Foundations of Wavelets, Filter Banks and time Frequency Analysis.
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Week-6.
Lecture - 15.3.
transform Analysis of Haar Wavelet Function.

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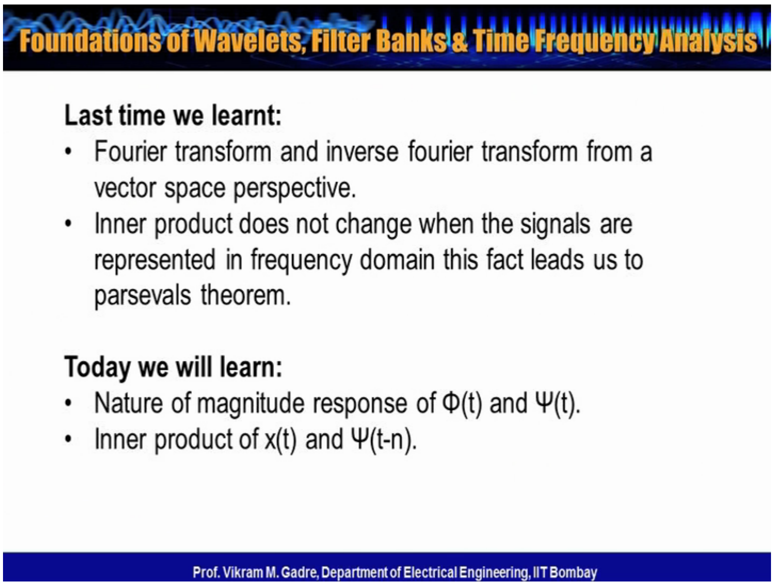

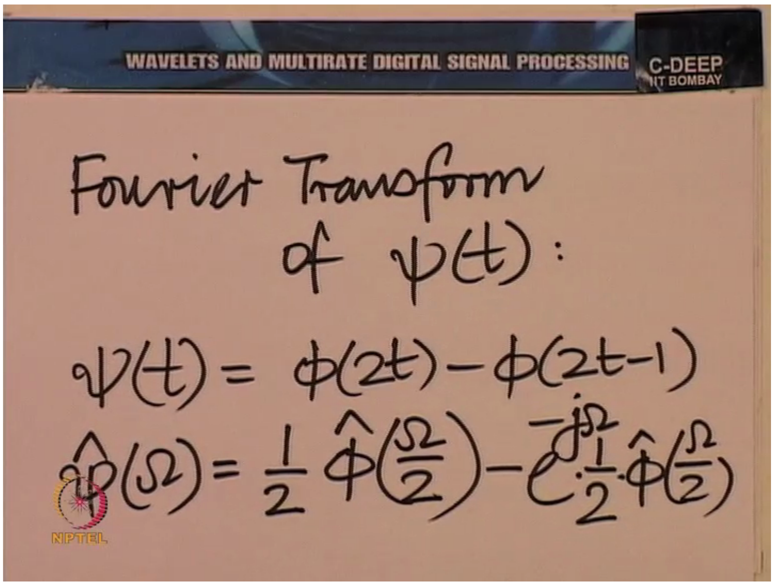

In other words here we have analysed the implications of taking the projection on one of those subspaces in the ladder. What happens when we project on one of the incremental subspaces? that is also an interesting question. So for that we must 1st consider the Fourier transform of Psi t and that is easy to do. Psi t as you know is Phi of 2t - phi of 2t -1 and you can easily find its Fourier transform. Psi cap omega is therefore going to be equal to, well, when you multiply by 2 here you will be dividing by 2 in the other domain and of course here you need to take care of the -1, so e raised to the power - J Omega times the same expression, we can evaluate this.

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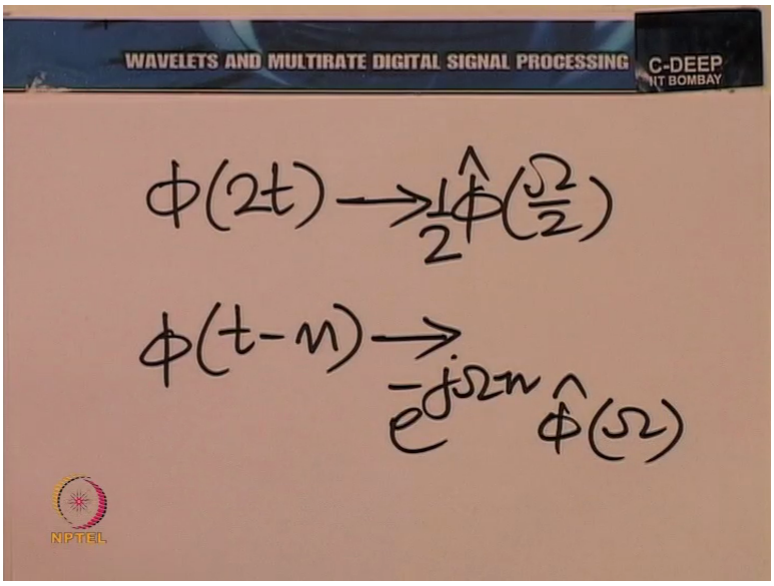

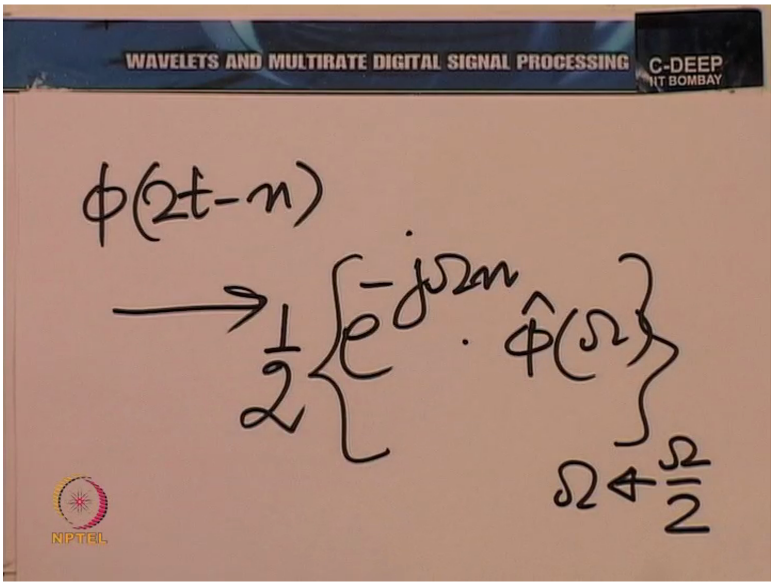

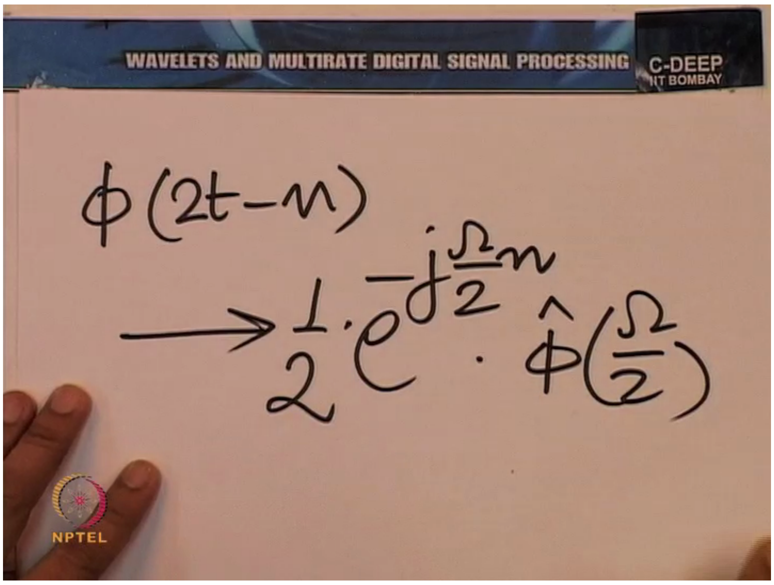

So, you know it is, maybe we should, it would be better to come to this a little more systematically. So Phi 2t would have the Fourier transform Phi cap omega to multiplied by half. Now, Phi of t - n would be general have the Fourier transform e raised to the power - J Omega n times Phi cap omega. And when we consider Phi of 2t - n in general, so here we are going to replace t by 2t, we should take this, replace Omega by omega by 2 and then multiply by half. So this replacement must be in the whole thing and therefore Phi of 2t - n would have the Fourier transform half e raised the power - J Omega by2 times n Phi cap omega by 2.

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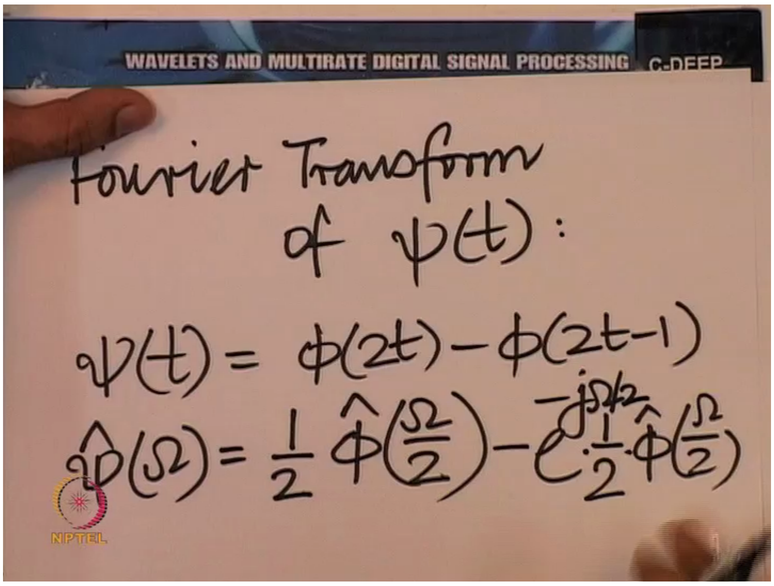

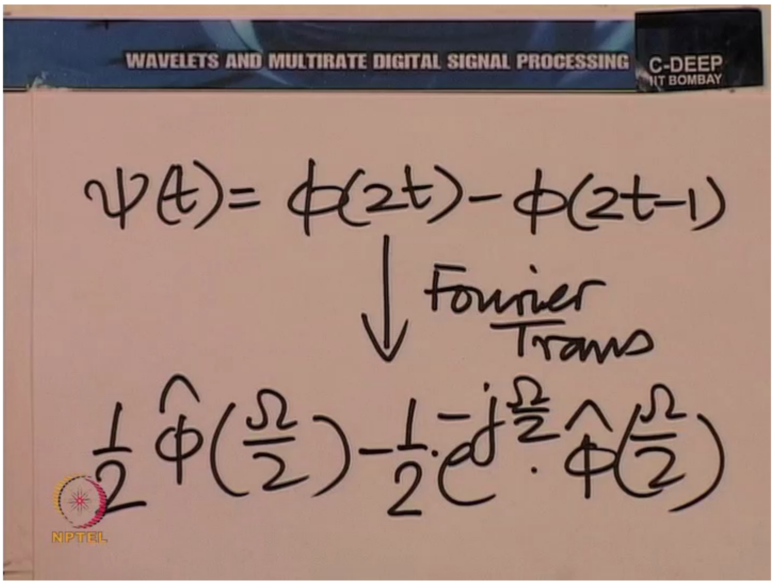

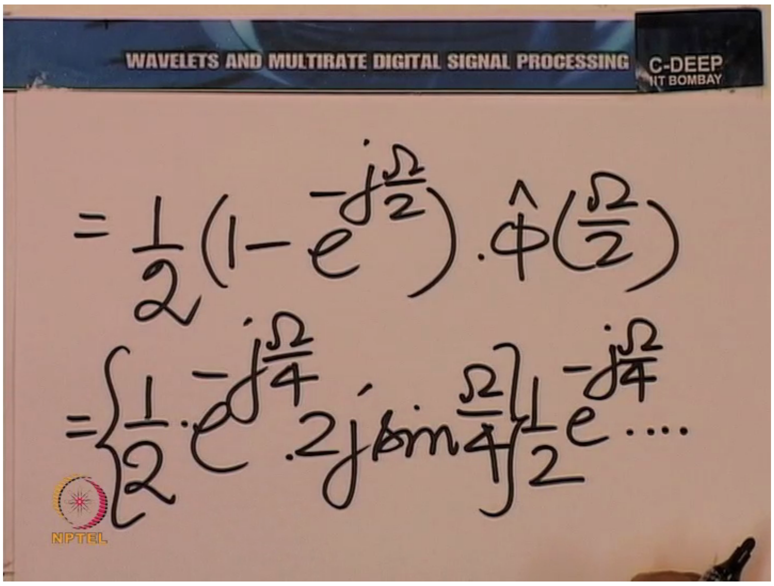

And therefore we need to make a little correction here. The correction is the need to replace Omega by omega by2 here, so in fact let me rewrite that part of the expression, it is more convenient. Psi t which is Phi 2t - phi 2t - 1 has the Fourier transform Phi cap omega by 2 - e raised to the power - J Omega by2 times Phi cap omega by 2. And if we aggregate terms, we have Phi cap omega by2 and this is easy to write, this is half, well, we can play the same trick, we can extract an e raised to the power - J Omega by 4 common from here and get 2 J, I am skipping a couple of steps, sin Omega by 4 and we know the expression for this.

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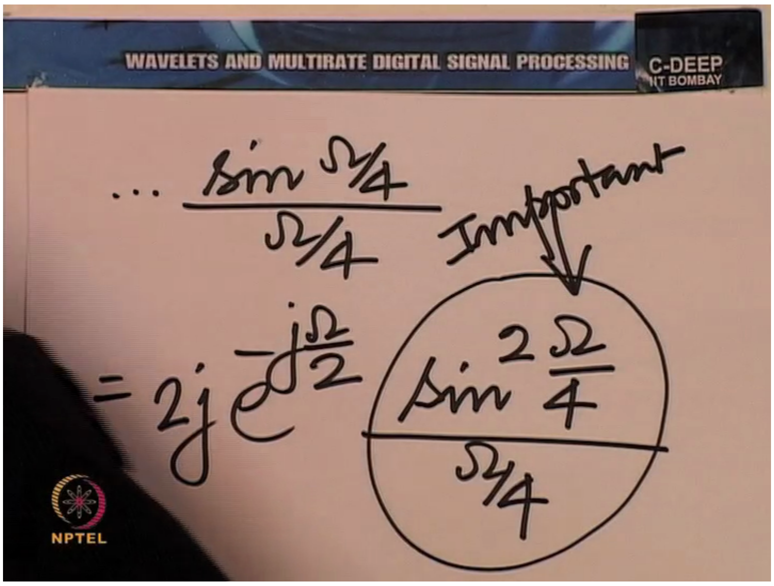

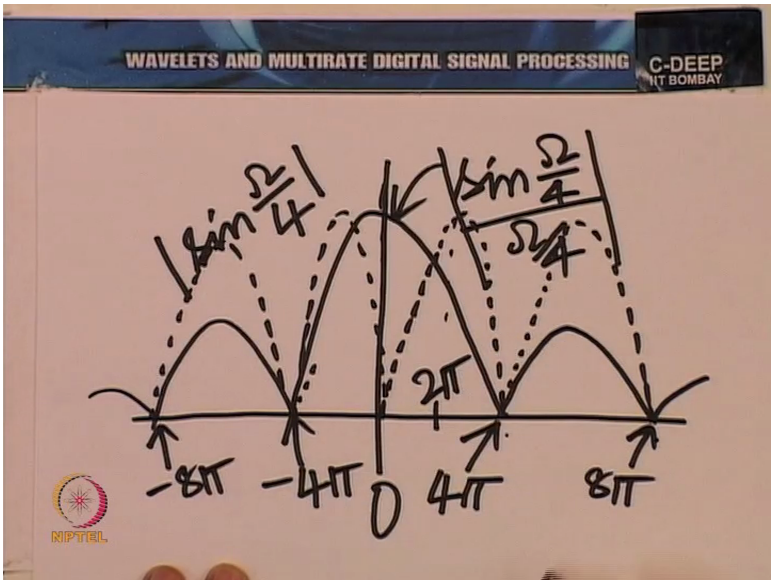

This is essentially 1 by 2 e raised to the power - J Omega by 4 times, well, I will continue on the next page, it is a little complicated but not too difficult. Sin Omega by 4 Divided by omega by 4, in fact let us multiply all this and put it together. So I have e raised to the power - J Omega by 2 coming together J there sin squared omega by 4 divided by omega by 4 and a factor of 2 remains outside. So what we need to focus on here is essentially this part, this is important, the rest of it is not, because it is this that really affects the magnitude seriously, rest of it is essentially a constant magnitude.

So let us look at the magnitude of sin square omega by 4 by omega by 4. 1st let us look at sin Omega by 4 by omega by 4, that of course would look like this, you see this is the point 4 pie, - 4 pie, and so on. And then on the same graph I will draw, this is mod sin omega by4 by omega by4 and on the same graph I will show in dotted mod sin omega by4 once again. So that is going to look like this, it is going to have a maximum at pie by 2 and then it is going to, see omega by4, so one cycle will be completed when omega by4 equal to pie, that is omega equal to 4 pie, so one cycle is being completed here, Mod sin is going to look like this, so in a span of 4 pie you have one cycle, one half cycle being completed.

Right, so this is one of cycle, now in the next span of 4 pie again you have one more half cycle being completed like this, this is the situation. Now look at the situation, this is, this was the zero Frequency here, the solid line has a maximum at 0 and in tapers off up to 4 pie on both sides. The dotted line reaches the maximum in between at 2 pie, then omega by 4 is equal to pie by 2, all right, so omega is to pie, this point, the maximum here occurs at 2 pie. And therefore when you multiply this decreasing function by an increasing function here, there is going to be some point of maximum in magnitude somewhere in between 0 and 4 pie and it is going to taper off to 0 at 4 pie again.

So let me focus, if you see in the other side lobes it is easier to understand. For example between 4 pie and 8 pie, it is very easy to understand. You are multiplying essentially to similar looking functions and so you can see there is going to be maximum somewhere, it is going to taper off. And it is also clear that the maxima in the other side lobes are going to be bigger than the maxima in the main lobe. All this, so when you multiply the dotted function and the solid function, you are going to get pattern something like this, which I now sketch.

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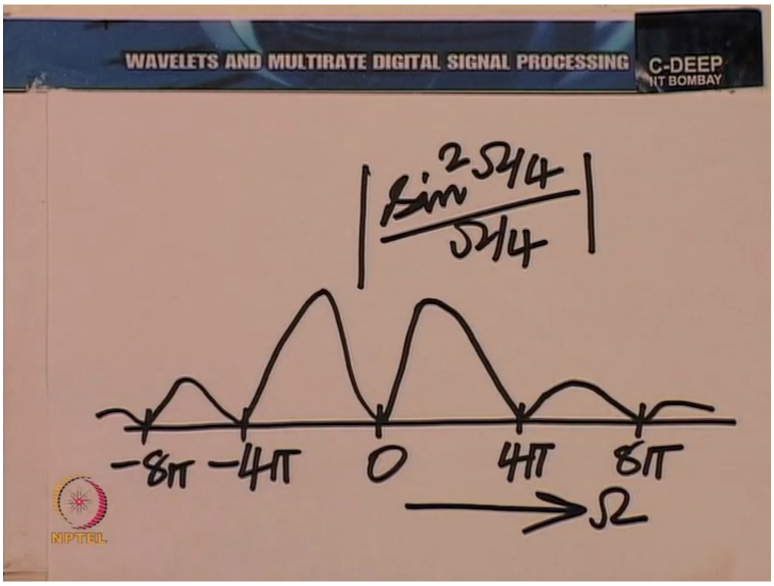

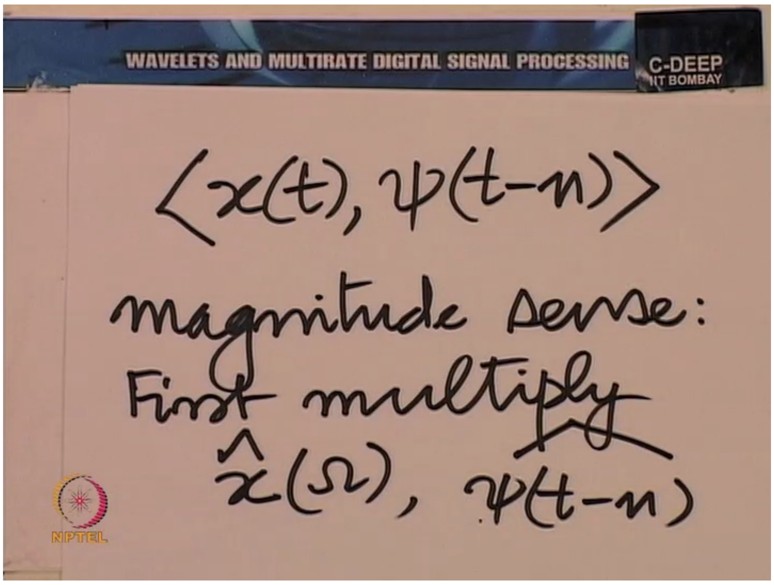

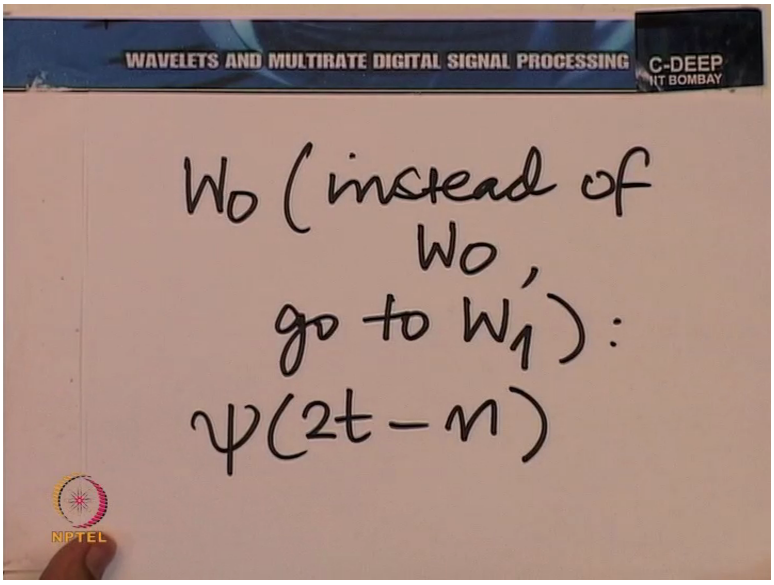

Something like this, this is going to of course be mirrored on this side and then this way and this way. So this is mod sin square omega by4 by omega by 4 as a function of capital omega. Let us take a minute to look at this. When we take an inner product, an inner product of a function xt, now here I am writing t to explain that the inner product in time with say Psi of t - n. In the magnitude sense what are we doing? We are 1st multiplying the Fourier transform of xt with the Fourier transform of Psi t - n, this should be understood to mean the Fourier transform of Psi t - n, a cap on the whole function.

So when you multiply them and then integrate, that part of the Fourier transform of Psi t - n which is significant in magnitude is going to be extracted out of the Fourier transform of the original function x and what is that significant part, let me put it back here for you. It is therefore going to extract, give emphasis to a band now, not a band around 0, a band around some other Frequency here. Of course there is symmetry, there is always symmetry in frequency, so you could focus on the positive side of the frequency axis. But what is going to be done is to emphasise a band here.

And instead of Psi of t, if you take Psi of 2t for example, so if you take, say instead of W0 if you go to, instead of W0, go to W1. In which case you would have Psi of 2t - n and how would the Fourier transform of Psi of 2t - n look? So if you take the magnitude, if you take the Fourier transform of Psi of 2t - n and plot its magnitude, its appearance would be something like this, it would be stretched. So instead of going from 0 to 4 pie now, you would have a band between 0 and 8 pie, of course symmetrically on the negative side.

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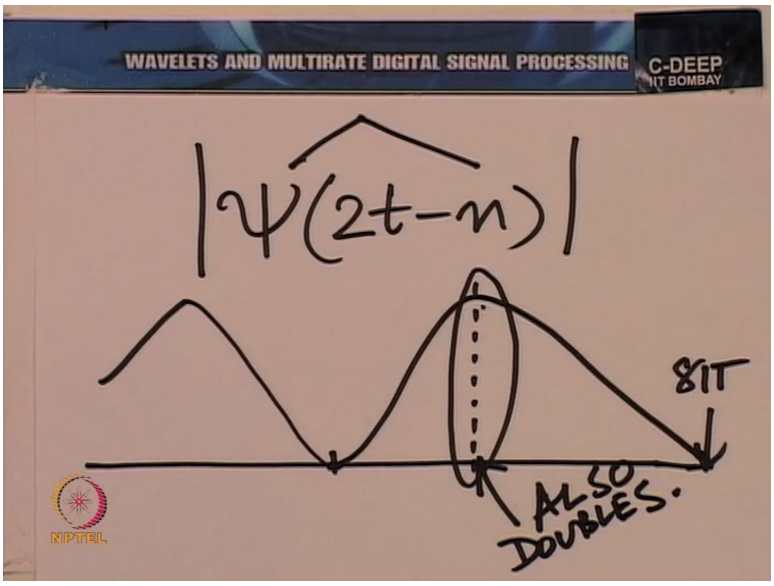

Now there are 2 things, you see, you must keep in mind that unlike the case of Phi, in Psi there are 2 changes taking place. One is that the band expands, the main lobe expands and each of the side lobes expand. But 2nd is that the Centre frequency, the point where this was a maximum here, you see this Centre frequency also doubles, both the band and the Centre frequency double. So that means now you are emphasising the different band, the Centre frequency is different. Each time you go to a different omega, different W, you take W0, you are emphasising one band, when you take W1, you are doubling the Centre frequency, so you are emphasising a different band and of course the band itself has doubled also.

When you take W2, you are again doubling the Centre frequency so you are emphasising a different band and again the band has the world. So each time you are doubling the band and you are doubling the Centre frequency. So in a certain sense that idea of complementarity, you know each time you put one incremental layer, you are putting one more band and the band says is doubled here. Now, where in all this is our discontent with the Haar, why are we discontent? The discontent is because even if we say we are emphasising a band, it is only true to a certain extent.

Let me put back the Fourier transform and illustrate. We might say to an approximation that we are emphasising this band, but that is only approximate. There also to some extent keeping this band and this and then of course the negative corresponding pieces. And that is where we are not content with the Haar. We want to keep a certain band, focus our attention on it, we do not want interferences from the other side lobes that are there. And in going to other multiresolution analysis we are essentially trying to reduce that unwanted presence of the side lobes as much as we can. We have given a feel of what our discontent is like, we shall build on this further in subsequent lectures. Thank you.