Foundations of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-6.
Lecture -15.2.
Revisiting Fourier Transform and Parseval's Theorem.

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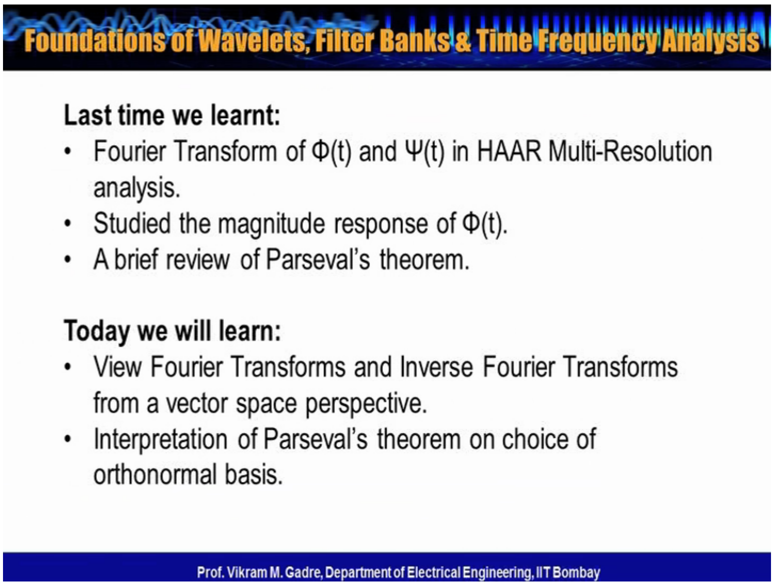

You are representing a function with respect to the basis formed by the rotating complex exponentials or the phasors for different frequencies Omega. The Fourier transform is essentially a projection of a function, in this case a function in L2R on a particular element of that basis, a particular rotating phase. The inverse Fourier transform reconstructs the original function from its components. It is worth recalling some of these points, finer points because it helps for them to be firmly embedded in our consciousness in a course like this. So even if it means a little bit of repetition, let us emphasise those points again.

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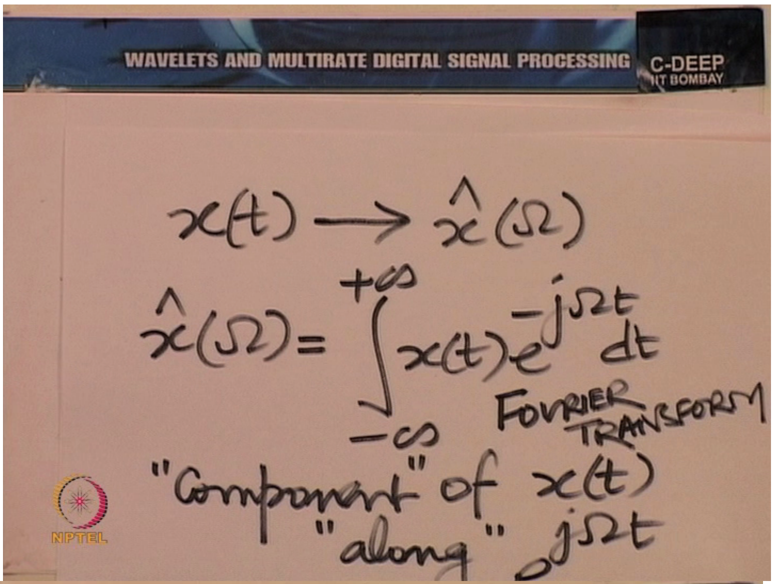

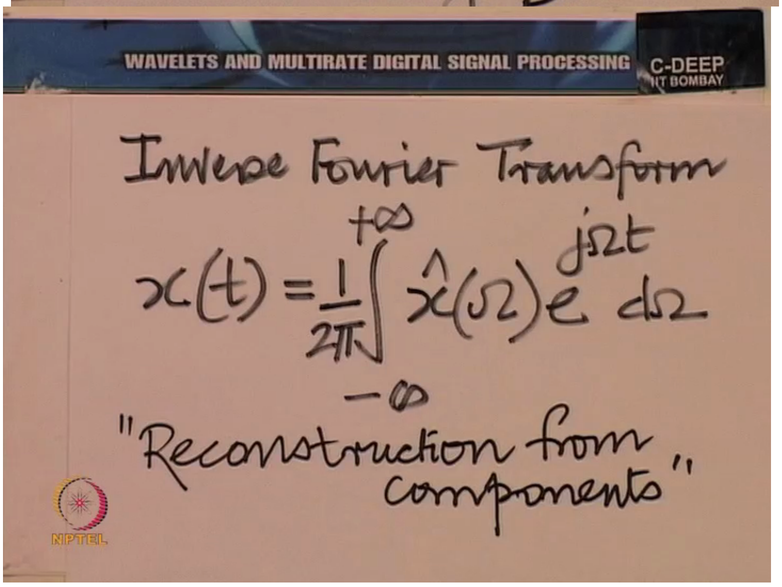

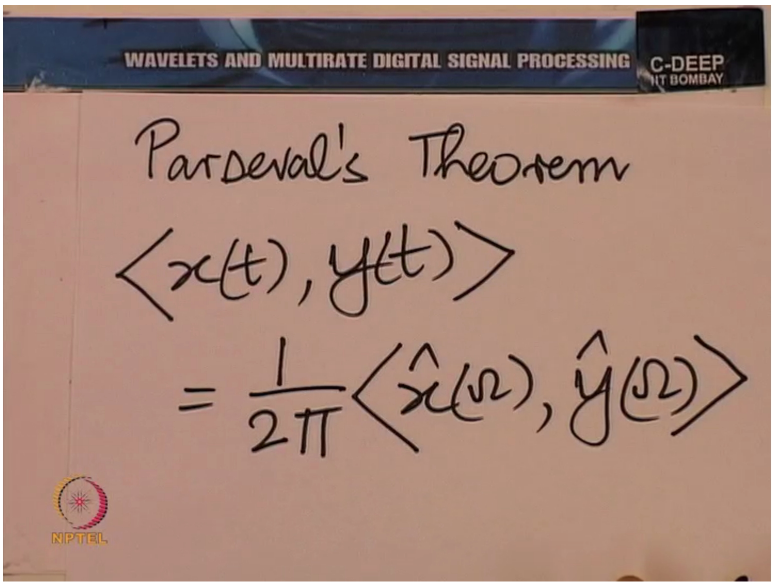

What we said was if I took the Fourier transform, so if I take xt and its Fourier transform X cap Omega so to speak, then X cap omega is essentially a projection xt e raised to the power - J Omega T DT. A component of xt along e raised of the power J Omega T. This is a Fourier transform and the inverse Fourier transform, this is, so let us write that down, this is the Fourier transform. And we have the inverse Fourier transform which reconstructs xt from its components with a factor of 2 pie here. And you will recall the interpretation that we gave this.

We had said that essentially, this is the component along a particular omega and this is the so-called unit vector with the factor of 2 pie. So if I take this and this together, it is like a unit vector here. And what we are seeing here is the original function is essentially the component multiplied by the unit vector integrated over all the components. Reconstruction of a vector, reconstruction from components, that is the interpretation. Now, with this background what we have said in Parseval's theorem is the following. These are products of xt and YT, so, you know you are talking about 2 different domains is equal to the inner product of X cap omega and Y cap omega but with the factor of 1 by 2 pie.

In the language of components, what is the interpretation? The interpretation is that it in calculating the inner product it does not matter whether one is using one orthonormal basis or another, the result is the same. So the inner product is not a function of which orthonormal basis you use, you could be using a time basis, you could be using a frequency basis, you could be using impulses in time, you could be using the complex exponentials. The inner product has nothing to do with the choice of basis. The inner product between 2 vectors remains the same whatever basis we choose to express the vectors, that is the statement being made in Parseval's theorem here.

We are saying represent the functions in the natural basis of impulses or represent them in its Fourier basis, the inner product is the same. Of course to the within a factor of constant, this constant appears because of the angular frequency radians, radians per second I mean, otherwise if you were to take heard frequency, this factor would be absent as well. Anyway, this was an important result that we had seen when we looked at functions from a perspective of vectors. And now we shall use this Parseval's theorem to interpret this idea of projection onto VM.

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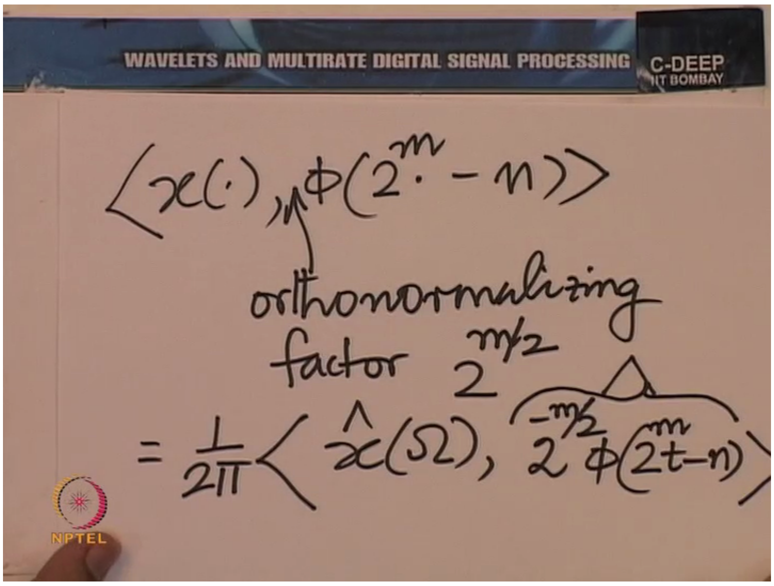

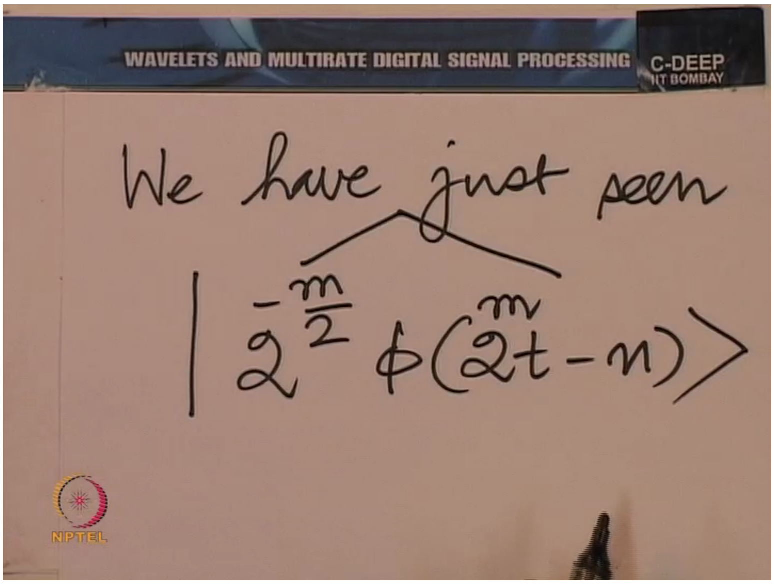

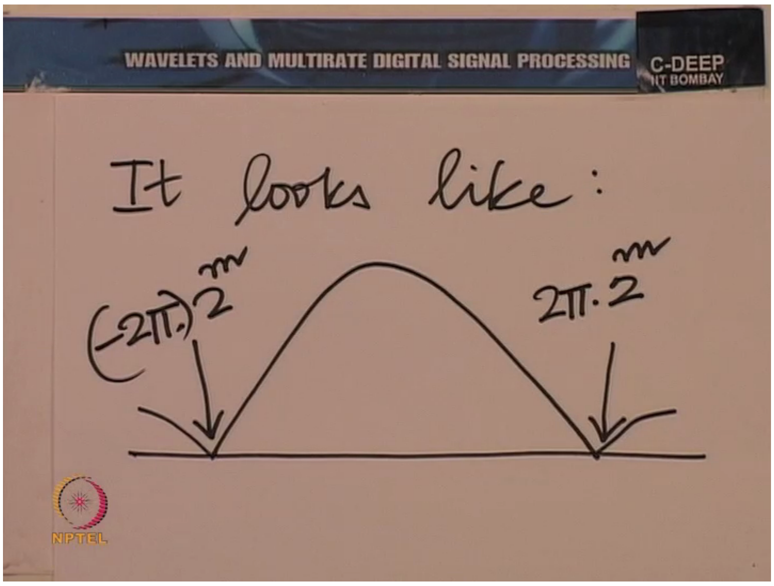

So when we consider the inner product that we were doing a few minutes ago, the inner product of X with Phi 2 raised to the power m t - n and now if you wish to make it orthonormal, then introduce the orthonormalizing factor, 2 raised to the power m by2, this is going to be equal to 1 by 2 pie times the Fourier transform of each of them. So the Fourier transform of X, of course with its argument, the argument remember is going to be omega here. And I shall write here the Fourier terms of this whole thing, so please do not misunderstand what I am writing to be the expression itself but understand it to be the Fourier transform of this.

So you have 2 raised to the power mt - n but take the Fourier transform, so I am saying Fourier transform of this whole thing here. Now I would like to interpret this graphically 1st. What are we going to do when we take the inner product in the Fourier domain? So what is this going to look like? We have just seen the magnitude, we have just seen the magnitude of this quantity and that looks like this. This is the place where you have 2 pie multiplied by 2 raised to the power of m. So in effect you have expanded that main lobe and all the side lobes by a factor of 2 raised to the power of m, in particular which m equal to +1 you have expanded by 2, if m is equal to -1 you would have contracted it by factor of 2 and so on.

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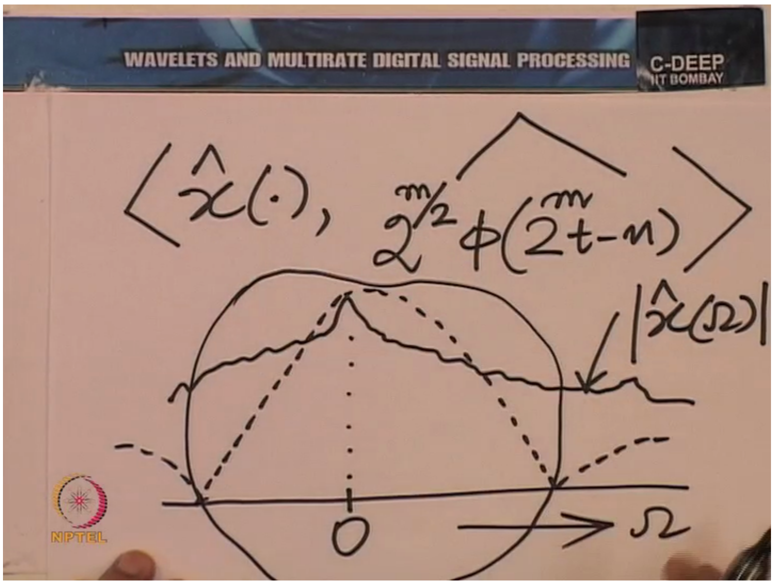

So when we take the inner product, in the inner product X cap and the cap of this, what are we going to do, we have the Fourier transform, so let us understand it graphically as I said. Let us say this is the Fourier transform of X, whatever it be, I mean let us understand in the magnitude sensors. So let this be the magnitude of the Fourier transform of X, this is 0 let us say here and this is the magnitude of the Fourier transform of the other argument, I am showing only the main lobe and a part of the other side lobes. When you multiply them, their magnitudes are going to get multiplied and therefore essentially you are going to extract this band so to speak.

In a notional sense we are going to emphasise most the area of the Fourier transform around the main lobe of Phi. You see after all it is the magnitude which plays a significant role here. When we take the dot product we are going to multiply the 2 Fourier transforms and integrate over all frequencies. Where the magnitude is larger, the contribution will be larger, where the magnitude is smaller, the contribution will be smaller. So the side lobes would kind of suppress that part of the Fourier transform and the main lobe will emphasise the corresponding part of the Fourier transform contained under the main lobe.

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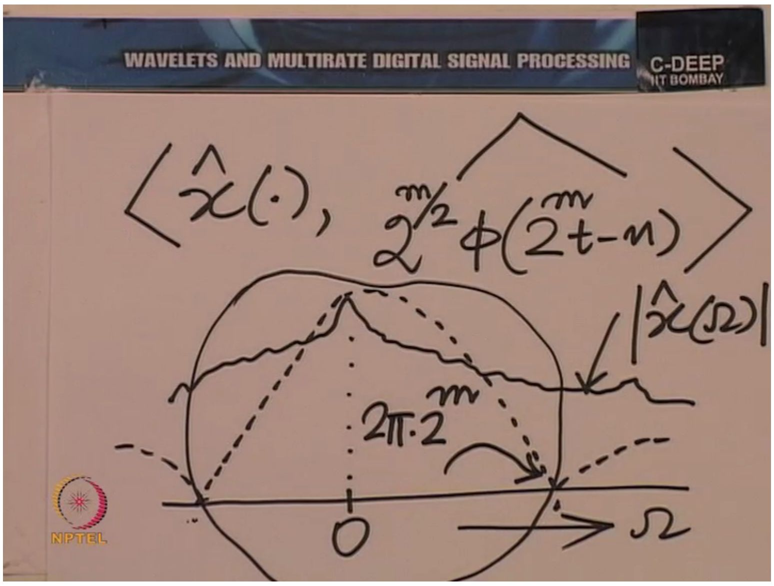

What we are saying is that part of the Fourier transform of the original function X which is contained in the main lobe is emphasised as again all the rest in calculating the area. Now this also gives us an interpretation of what happens when we increase or decrease m in frequency. In fact let us look at this drawing once again. This point is 2 pie multiplied by 2 raised to the power of m. So take for example m equal to 0, you are essentially, this dotted line here is the magnitude of the Fourier transform of the appropriate dilate of Phi and of course we do not care so much about the translate, the effect of the translation is only to change the angle and it does not reflect in the magnitude.

So for m equal to 0 we are essentially emphasising a region of the frequency axis broadly speaking between - 2 pie and +2 pie. When we take m equal to1, we are emphasising region between - 4 pie and +4 Phi. When we take m equal to -1, we are emphasising a region between - pie and + pie and so on so forth ad infinitum. What are we saying? When we increase m, m1, 2, 3 and so on, we are effectively keeping more and more information around the 0 frequency, we are broadening it. Of course we are narrowing in time but we are broadening in frequency, so we are keeping a larger band of frequencies but all around the 0 frequency. Now, what happens when we consider psi t that is an equally interesting interpretation, let us do that.