Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-7.
Lecture-17.1.
Variance from a slightly different perspective.

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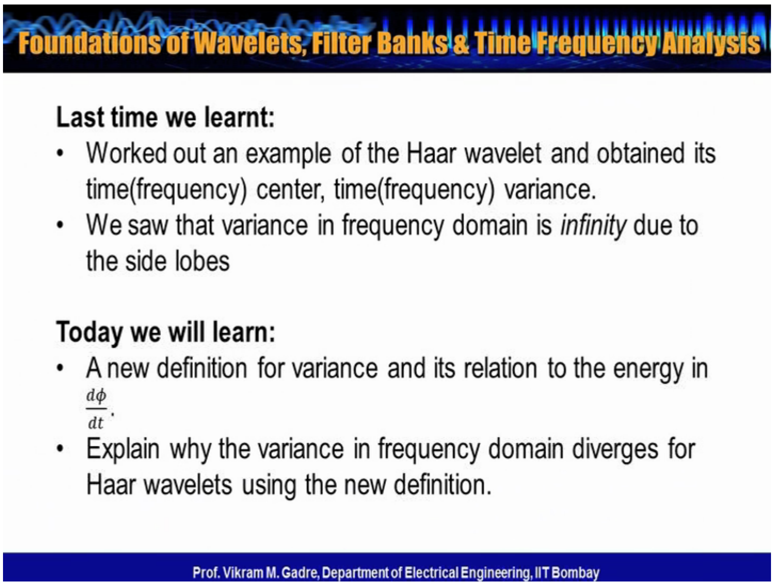

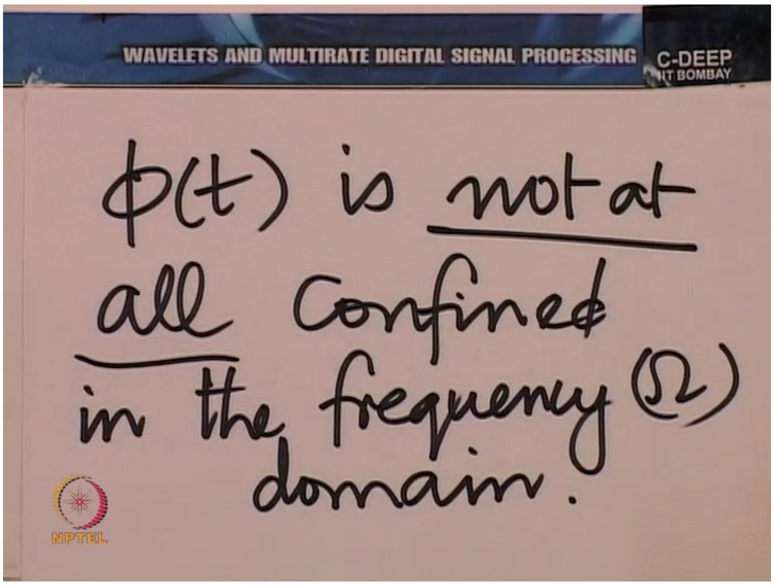

Now all this while in our discussion when we talked about time and frequency together and so on in the previous lecture, we have been worried about the side lobes as we call them. We said, well it is all right to look at the main lobe and talk about presence in the main lobe but then we have these side lobes and the side lobes are falling off only by the factor 1 by omega in magnitude. And as you can see the side looks have created a problem.

After multiplication by omega square in the calculation of variance, the side lobes create a periodic function to be integrated, a periodic nonnegative function and we are in trouble. So this tells us again why we have to go much beyond the Haar. We have been asking again and again why we cannot be content with the Haar multiresolution analysis, now we have one more formal answer. If we look at the scaling function for the Haar multiresolution analysis, its variance in frequency domain is infinite.

It is not at all confined in the frequency domain in this sense. Now, it is a natural question to ask what is it that made this variance infinite. Why did we have a divergent variance here? In fact, we can answer that question if we only care to make a slight adjustment of the expression for variance. the variance of phi cap is finally as you can see given by omega squared P phi cap omega d omega.

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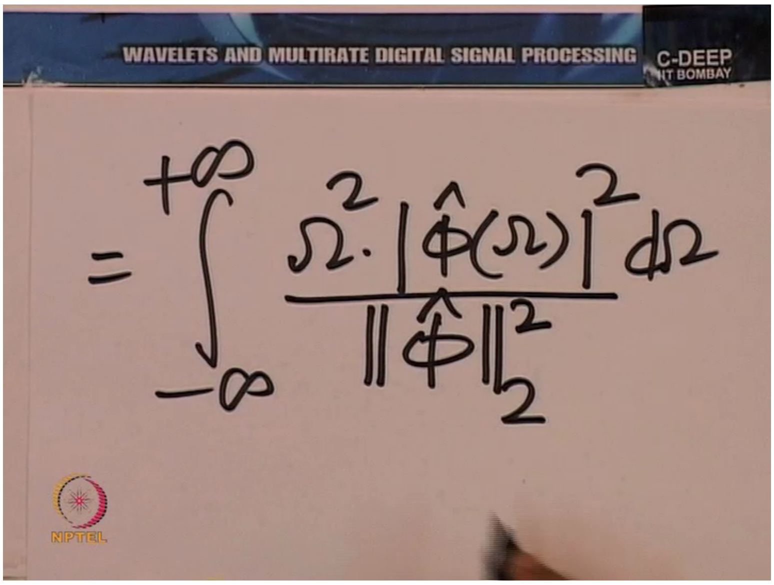

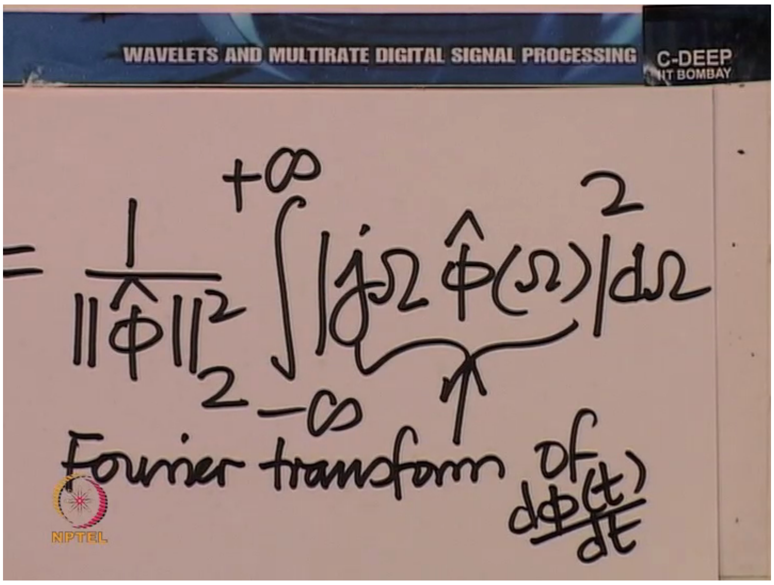

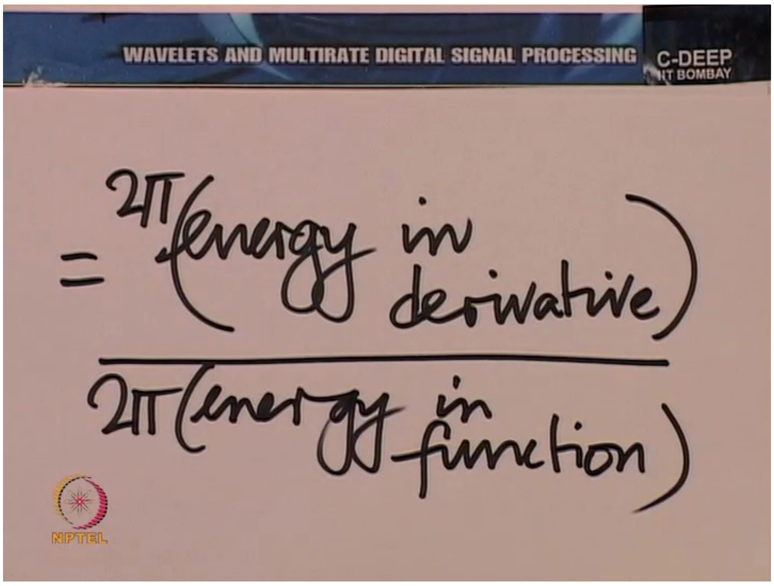

And this can be written as omega squared times phi cap omega mod squared d omega divided by the norm of phi cap in L2R the old squared. Now this norm is a number, it can be brought out of the integral. So I can rewrite this as 1 by norm of phi cap squared integral and I will also do a little bit of rearrangement in the integral. I will write the integral as J omega phi cap omega the mod whole squared d omega.

Notice that if I take the modulus of J omega phi cap omega, it is essentially modulus omega squared times modulus phi cap omega square. And modulus omega square and omega squared are the same for omega real. But then when we write it like this, this has a meaning. It is essentially the Fourier transform of d phi t dt, Fourier transform of the derivative of phi. So essentially what we are saying is this variant is actually the energy in the derivative.

Of course remember you would have a factor of 2 pie there because this is the energy in the derivative but for a factor of 2 pie. So this would be 2 pie times the energy in the derivative divided by 2 pie times the energy in the function. And please note that this inference that we have made is independent of what function we consider. As long as the function is real, the variance in frequency is going to be this ratio, the energy in the derivative divided by the energy in the function.

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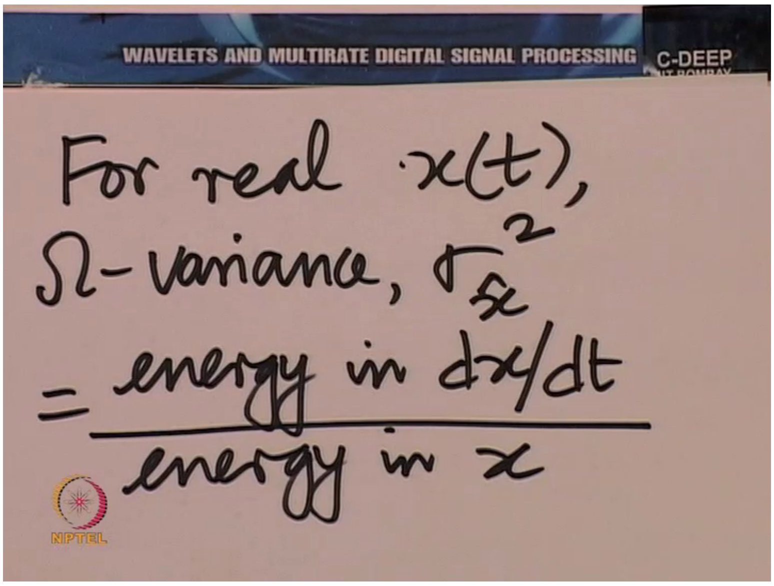

Let us make that remark, for real functions, for real xt the frequency variance, omega variance or Sigma x cap square is essentially energy in dx t dt divided by the energy in x or the L2 norm square and of the derivative of x divided by the L2 norm of x. And now we have the answer why we ran into a problem for phi t. As you can see phi t is discontinuous. So when its derivative is considered, there are impulses in the derivative and impulse is not square integrable and therefore the numerator of this quantity diverges, if you look at it from that perspective.

The moment we have a good discontinuous function, we have an infinite frequency variance and there we are. With this note then we realise that if we want to get some meaningful uncertainty, some meaningful bound, we might at least consider continuous functions and we shall proceed to relearn this concept further in the next lecture. Thank you.