there again people really fight hard to realise a Gaussian filter. So you see that the Gaussian is difficult to realise in physical systems, can only be approximated. So although it is good news that we know what the optimal function is, it is bad news that we cannot easily realise this optimal function by using physical systems. Well then, that is bad news. Let us bring some good news. If not the Gaussian, then can we use a reasonable function which we could probably realise with the cascade of 2 simple systems or something of the kind and go close to the Gaussian.

So in other words, when we started with the Haar, we had a terrible time bandwidth product, infinite. Now can we do a little better?