Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
Professor Vikram M. Gadre.
Department Of Electrical Engineering.
Indian Institute of Technology Bombay.
Week-7.
Lecture-17.2.
Defining Mean, Variance and "containment in a given domain".

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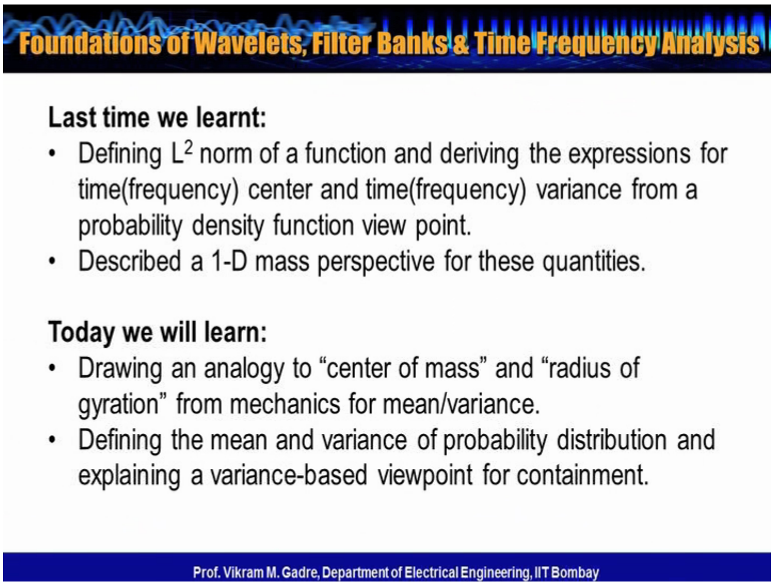

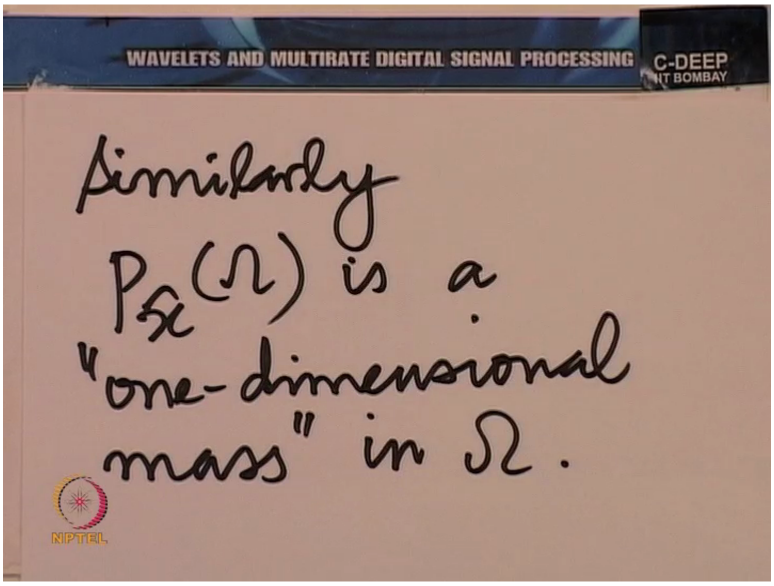

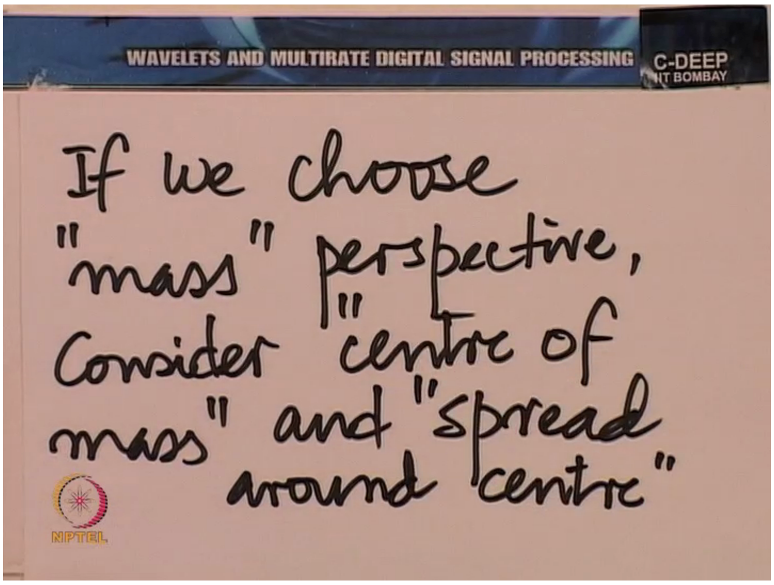

Now, once you take the mass perspective, then immediately you have the notion of centre of mass, centre of gravity if you like to call it that. And when you take a probability density perspective, you have the notion of mean. Either of them is okay, they are equivalent. So, let us make a note of that. If we choose the mass perspective, consider the centre of mass and spread around the Centre. Now incidentally as I said, the spread around the Centre in mechanics is often measured by a quantity called the radius of gyration.

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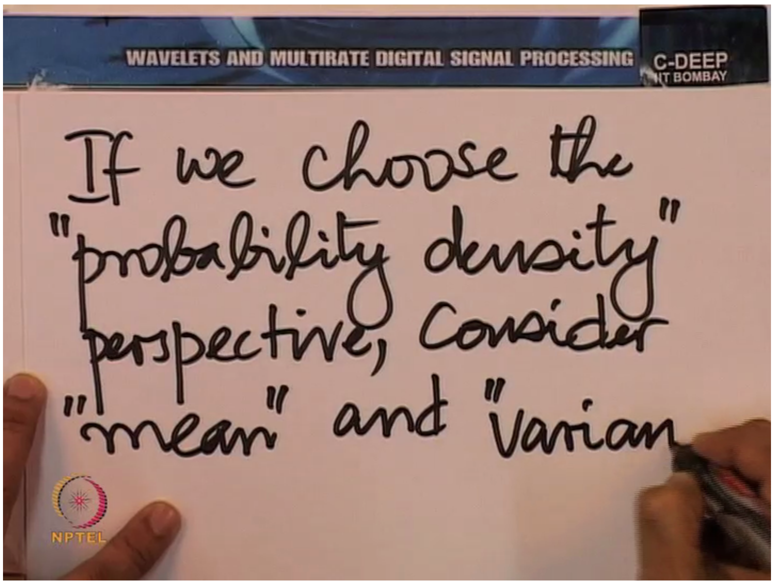

If we happen to take the density perspective, consider the mean and the variance. Now we must of course assume that these quantities can be calculated and we shall do that. It is possible that the variance be infinity, that is a subtle point. So we are not always guaranteed of finite variance and in fact that is not a contradiction to what we have been seeing so far.

We are trying to find a lower limit to where the quantities go in the 2 domains simultaneously.

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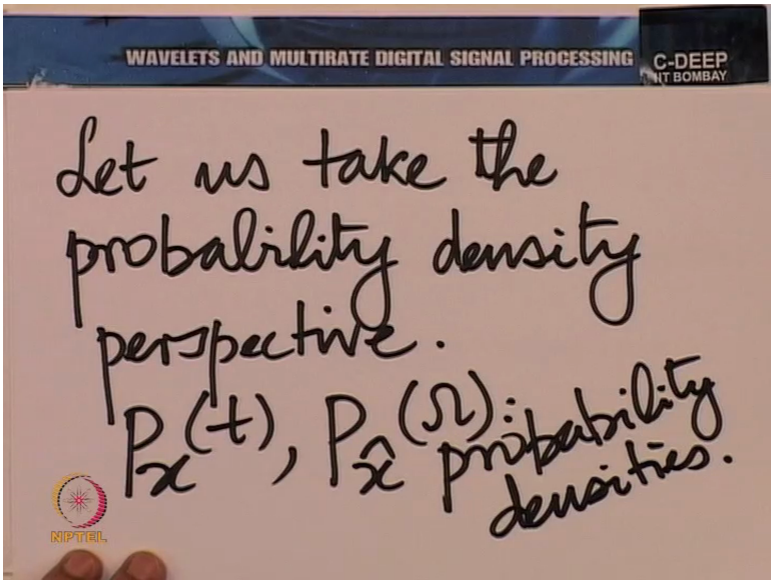

So of course if the variance happens to be infinite, which it will actually in some situations, we shall simply say that is the worst possible case that we can encounter. Anyway, so let us consider, given the function xt, we will prefer to take the probability density perspective. So we think of Px of t and Px cap of omega as probability densities. And we then write down their names. So indeed let Pxt have the mean t0, what would that mean?

t0 would then be integral t times Pxt over all t from - to + infinity. Simple, definition of mean, you will of course recognise the same definition to hold good for the centre of mass here, essentially you are calculating the moment by choosing the fulcrum to be 0 and therefore getting a different fulcrum or a point at which the moments are all balanced.

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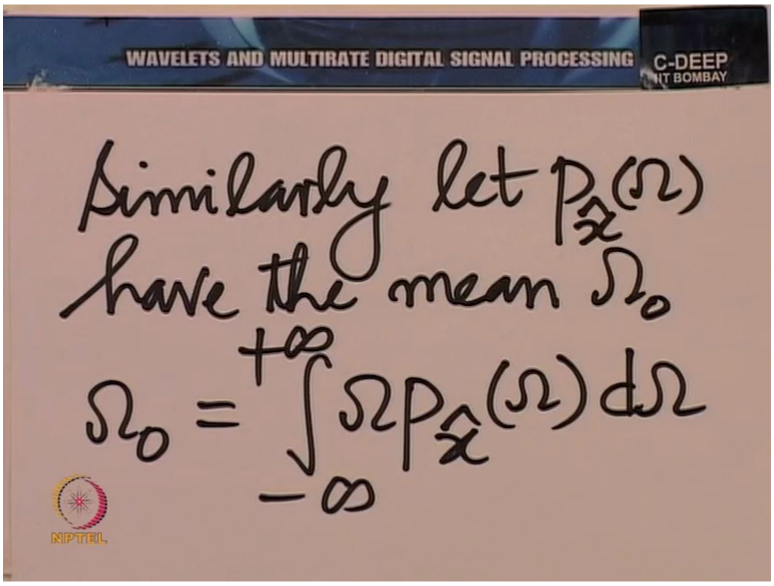

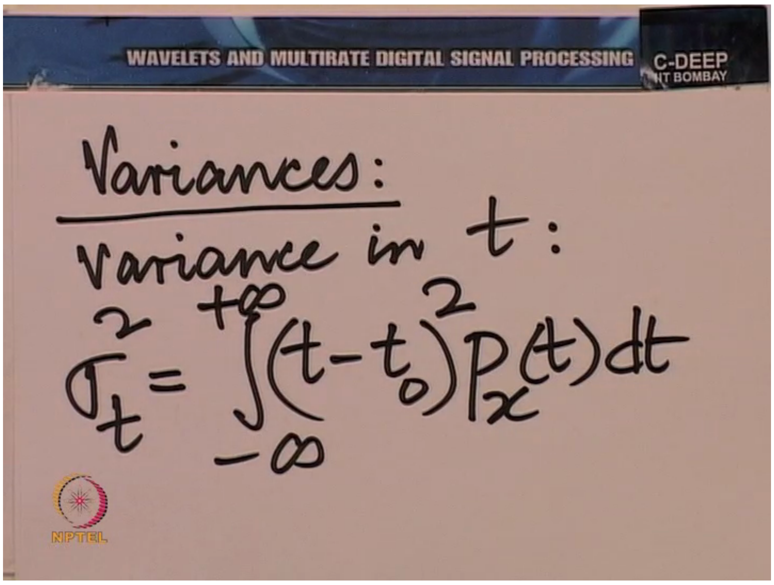

Similarly, let Px cap omega have the mean omega 0 whereupon omega 0 would be integral from - to + infinity omega Px cap omega d omega, again the centre of mass, if you like to look at it that way in the frequency domain. Now once we have the mean, and of course just as a tongue in cheek statement, we assume the means are finite, normally they should be, in some pathological we may have a problem, we are not looking at those pathological situations.

So assuming these means are finite, let us look at the variances. So the variance in t would then be given by and we will define it to be Sigma t squared. By definition this would be t - t0 the whole squared times Pxt integrated over all t. And similarly we could talk about the variance in frequency.

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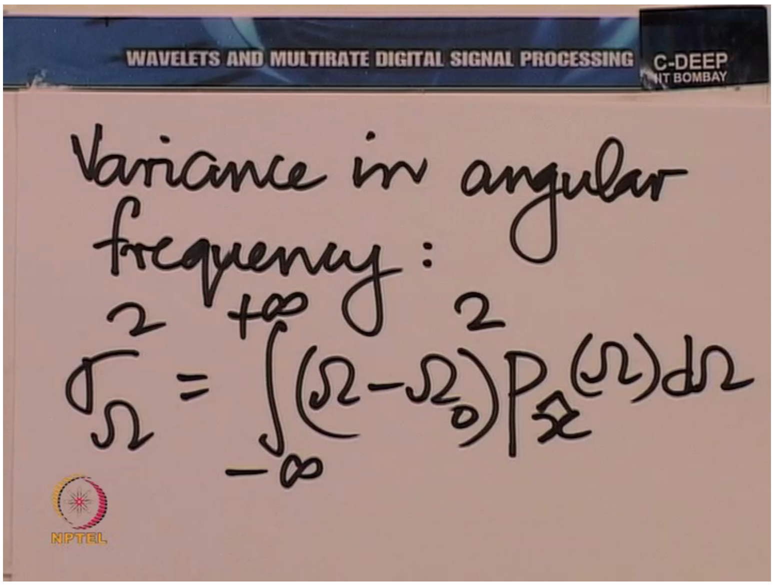

So variance in angular frequency, Sigma omega squared is integral from - to + infinity omega - omega 0 the whole squared Px cap omega d omega. Once again, tongue in cheek, we are assuming these variances to be finite, in any case, here we do not have such a problem. Even if the variances are infinite, we will accept it, we will say that is extreme and the worst case. So whatever they be, finite or infinite, we accept.

Now it is very clear, you see few look at a probability density or perhaps if you were to choose, to think of these as one-dimensional masses, it is very clear that the variance is an indication of the spread. So the larger the variance, the more the density is set to have spread around the mean. The smaller the variance, the more that density or that mass is said to be concentrated. So now we have a formal way to define containment. In fact we shall now make a very simple definition.

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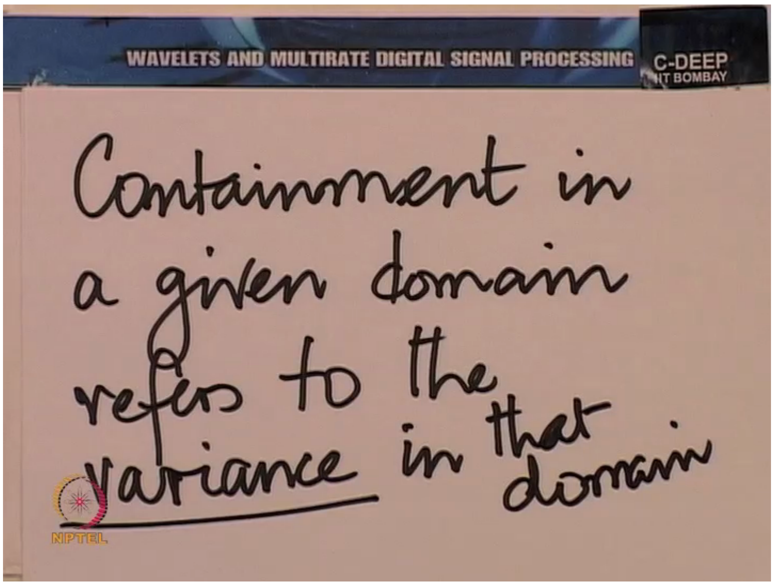

We will say containment in that particular domain refers to the variance, or if you like the square root of the variance, positive square root of course. So let us put down the statement formally. We will say containment in a given domain refers to the variance in that domain. So containment in time is essentially the Sigma t squared quantity and containment in angular frequency is essentially the Sigma omega squared quantity. Now, we ask ourselves how small can we make any one of these quantities for a function, for a valid function. And in a few minutes will be convinced that there is really no limit.