Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-5.
Lecture-14.1.
Effect of minimum phase requirement on filter coefficient.

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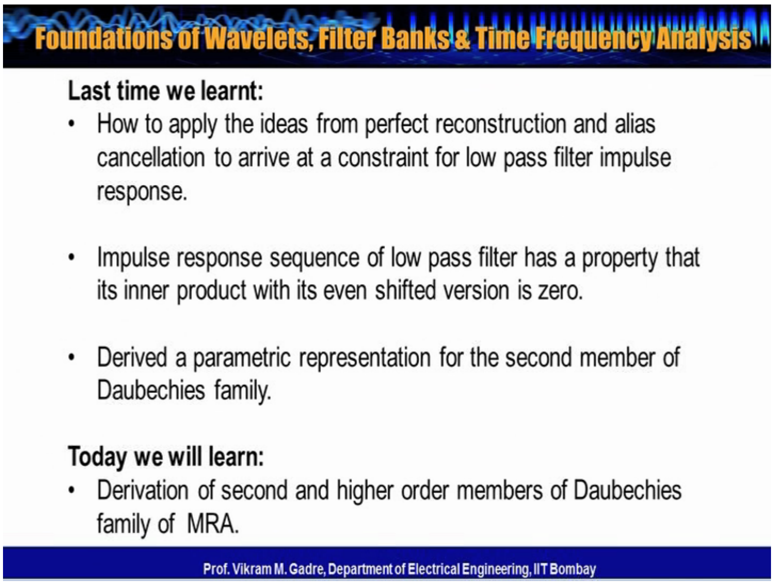

A warm welcome to the lecture on the subject of wavelets and multirate digital signal processing. We continue in this lecture to discuss the Dobash or Deubeshies filter bank which we had very briefly introduced in the previous lecture. I would like to put before you the salient points of that filter bank once again and then complete the design that I had begun in the previous lecture. So recall that we had based the construction of the series of Dobash filter banks on the idea of annulling polynomials of higher and higher degree.

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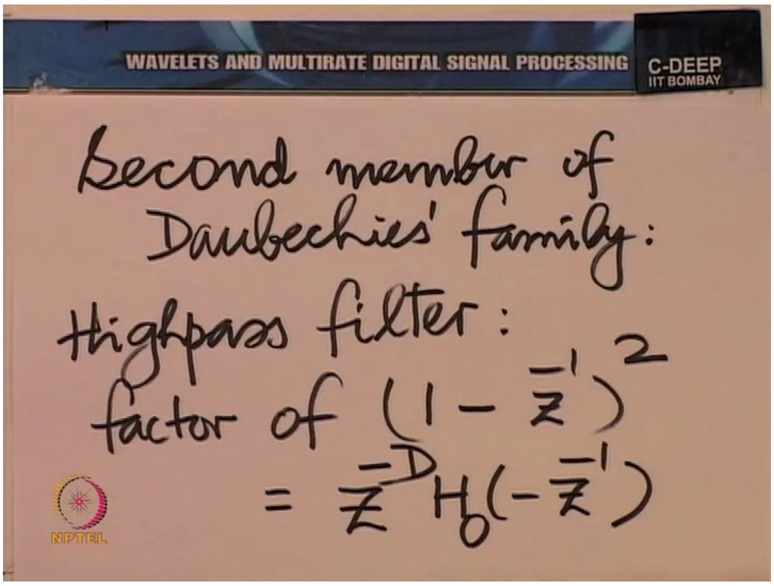

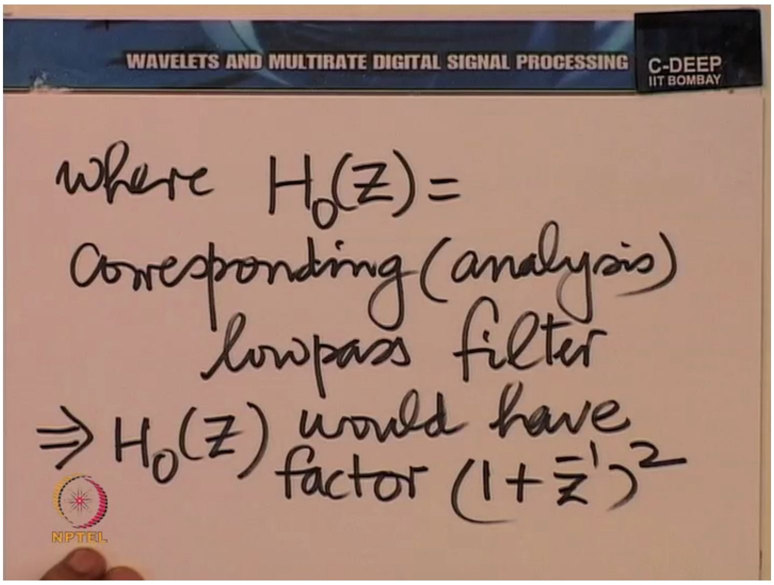

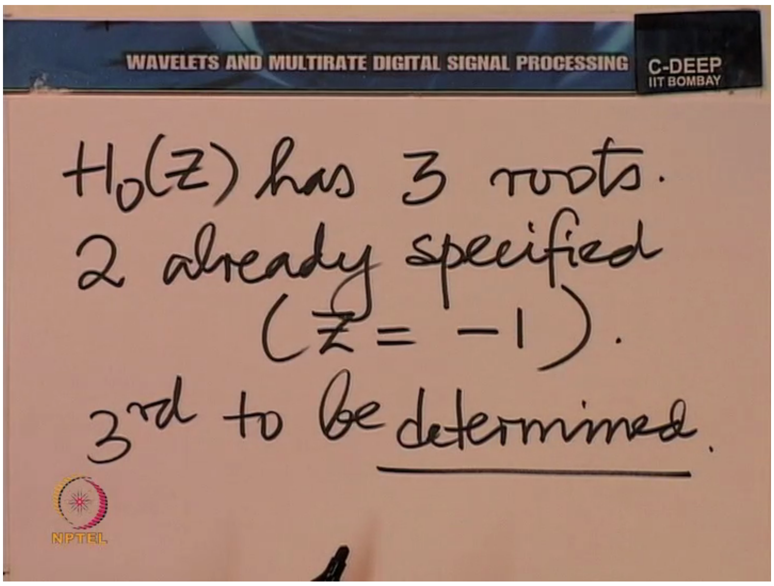

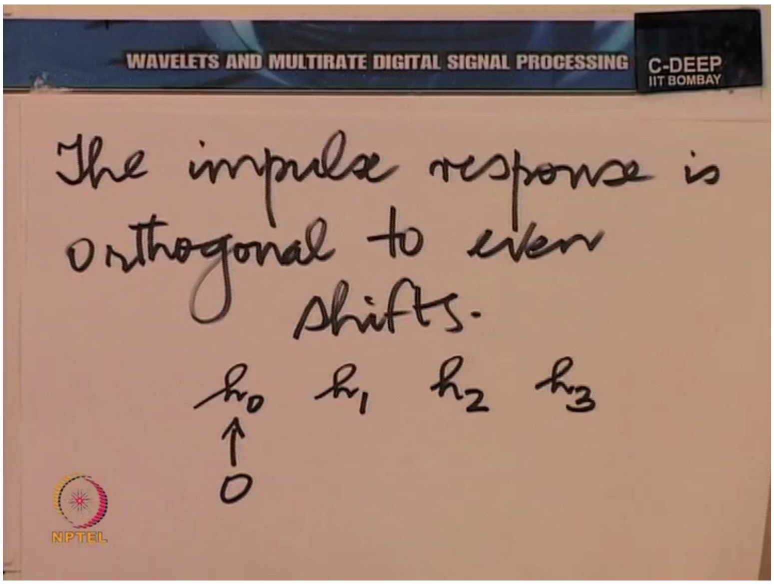

So we said that we wanted more and more factors of the form 1 - Z inverse appearing in the high pass filter. So for example, let me put down the structure of the next member in the family after the haar case where instead of one, we would have 2 factors. What we said was the 2nd member of the Daubeshies family would look like this. The high pass filter would have a factor of 1 - Z inverse the whole squared. Now therefore, if you look at the lowpass filter, remember the high pass filter was essentially of the form Z raised to the power - D H0 - Z inverse.

Where H0Z is the corresponding lowpass filter. Of course I am talking about the analysis side. So essentially the analysis lowpass filter would now have a form like this, H0Z would have the factor 1+ Z inverse squared. And then we recall that we had even length for the filter and therefore we have a situation H0Z has 3 zeros, 2 already specified, obviously they belong to -1 and therefore the 3rd to be determined. And why do we determine this 3rd root from? Well, we go back to the requirement of orthogonality to even shifts.

So recall that we have said that the impulse response is orthogonal to even shifts. That means if I assume that this filter has the impulse response h0, h1, h2, h3 at 0, then this is orthogonal to its shifts by 2, 4 and so on.

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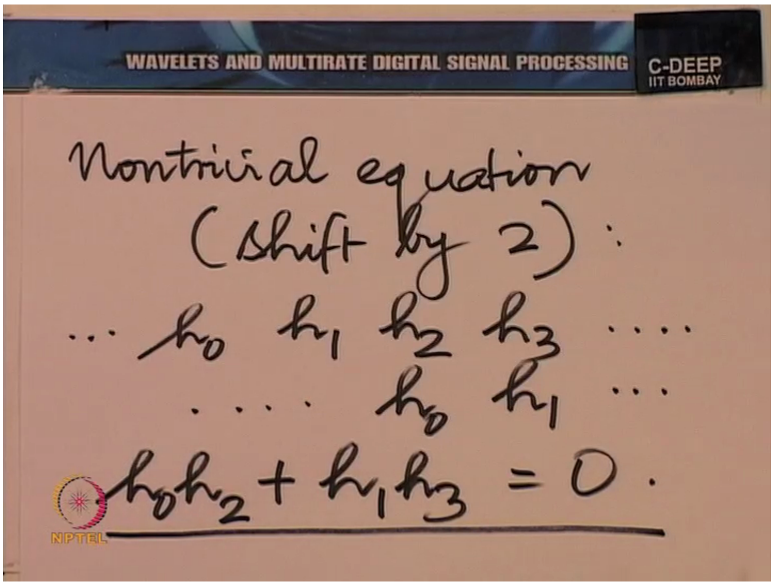

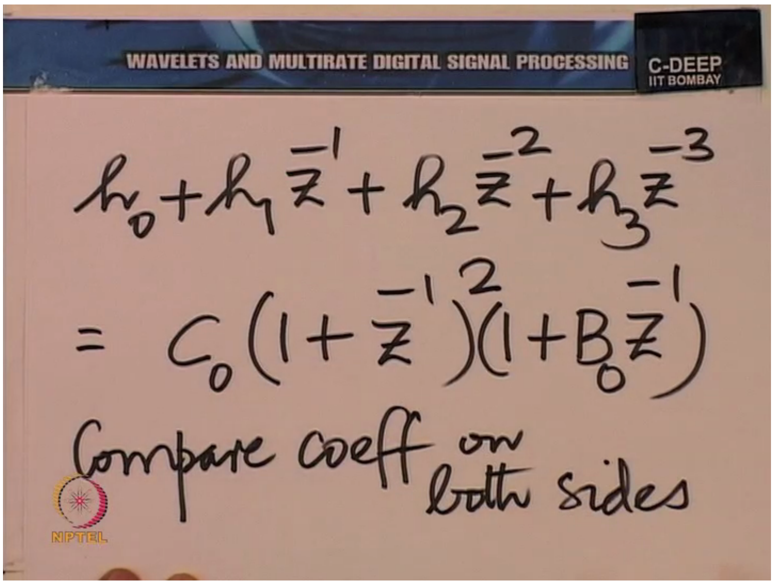

So the nontrivial case, the only nontrivial equation that we get is the following. It comes from a shift by 2 h0, h1, h2, h3 and when it is shifted by 2, you have h0, h1 and then you know there are all zeros after this, there are zeros before and there are zeros before. So all that you have is h0 h2 + h1 h3 is equal to 0, this is the only nontrivial equation that we get. Now from the location of the zeros and the free parameters, we need to express h0 through h3 in terms of the free parameters. So what do we have, we have h0 + h1 Z raised to the power - 1+ h2 Z raised to - 2 + h3 Z raised to - 3 is of the form some constant, let us say C or C0 if you please, times 1+ Z inverse the whole squared times 1+ B0 Z inverse.

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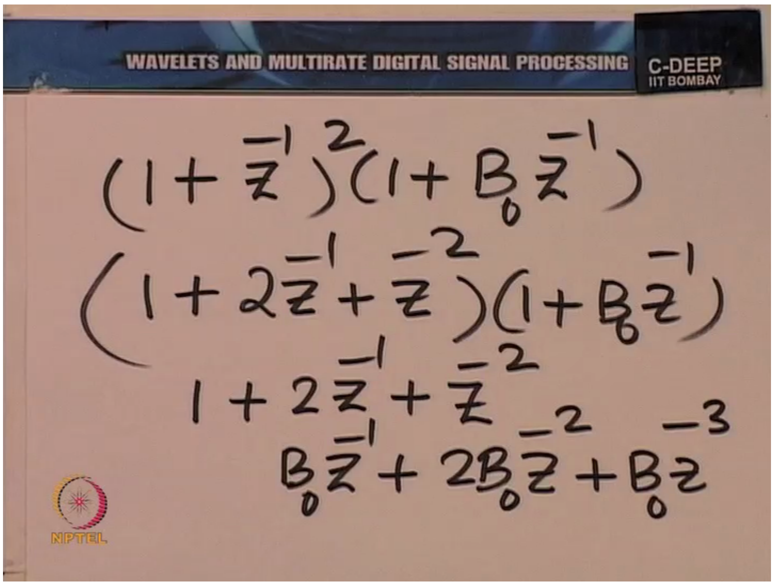

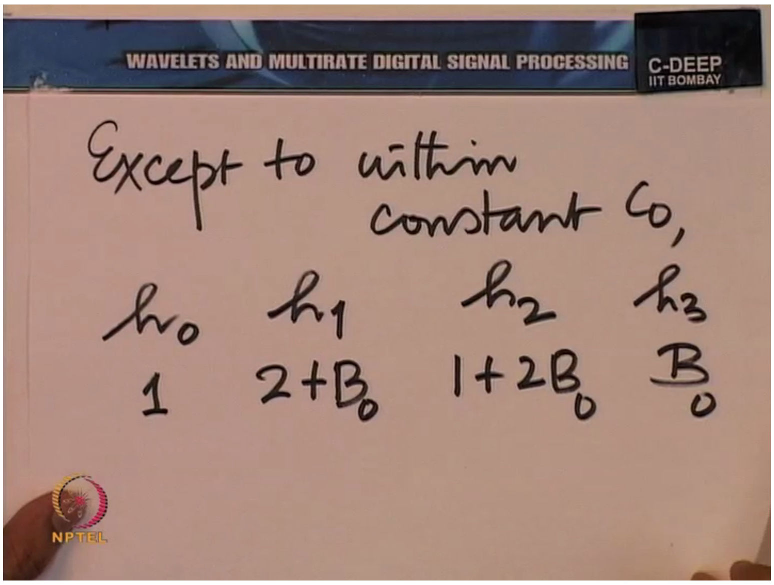

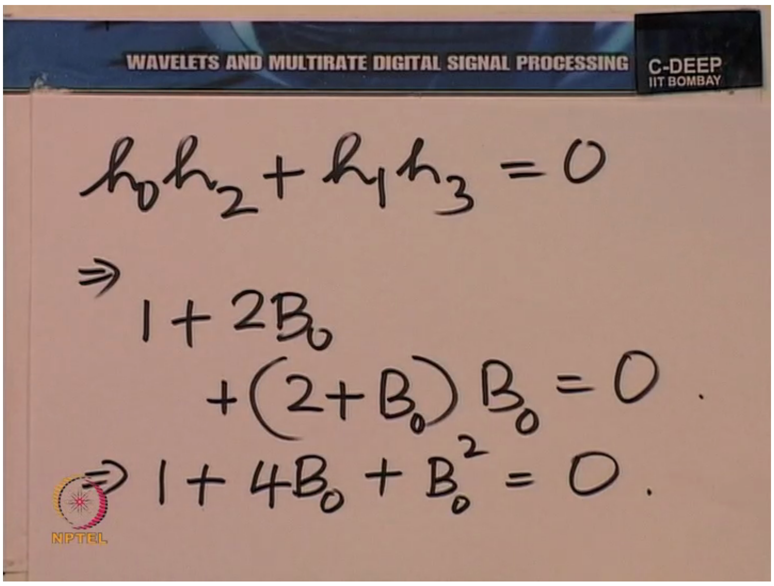

And we need to compare coefficients on both sides. Now the constant C0 does not affect orthogonality. So we shall just focus on the rest of the equation because we 1st need to satisfy the requirement of orthogonality, orthogonality to even shifts I mean. And from there we need to write mean the constant C0. We will see later what helps us determine C0. So in fact if you look at it, what we are asking for is the following expanded term 1+ Z inverse the whole square times 1 + B0Z inverse where B0 needs to be determined. And this as we said could be expanded as 1 + 2 Z inverse + Z raised to – 2 times 1+ B0Z inverse and if we collect terms that gives us 1+ 2Z inverse + Z raised to – 2 + B0 Z inverse + 2B0 Z raised to - 2 + Z raised to the power - 3 times B0.

So we aggregate terms together here, we will take the coefficients. So essentially, you know, except for the constants, h0 is essentially therefore 1, h1 is of the form 2 + B0, h2 is of the form 1 + 2B0 and h3 is of the form B0. And now we can write down the orthogonality equation that we seek. It says h0 h2 + h1 h3 must be 0 implying that 1 + 2B0 + 2+ B0 into B0 is 0, where we are effectively saying 1+ 4 B0 + B0 squared is 0. So there we have an expression for B0, it is easy to determine, so B0 has got constraint as you see, not surprising.

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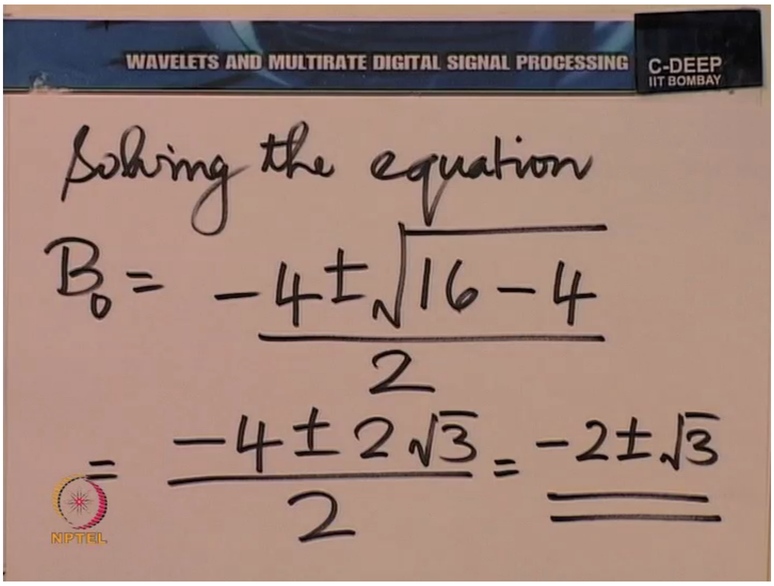

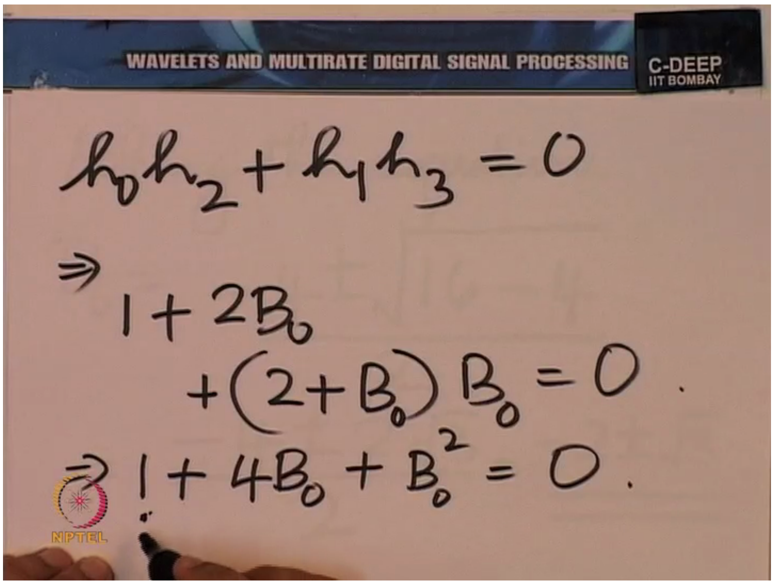

There was one free parameter and one nontrivial constraint. Solving the equation we have B0 is - B + - root B squared - 4 AC by 2 A, that gives us – 4 + - 2 root 3 divided by 2. And that gives us 2 solutions, - 2 + - square root of 3. Now we have 2 solutions here, which of them should we choose? Well, what distinguishes these 2 solutions? It is very clear from the quadratic equation and let me go back to it. You see if we look at this term, this tells us the product of the roots, the term one.

And the product of the roots clearly has a magnitude of 1. So if one of them lies inside the unit circle, the other must be outside. In fact they cannot lie on the unit circle because none of them has a magnitude of 1. And therefore one must lie inside the unit circle and the other outside. Let us take note of that, in fact it is very easy to see which lies inside and which lies outside. If you look at the - 2 - square root of 3 solution, it clearly has a magnitude greater than normal one and the other one has a magnitude less than 1.

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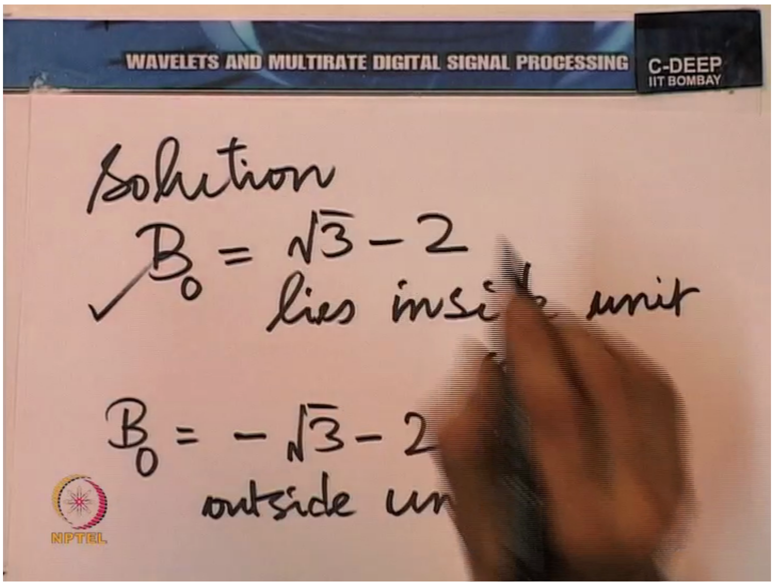

So among the solutions B0 is square root 3 - 2 lies inside the unit circle and B0 is - square root 3 - 2 lies outside the unit circle. Now we have a choice, we could either choose this or this, we shall choose this and there is a reason for it. You see, very often we like to choose what is called the minimum phase solution, there is this idea of minimum phase. I shall not dwell too much on it at this moment. Suffice it to say that minimum phase essentially means choosing all zeros inside the unit circle wherever possible.

In fact the idea of minimum phase comes from what is called minimum phase delay. So when we you know after all one interesting thing is that whether we choose the roots inside the unit circle or outside, the magnitude of the frequency response would be the same, the magnitude would not be different, what would be different is the phase. And if we put the root outside the unit circle, there is going to be an increased phase delay. And very often that would also point to an increased group delay, increased phase delay, increased group delay in the filter.

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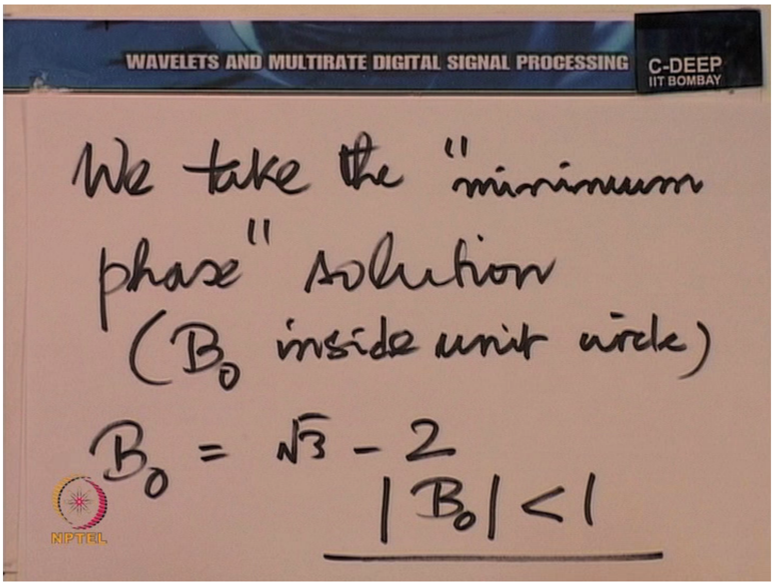

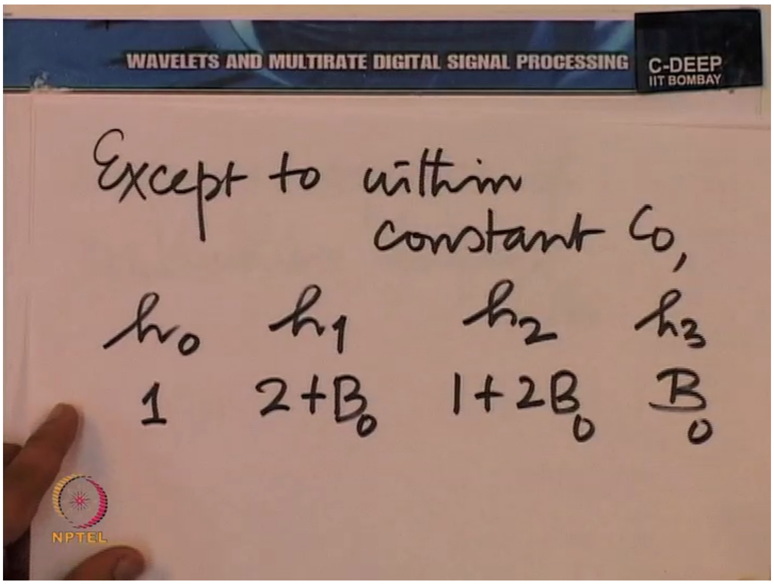

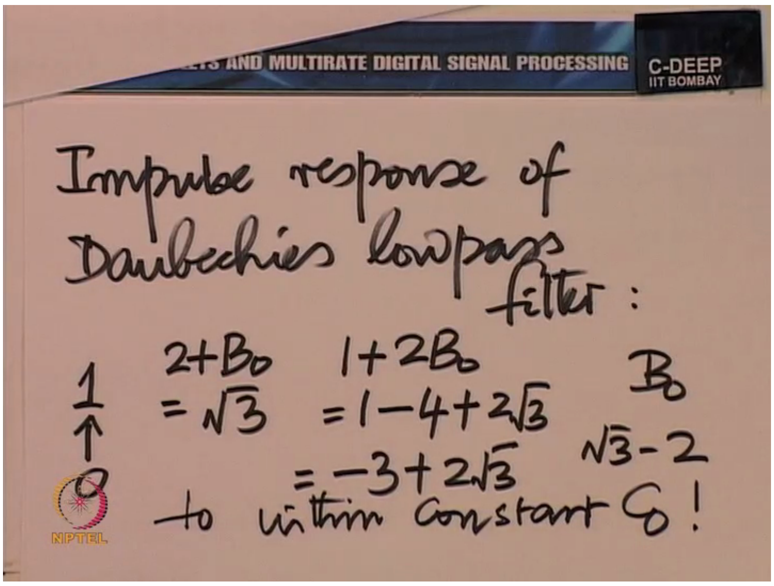

And when we take the solution inside the unit circle, we are reducing the phase of the group delay as much as we can in the filter. So it is essentially a question of choosing the better solution in terms of phase. So let us summarise that. We take the minimum phase solution, that is B0 inside the unit circle and we have B0 is square root 3 - 2, obviously mod B0 is less than 1 here. And one can also evaluate this approximately. If you recall square root of 3 is about 1.73, so this would be 1.73 - 2 and one can come up with an approximate value, that is not really an issue.

Anyway, we can now put down the impulse response of the Daubeshies lowpass filter therefore. It is essentially, well let me read off the impulse response from here. Let me flash the impulse response before you once again and then we will substitute. So this was the impulse response, now we substitute for B0. So 1, 2 + B0 which is essentially square root 3 and then you have 1+ twice B0 which is 1 - 4 + 2 root 3 or – 3 + 2 square root 3 and finally B0 which is just square root 3 - 2. So this is h0 of course, well, please remember this is to within a constant. So there is still that constant C0 that needs to be determined.

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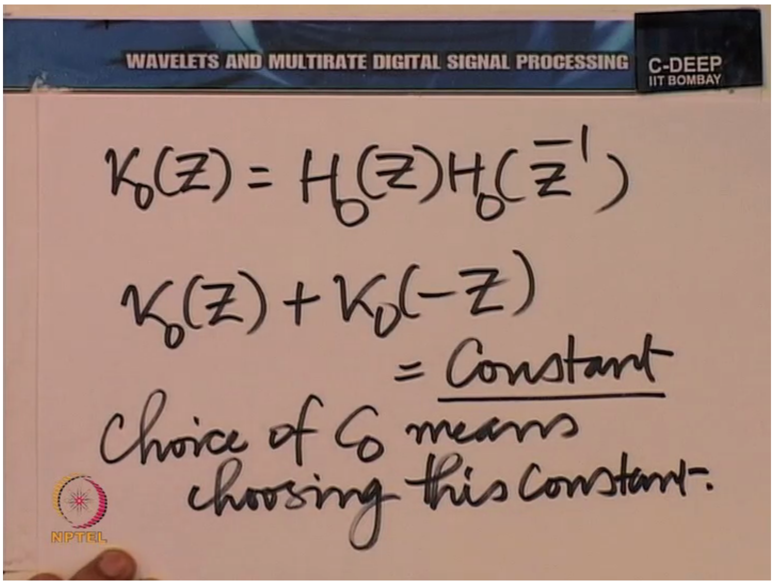

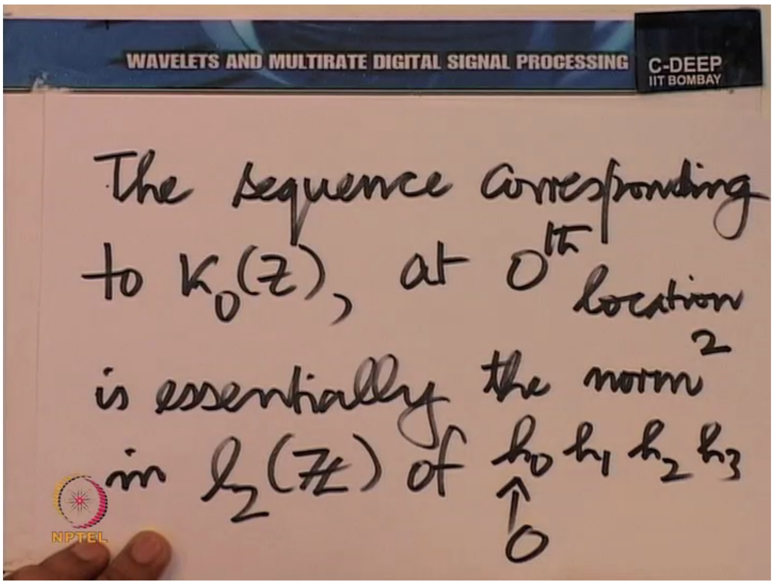

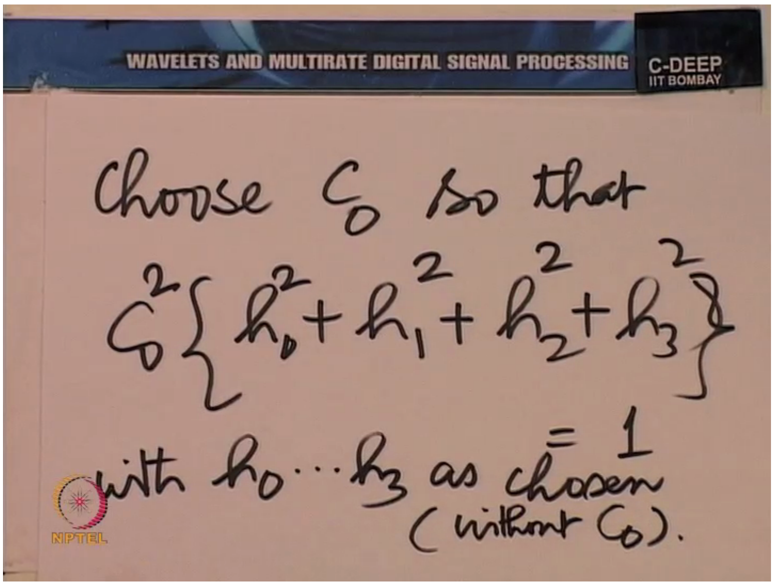

How do we determine the constant C0? Well, we go back to the original equation of Kappa 0Z that we had. So we wanted Kappa 0Z defined by H0Z times H0Z inverse to obey the following Kappa 0Z + Kappa 0 - Z is a constant. And in fact what it really means is we now need to choose this constant, choice of C0 means choosing this constant. The easy thing to do is to ensure that the impulse response has a unit norm in the sense of L2. Because if you look at it, the dot product of the impulse response with itself, you know what we are saying essentially is Kappa 0Z, you know the sequence corresponding to Kappa 0Z at the 0^{th} location is essentially the norm of in L2 Z, the norm square actually in L2 Z of the impulse response.

It is a dot product of the impulse response with itself. And we could as well make that 1 for convenience. So now we shall choose C0 in order that this becomes one. And in fact I do not need to carry out the computations, essentially what I am saying is choose C0 so that C0 squared times h0 squared + h1 squared + h2 squared + h3 squared is equal to 1 with h0 through h3 as chosen before, without C0. So I leave that little calculation for you to do and I would strongly recommend that students look at various texts that list the Daubeshies filter responses and verify that the Daubeshies filter response for a filter length of 4 exactly coincides with what we calculate from here.

So much so for the Daubeshies filter bank. Now the next step is to build Phi t and psi t, how would you do that?