Foundations of Wavelets, Filter Banks and Time Frequency Analysis.
Professor Vikram M. Gadre.
Department Of Electrical Engineering.
Indian Institute Of Technology Bombay.
Week-5
Lecture - 14.2.
Building Compactly Supported Scaling Functions.

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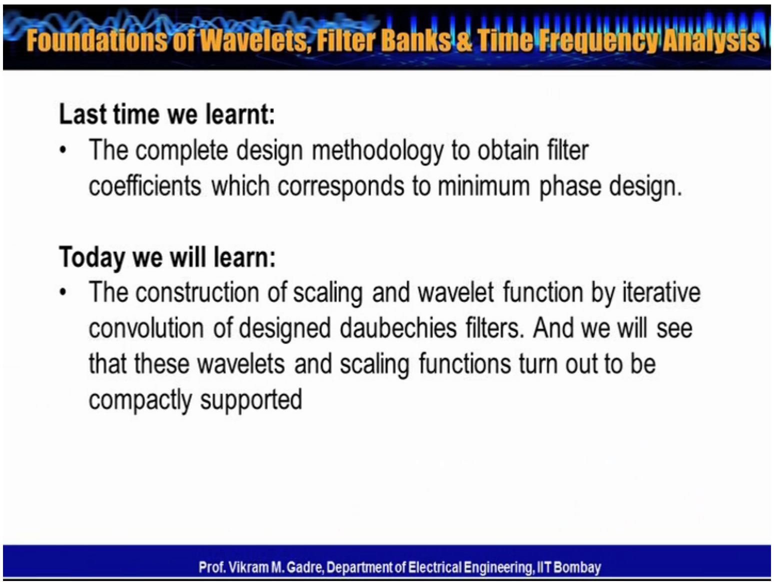

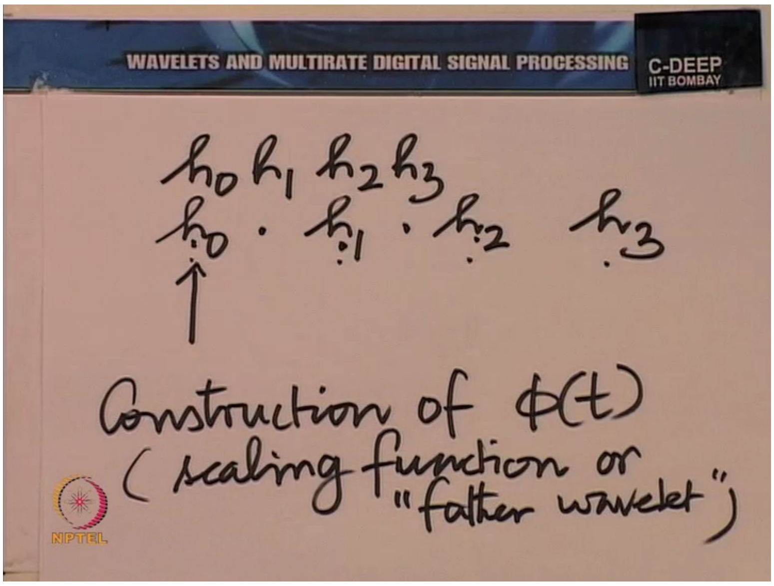

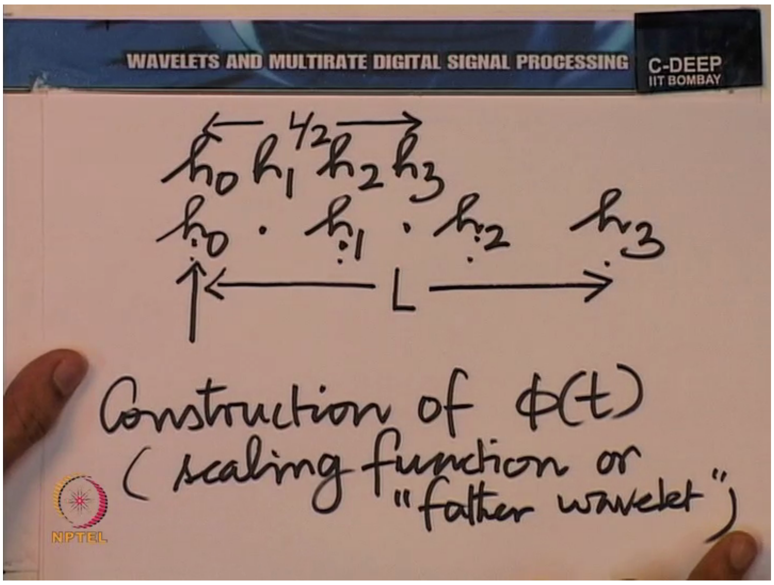

So there again we go back to that iterative convolution that we talked about. So essentially we would have a situation where you 1st construct h0, h1, h2, h3 lying on, you know so the situation is you need to compress and convolve, compress and convolve, as we did in the haar case as well. So we would start essentially by putting h0, h1, h2, h3 like this, then, now remember what we are doing here is to construct Phi T, the scaling function or the so-called father wavelet.

If you recall the basic step everytime was 1st to take the sequence and then to take the sequence squeezed on the time axis by a factor of 2. So you know if you put the sequence at these locations, now put the sequence at the following locations h0 h1, h2 h3, here, these locations are midway between these 2 locations here. So midway between these is this and midway between these is this and put h0, h1, h2, h3 here. Visualize these to be impulses of strength h0, h1, h2, h3 and here again impulses located at these places, again which spreads h0, h1, h2, h3 and convolve this with this essentially.

So you know it is an iterative convolution. And you can visualize that you would get impulses in one step of this convolution from 4 you will get 8 and in fact this would stretch a little beyond as you can see. In the next one you would again get this squeezed by a factor of 2. Now in this case it is much more difficult to visualize where this convolution is leading. What I have given you is the mechanism and in fact it is a very important exercise, a simple but very effective computer exercise in this course to actually carry out this convolution. What kind of Phi T would result?

The answer is not trivial. It is unlike the haar case where you had a very neat answer, you got a nice, beautiful rectangle Pulse of height 1. That is not the case here. The impulse response coefficients are somewhat complicated and when we start involving them, you have 4 of them, you are convolving 4 with 4 squeezed by factor of 2 and this is going to go on. What you can see of course is that ultimately all these impulses are going to come over a finite area, that is a point of observation. I want to explain this point to you. It is a subtle point but not very difficult to understand. You see, let me go back to this drawing. The 1st time you carry out this convolution, this with this, you are going to have an extension of the length over which these impulses life.

So after this step you can visualize h0 coming here, so the spread would be up to 3 of the samples going beyond h3. The next time you would have half of this interval getting added on there. So if I call this interval L, then you have interval L by 2 here, let me draw the situation carefully, let me draw it for 2 or 3 steps.

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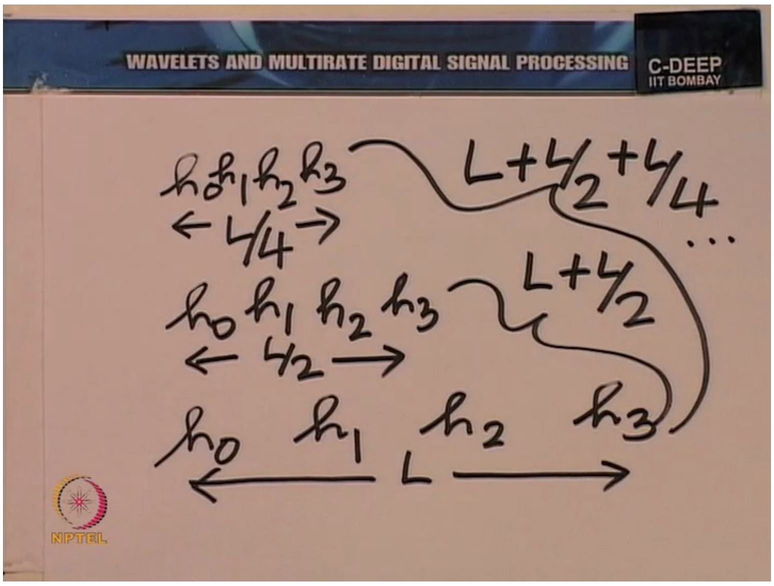

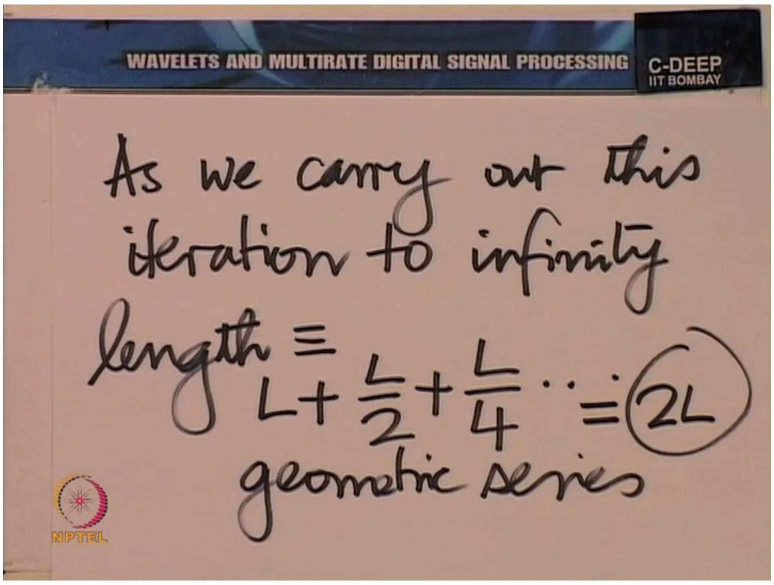

So what I am saying is, it is a convolution something like this. You start with a train of impulses which is spread over a length of L, the next time the same train is squeezed by a factor of 2, so it spreads over interval of L by 2. The next time it is squeezed again by factor of 2, so you know I will draw them squeezed together like this and this L by 4. And when you convolve this length L with length L by 2, you get at this stage length L + L by 2. And when you convolve this with this, you would get L + L by 2 + L by 4 and so on so forth.

So you can see that the length is not going to go to infinity but it is going to converge. And what is it going to converge to? Very simple, as we carry out this iteration to infinity, the length is going to converge to L + L by 2 + L by 4 and so on, it is a geometric series. And the sum is very easy to calculate, it is 2L. So what we can see for sure is that this leads to what is called a compactly supported Scaling functions. When we iterate these impulses with a contraction everytime, we ultimately get a function lying on a compactly supported region.

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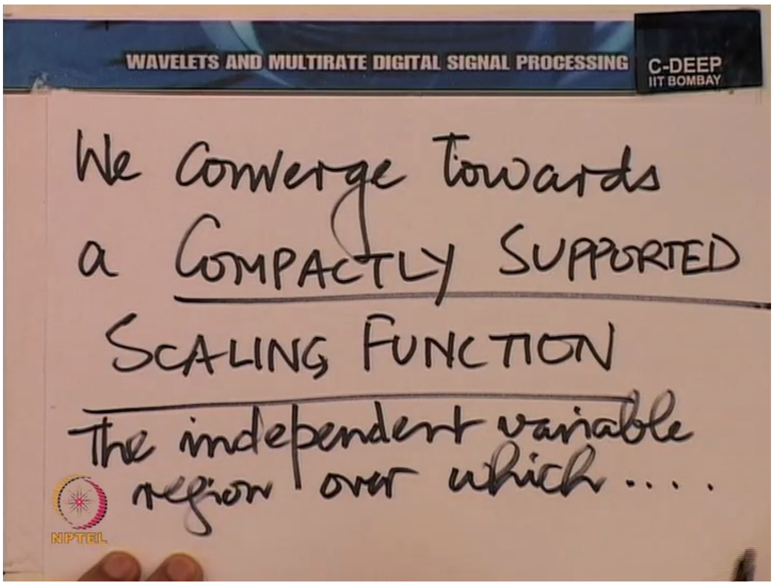

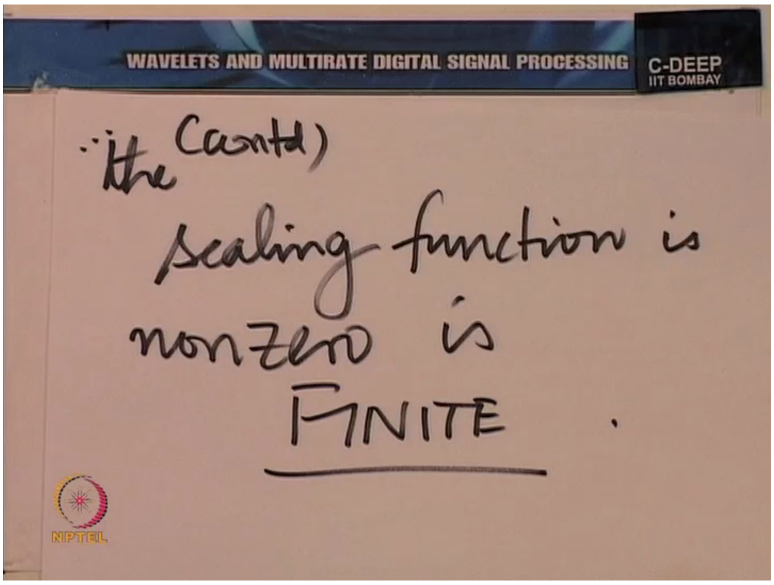

Or a more accurate way of saying that is the function, the scaling function is compactly supported, it has a finite part of the time of the independent variable axis on which it is nonzero. This is an important thing that we should write it down, we should emphasise it. We converge towards a compactly supported Scaling functions. And what does that mean? We are essentially saying the region, the independent variable region over which it is nonzero, so I will continue this over which the scaling function is nonzero is finite.

Now you know this is the most important contribution that Daubeshies made. Before Daubeshies introduced this set of filter banks, the idea of neatly constructed a family of compactly supported multiresolution analysis was not existent to the literature. So I would put this as a very very powerful contribution. That does not mean that the scenes were not there, in fact the subjects of wavelets and filter banks, filter banks as a multirate signal processing paradigm had almost developed in parallel. But using filter banks in an effective way to construct compactly supported Scaling functions and wavelets is an important contribution emerging out of Daubeshies work, the work of Dobash Daubeshies.

Not only just compact support, successive scaling functions here have some additional property in terms of their behaviour with respect to polynomial as you can see. For example, what would happen in the 2nd member of it Daubeshies family, let us reason it out a little better.