Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-7.
Lecture-19.1.
Introduction.

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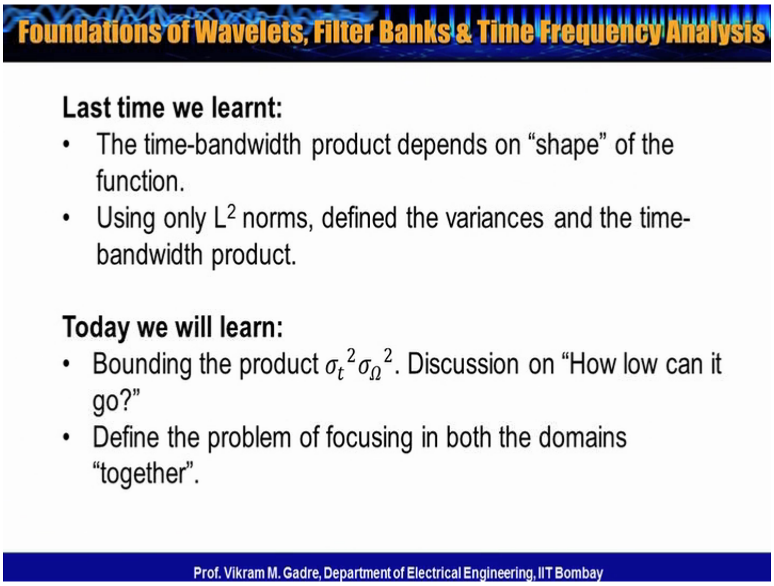

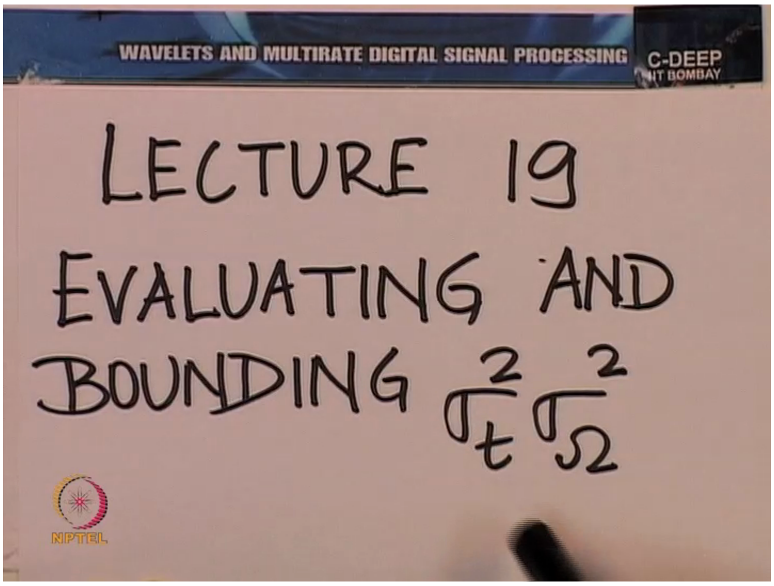

A warm welcome to the 19th lecture on the subject of wavelets and multirate digital signal processing. In this lecture we shall go further on the uncertainty product or the time bandwidth product as we called it in the previous lecture to proceed, to evaluate and to bound it, to put certain fundamental limits on it. And then to see how close we can get to those limits. Therefore I have titled the lecture today as evaluating and bounding Sigma T square Sigma omega squared.

To put the discussion in perspective, let us recall very briefly what we did in the previous lecture. You will recall that we had talked about these quantities Sigma T square, Sigma omega square, we had also noticed that this product Sigma T square Sigma omega square is a very strong invariant. It is invariant to translation, it is invariant the modulation, both in time, translation in time, modulation in time, it is invariant to multiplication of the dependent variable by a constant or scaling the dependent variable or in other words multiplying the function by a constant.

Most interesting of all is that it is invariant to a scaling of the independent variable, that is striking. So you see ultimately what is left is just the shape, the time bandwidth product is a direct function of the shape, the shape can then get compressed or expanded, it can get scaled in the vertical direction, it can be shifted, it can be modulated by an E raised to the power J Omega T kind of term, rotating complex number or a phaser. All these do not affect the time bandwidth product, Sigma T squared Sigma omega squared as we called it.

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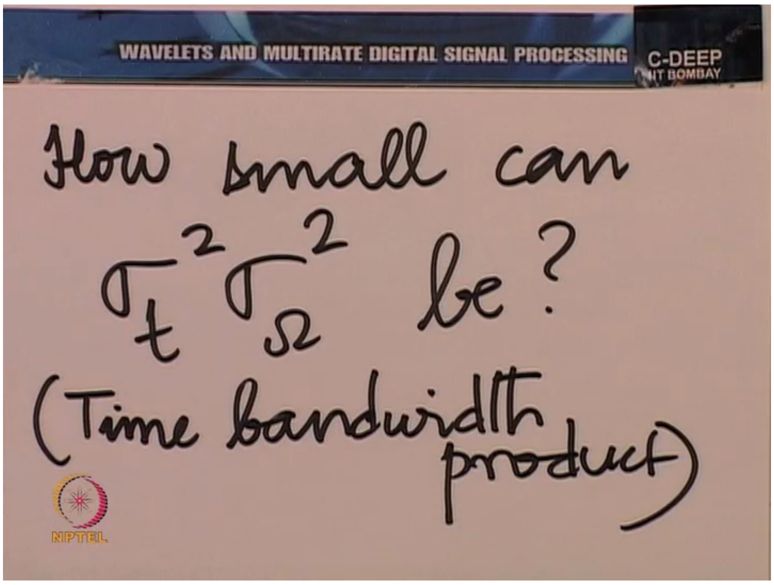

Now our objective today is to ask a very important question which lies at the heart as I said of wavelets and multirate digital signal processing, namely how small can we make this product. Let us put down the question 1st. Essentially, how small can the time bandwidth product be? And once again we need to emphasise a couple of points. Why are we talking about the time bandwidth product instead of just the time variance or the frequency variance?

We must understand that making the time variance small or the frequency variance small is not a difficult job at all. In fact one can do it by scaling the independent variable. If you compress a function in time, the time variance is compressed by the square of that factor, if you compress a function in frequency, the frequency variance is compressed by the square of that factor. So compressing in time or frequency separately is not a difficult job at all. In fact, the problem is that when you compress in one domain, you are expanding on the other domain by the same factor, there is a cancellation effect as far as the time bandwidth product goes.

So compressing or expanding a function does not change its time bandwidth product but it definitely changes the time variance and the frequency variance individually, this is what we saw in the previous lecture. In that sense physically what it means is that nothing stops you from narrowing down as much as you desire in one of the domains. If you desire to focus on a small region of time, you can do it on as small a region of time as you desire.

If you focus on certain region of frequency, you can make that region of frequency on which you focus as small as you desire, there is no problem. So focusing in one domain is not a problem, you can do it as much as you desire, the problem is focusing in both the domains together. Any tool that we use, by tool I mean a function used for analysis, any tool that we use has a time bandwidth product essentially based on its shape.

So when you use the same shape, you are bound by that number, no matter how much you squeeze or expand, you are not going to be able to focus in both the domains simultaneously, in fact by focusing in one domain, you are going to compromise on your ability to focus in the other. This is the general tussle as we said between time and frequency. Now, the next natural question to ask is how small can we make this tussle.

If Sigma T squared Sigma omega square could have been 0 for example, could have been, it would be wonderful, nothing needed to be done. You could make any of them as small as you desire and the other one as small as you desire too. So you could focus in 2 domains simultaneously. But as I said nature does not allow this and it is a fundamental property of nature that it does not allow this. What we are now going to answer is what is the lower bound on this product, meaning to what extent no matter what tools you have can you really focus in both the domains together?

So what is the ideal towards which we must strive and why is it that we have a fundamental limitation to work with which always poses a challenge before us.