Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-5.
Lecture-13.2.
Applying perfect reconstruction condition to obtain filter coefficient.

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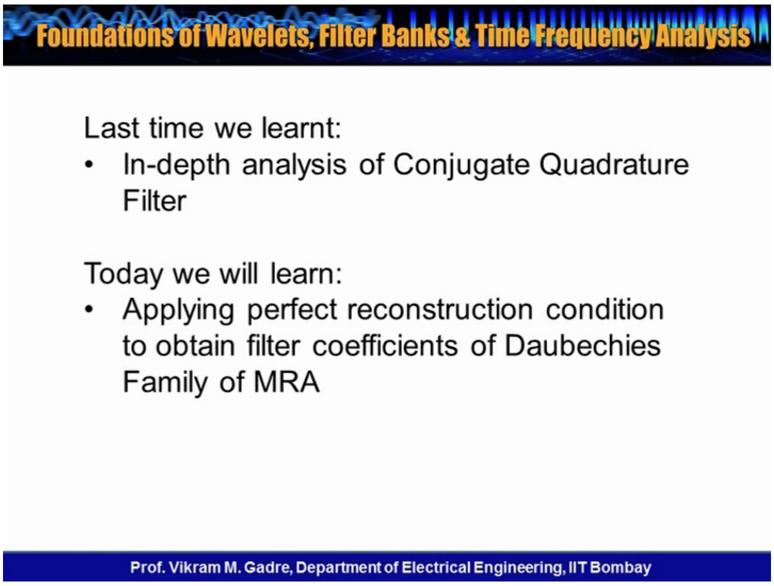

This is the motivation for that so-called quote unquote peculiar choice of h1. Now we will see things falling in place. The Z inverse was required to bring this complex conjugation, replace omega by - omega. And of course as you see for a real impulse response, it had no, no effect on the magnitude, but we could remove the phase. So it is a strategic choice of analysis high pass. You could have chosen h0 - Z or something like that but we chose h0 - Z inverse because you wanted that complex conjugation.

And then you put a Z raised to the power - D because you wanted to make it causal. So Z raised to the power - D is to introduce causality, the Z replaced by Z inverse is to bring in this complex conjugation, to bring in power complementarity and finally the -, I mean - Z inverse instead of just to that inverse is to convert the lowpass to high pass. So now it all falls into place and we have justified our choice. And now we also know what we demand of X0Z, so that we get perfect reconstruction.

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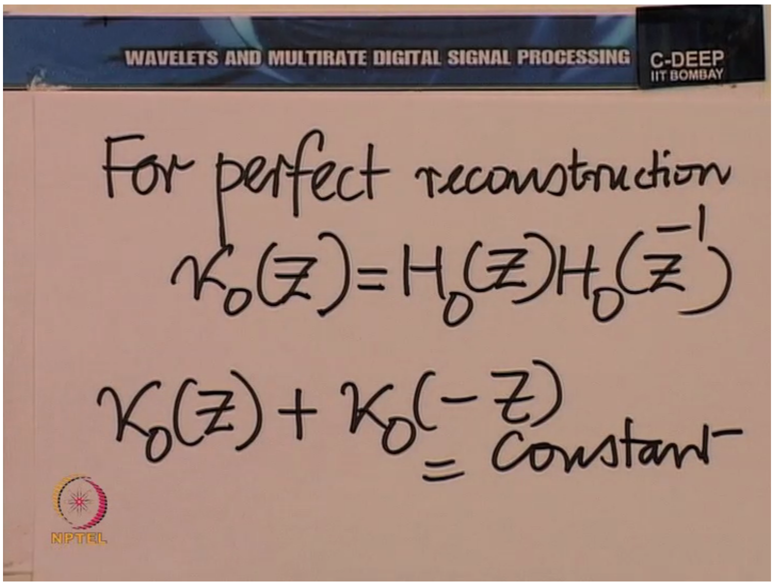

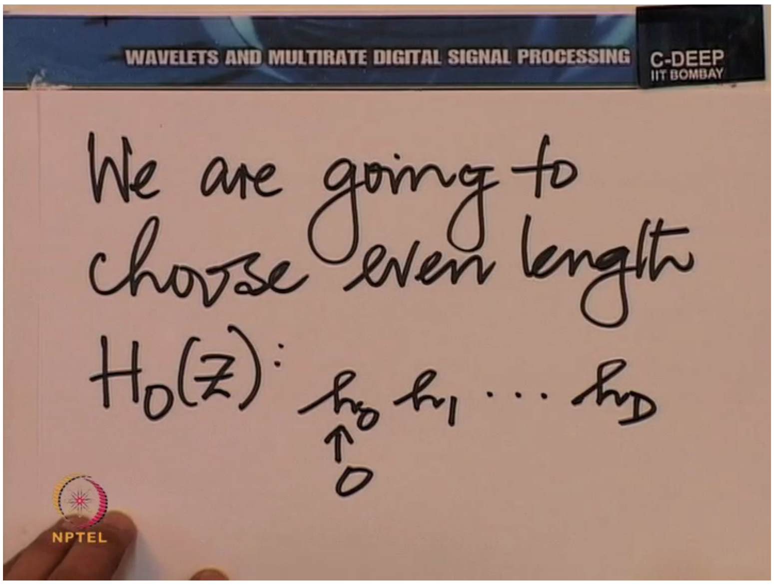

Let us look at that condition once again. That condition tells us and let me write it slightly differently, that condition tells us for perfect reconstruction some interesting intermediate filter which we shall define by Kappa 0Z, so let us define Kappa 0Z as H0Z H0Z inverse. What we are saying is that for perfect reconstruction we require Kappa 0Z + Kappa 0 - Z to be a constant. Now things are beginning to make even more sense. If we know the sequence that gives us H0Z, what is the sequence that gives us H0Z inverse?

Let us reflect a minute on this. So what I am trying to say is we have agreed that we are going to choose even length H0Z, something like an impulse response of the following form, h0, h1, and so on, h0 lies at 0 up to hD, remember D was odd. And therefore H0Z inverse would then correspond to the following.

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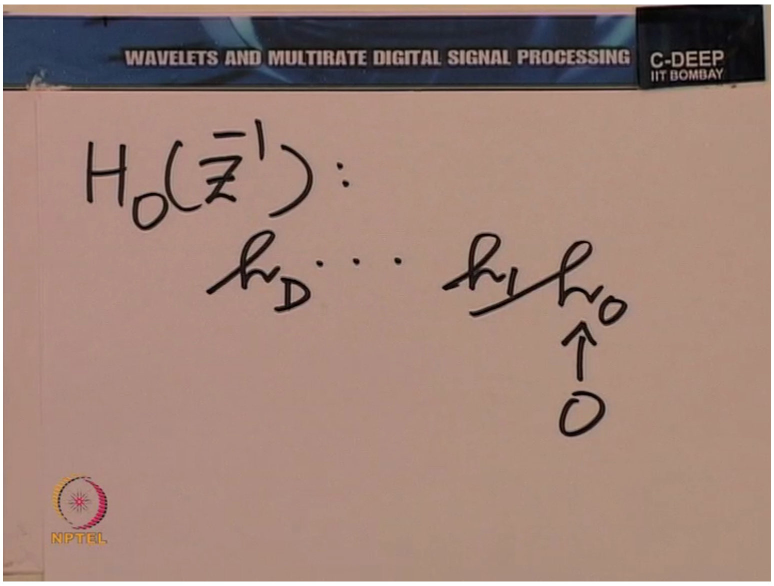

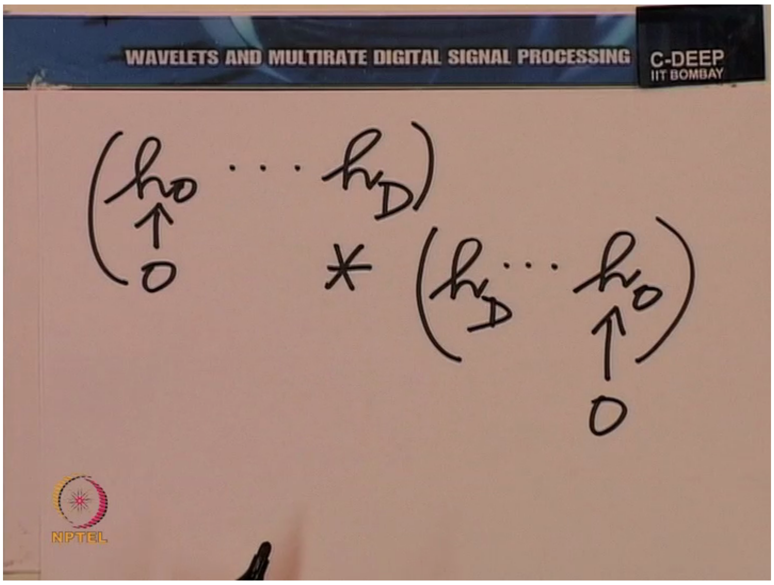

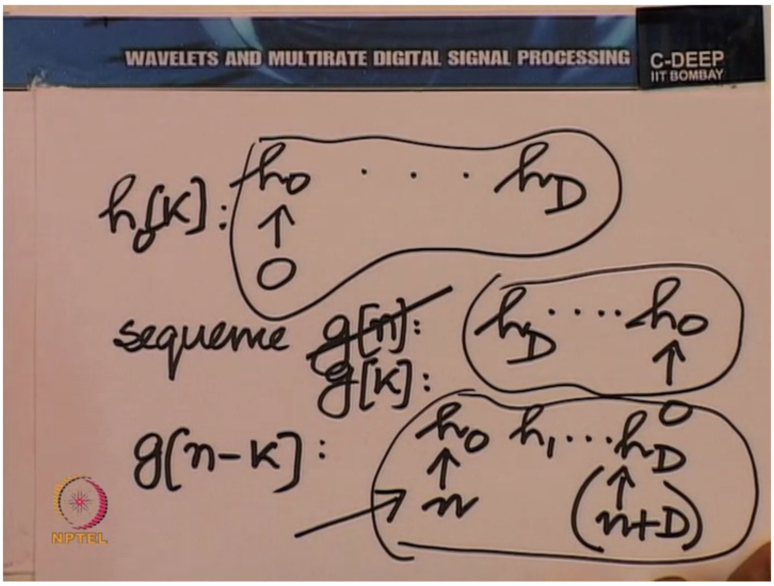

Quite clear, when you replace Z by Z inverse, you are essentially reflecting the sequence about the point N equal to 0, simple. Now H0Z times H0Z inverse corresponds to their convolution, you know when you multiply 2 Z transforms, the corresponding sequences are convolved and therefore we have this convolved with this, maybe I should put parenthesis here and indicate the zero clearly there. Now how do you convolve, well these are of equal length, so I could choose either of them as a static one and the other one at the moving one.

So just for convenience, what I will do is the sequence which we started with, the one corresponding to h0, we shall keep as a static sequence and the one corresponding to H0Z inverse we shall make it move. Now, what we are saying essentially is keep this static, so you have and make this move, so when you make the other one move, you are doing 2 things, you are bringing, you see you want to if essentially you have sequence 1, let us say sequence, let us call that sequence GN just for the time being, the sequence GN is this or GK if you like.

In which case the sequence GN - K, this is of course a function of K, so K equals to 0, it is h0 and so on. So G of N - K is going to look like this, the zero will go to N and whatever comes before 0 would go after N there. So you have h1 and so on up to hD. So this reaches the point N + D here, this is a sequence GN - K. And this you may of course call the sequence h0 of K if you like. So you are trying to convolve this sequence with essentially with this sequence but in that convolution you are going to move around this at different locations here.

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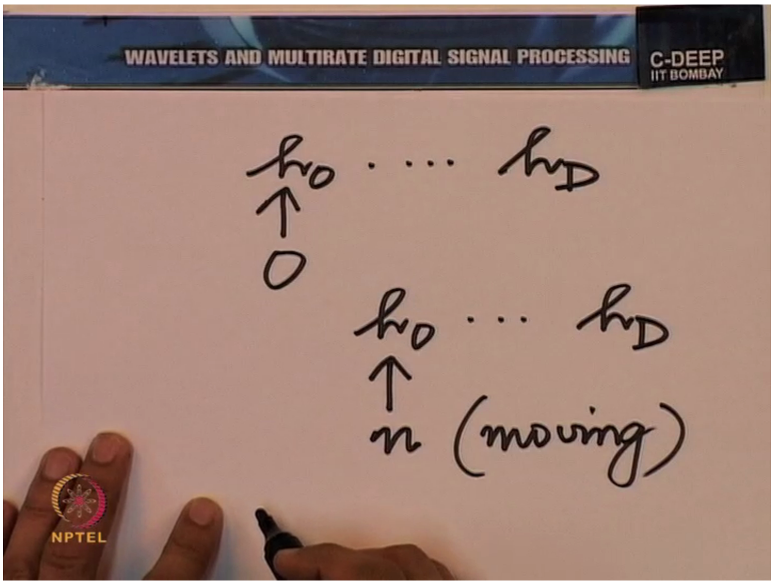

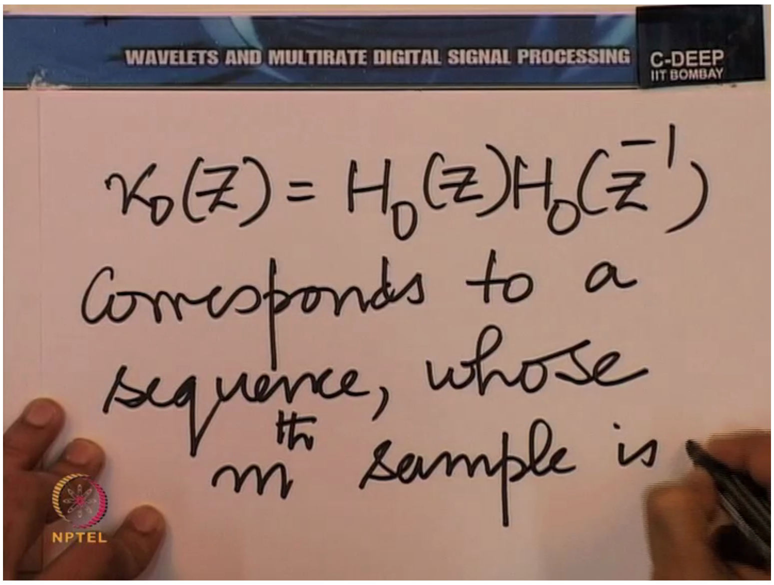

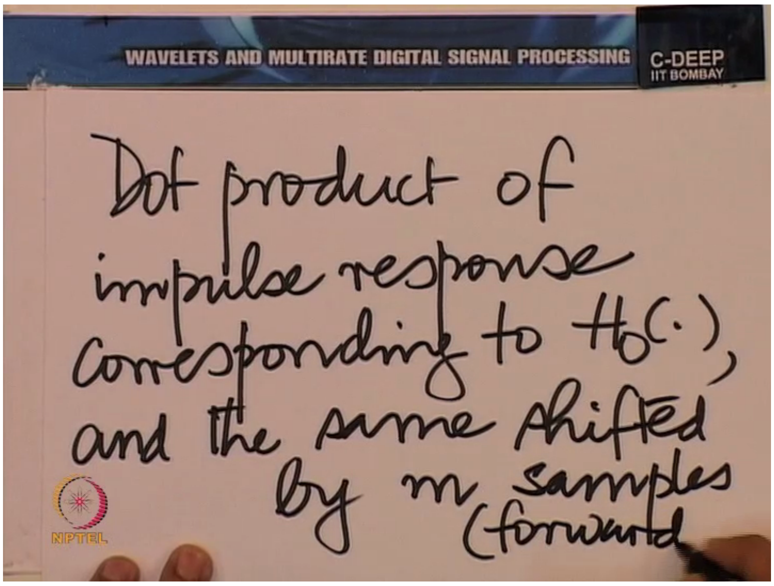

Now visualize, let me put that down clearly once again for you. We are saying we have this so-called static sequence and this is going to move around, N is moving, so you can visualize the situation. For different values of N, this lies at different locations with respect to the static sequence, for example when N is equal to 0, the samples actually coincide, when N is equal to1, then h0 clashes with h1 and of course hD has gone out of range, so it has gone to 0 sample here.

When N is equal to -1, you are here and then of course h1 clashes with h0, hD with that of D -1 here and so on so forth. So you see what we have is actually the dot product of the sequence and its own shifted versions, this is very interesting. What we are seeing is that the samples of Kappa 0 are actually dot products of the original filter impulse response shifted by different amounts of shifts, let us write that down, that is a very important conclusion.

Kappa 0, Kappa 0Z which is H0Z times H0Z inverse corresponds to a sequence whose Nth Sample is as follows. The dot product of the impulse response corresponding to h0 and the same shifted by M samples, if you want to be very specific, you should take M samples forward at that does not really matter.

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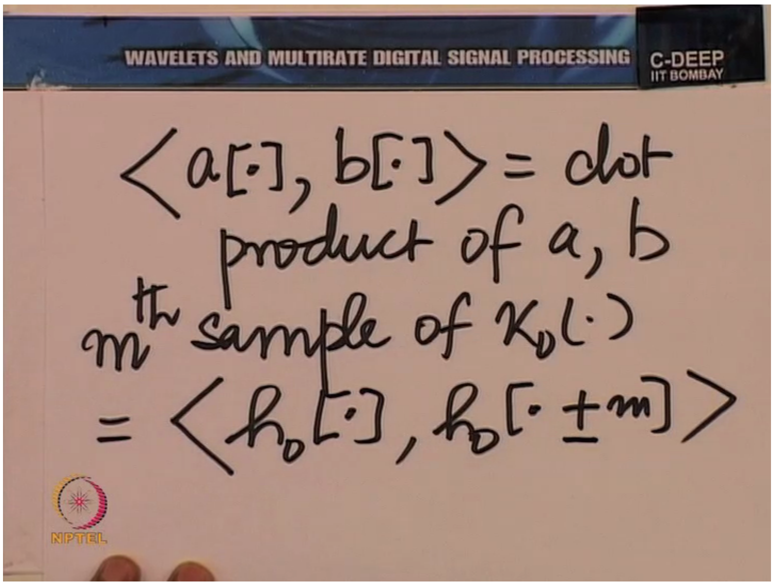

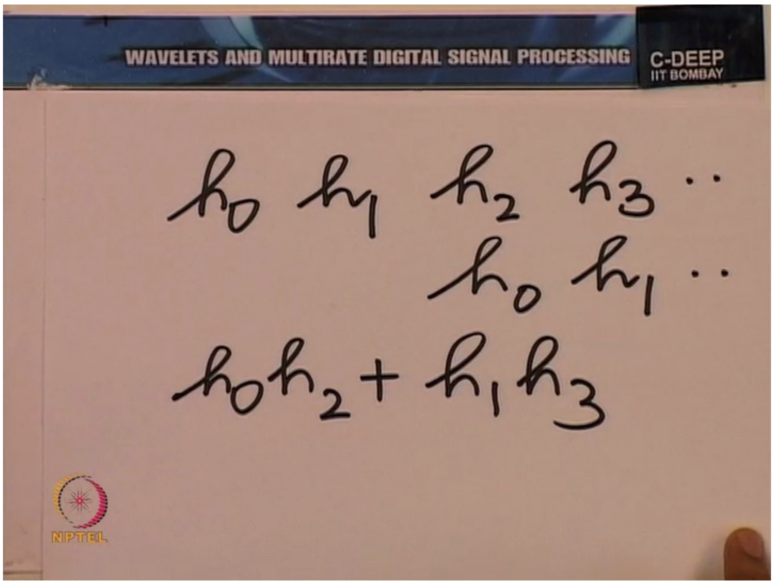

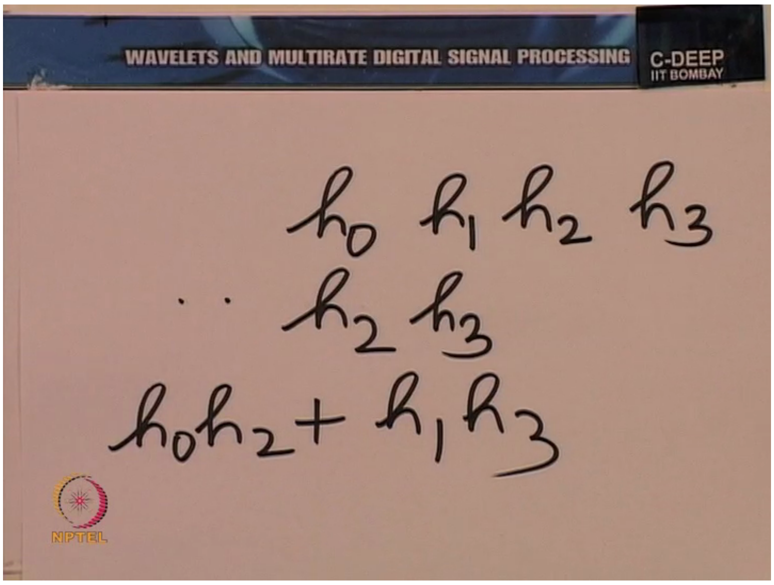

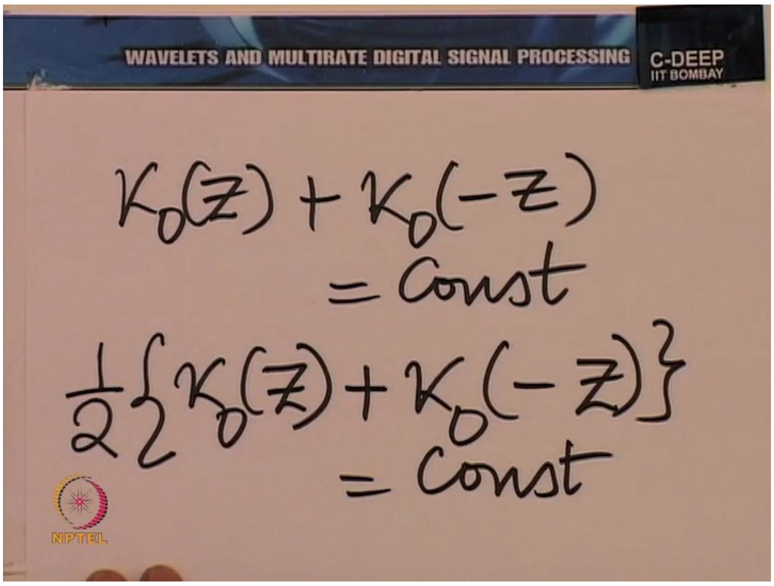

So if you want to write it down in the notation of dot product, what we are saying is that this denotes the dot product of sequences A and B, so you know A with an argument, integer argument, B with an integer argument, this is the dot product of A and B and we are saying the Nth Sample of the filter Kappa 0 is essentially the dot product h0 and h0 shifted by M, + or - is not really an issue. If you like you can make this -, there is symmetric.

You know you can visualize that, if you shift backward by 2 or forward by 2, it is the same, let us verify that for length 4 for example, you will see what I mean. So if you had a length 4 for example, you would have h0, h1, h2, h3 and if you took this and the same thing shifted by 2, you are talking about this dot product, the rest of it is 0 of course, so here again you get 0, then you do not need to write that. So the dot product is essentially h0 h2 + h1 h3. Now if you were to shift it backwards, so you have h0, h1, h2, h3 there any shifted it backwards and of course this is all 0.

So again the dot product would be h0 h2 + h1 h3, so as you can see shifting backward or forward by M is not an issue, however what we are saying here is something very interesting, we are saying that with this understanding of the samples corresponding to Kappa 0Z, Kappa 0Z + Kappa 0 - Z is a constant, and if we take the inverse Z transform now and if we only care to multiply by half on both sides, this is also constant obviously.

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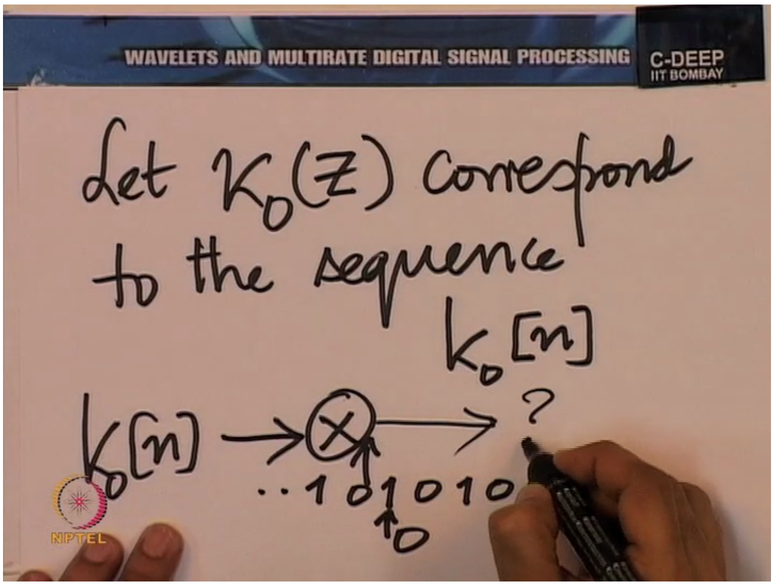

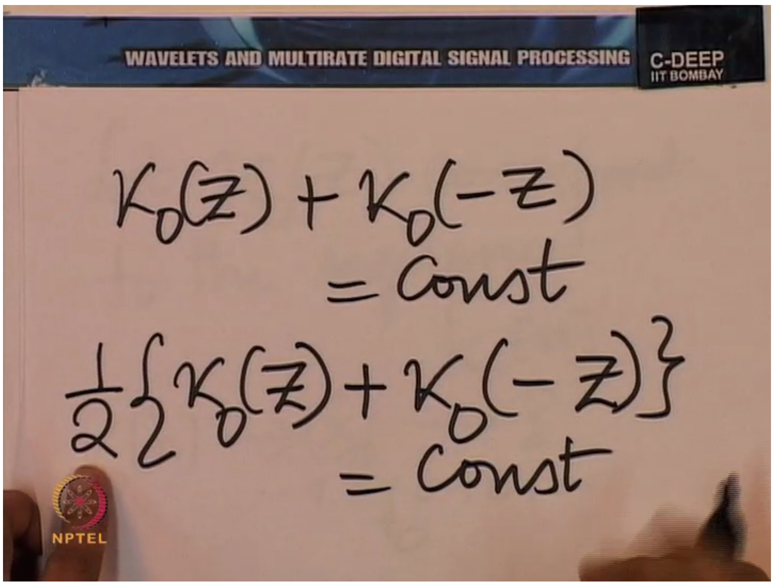

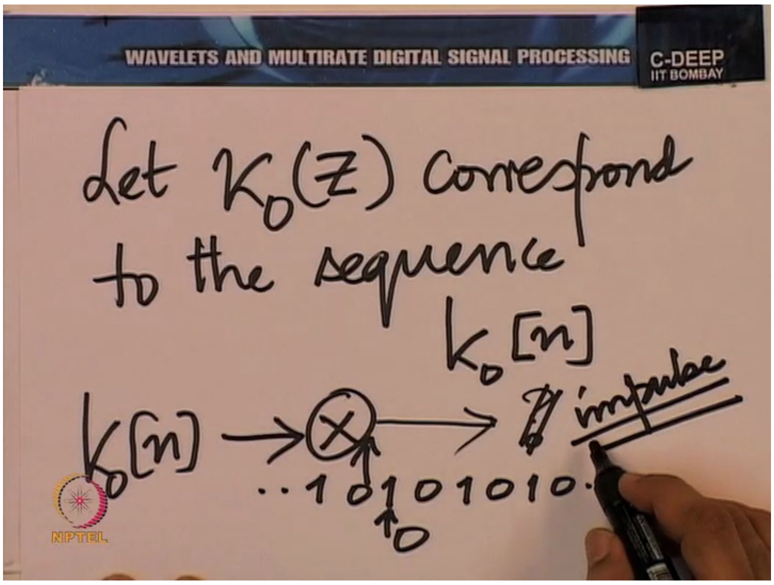

And this is something very familiar to us, we have encountered this when we did down sampling. So in fact if the original sequence corresponding to Kappa 0Z, so you know let Kappa 0Z correspond to the sequence, let us write small k0n, then what we are saying is that when this sequence is modulated by a sequence which is 1 at the even locations and 0 at the odd locations. So it is something interesting that we are doing, we are modulating this Kappa 0N by a sequence which is 1 at even locations and 0 at the odd locations.

This gives us a sequence corresponding to the inverse Z transform of a constant which is essentially an impulse. Now you know, this modulation is what we derived when we talked about the Z transform across a downsample. So remember when we go across a downsample by a factor of 2, it is like 1st modulating by a sequence which is 1 at the even locations and 0 at the odd locations. In general when you go across a downsample by a factor of M, it is like modulating with a sequence which is 1 at all multiples of capital M and 0 elsewhere, followed by an inverse upsampling operation.

So remember, downsampling by 2 was modulation by a periodic sequence with period 2 which was 1 at locations equal to multiples of 2 and 0 else followed by an inverse upsample by factor of 2. Inverse upsample means a compressor, throw away the zeros. Downsampling by a factor of M was essentially multiplication by a periodic sequence, period capital M, 1 at all multiples of M, 0 elsewhere followed by an inverse upsample by a factor of capital M which means throw away the zeros and compress.

So we, you see that throwing away the zeros was what made Z replaced by Z raised to the power half. So here in this expression, Kappa 0Z + Kappa 0 - Z, we are not writing Z raised to the power - half, so we do not do that inverse upsampling operation, but the rest of it is there and that the justification for this step here, modulation with the periodic signal. And now this is equal to a constant which means if we take inverse Z transform here, we are saying this is essentially the impulse.

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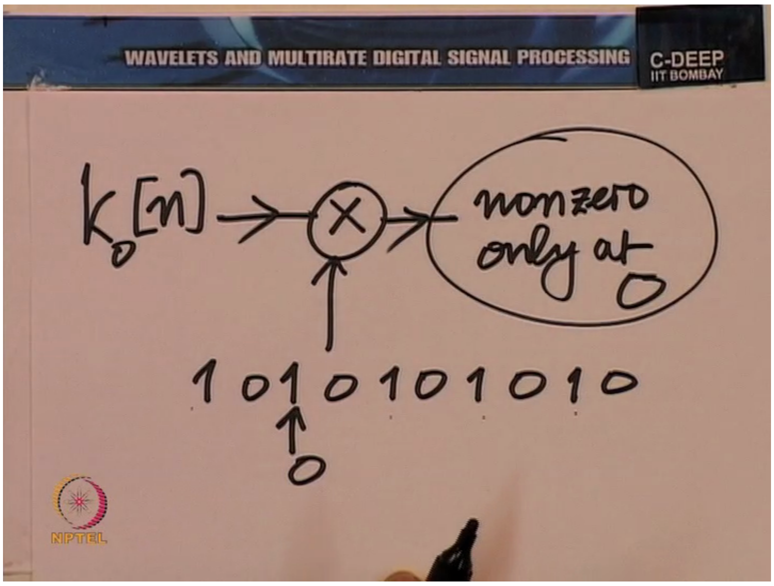

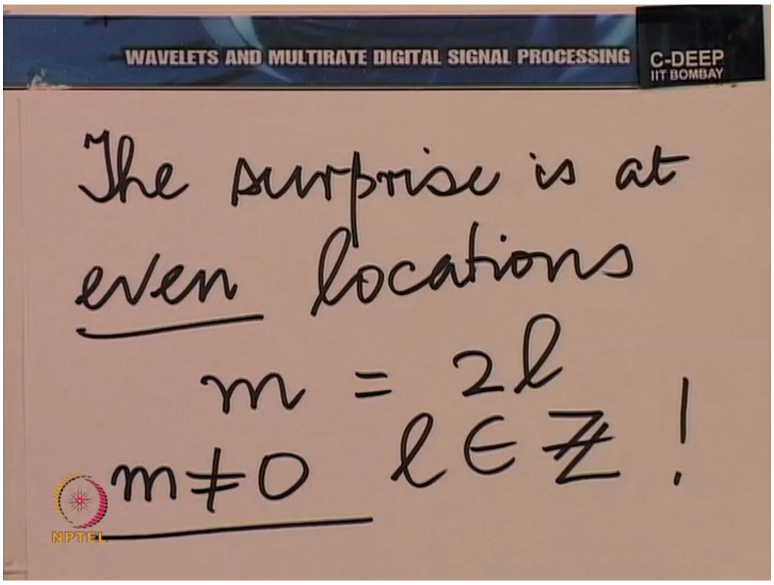

Which means this has a nonzero value at 0 but 0 everywhere else. So let us write that down. Kappa 0N when modulated with this periodic sequence with period 2 with the 1s that multiples of 2 and 0 elsewhere, results in a sequence which is nonzero only at Z equal to, N equal to 0, that is what we are saying. And obviously at the odd locations anyway day 0, so there is nothing very surprising here, it is at the even locations that we have surprising result there.

So the surprise is at the even locations. Of course M not equal to 0, so what we are saying is that if I take the impulse response of the lowpass filter on the analysis side, shifted by any even number of samples 2, 4, -2, -4, 6, -6 and so on and take the dot product of that shifted impulse response with the original impulse response, that dot product is 0. For those of us who are familiar with the idea of autocorrelation, what we are saying is that the autocorrelation of the impulse response of the lowpass filter is 0 at the even locations other than 0.

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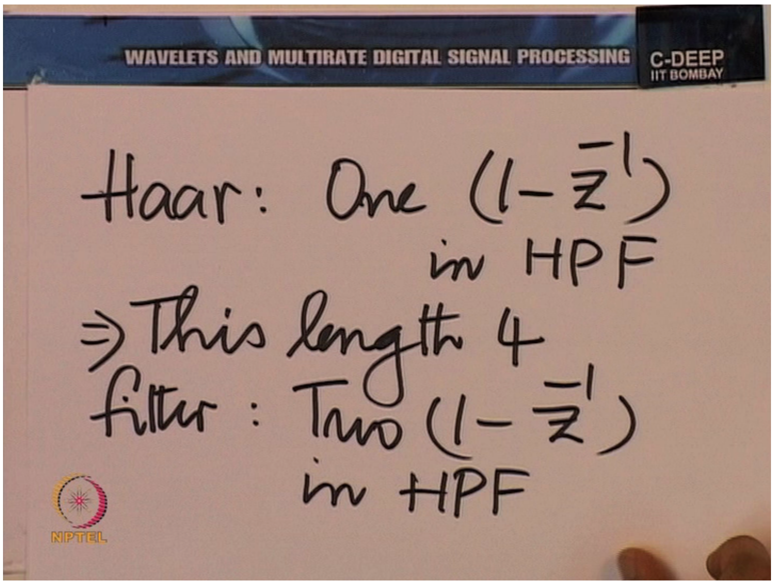

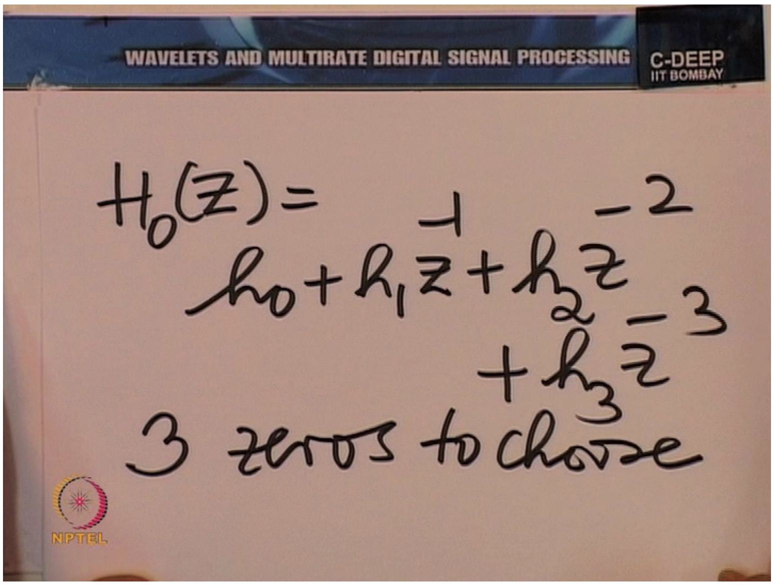

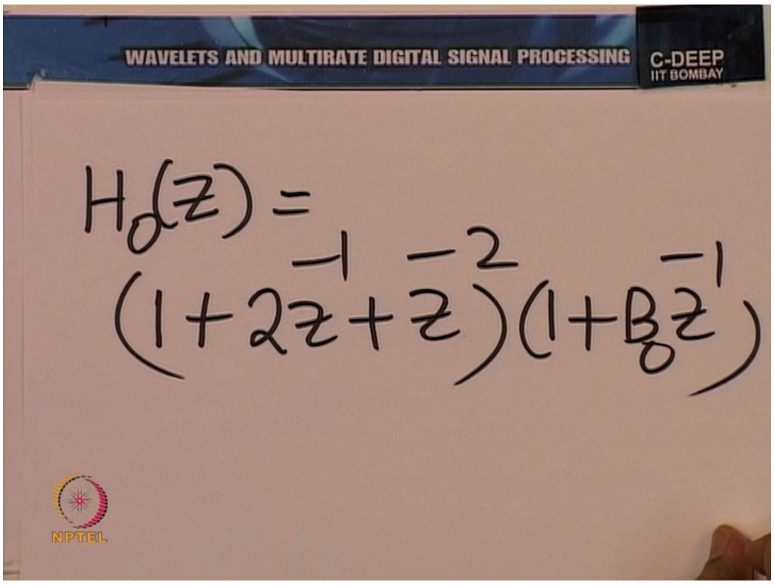

Now, let us use this to build the 1st of the family of Dobash filters. So, you see, well I should say 1st nontrivial, so this is 2nd in that sense, the 1st non-baby remember. Dobash filter with length 4 is going to look something like this, it is going to have an impulse response h0, h1, h2, h3. And recall what we did yesterday, we said that in this filter we would need to bring in one more factor of the form 1 - Z inverse in the high pass filter. So Haar had one 1 - Z inverse in HPF, so this length 4 filter would have 2 factors two 1 - Z inverse in the high pass filter.

And that means you see the high pass filter was obtained by replacing Z by Z inverse and then by - Z as well. So the Z inverse gets taken care of by the delay Z raised to the power - D but the Z replaced by - Z needs to be undone to go to the lowpass. And therefore the lowpass filter would have a factor 1+ Z inverse squared. Now when you say it has a factor 1+ Z inverse the whole squared, you have already constrained 2 of the 3 zeros that it has free to be chosen.

What I mean is if you look that is H0Z, it would have been h0 + h1 Z inverse + h2 Z raised to - to + h3 Z raised to the power -3, so there are 3 zeros to be chosen. Out of them we have already chosen 2, so we have only one free, let that free one be at B0. So in all it is very simple, we can take H0Z to be of the form 1+ Z inverse the whole square times 1+ B0Z inverse. What do we have then, let us expand this. We have X0Z is essentially 1 + 2Z inverse last Z is to the power - 2 times 1 + B0Z inverse.

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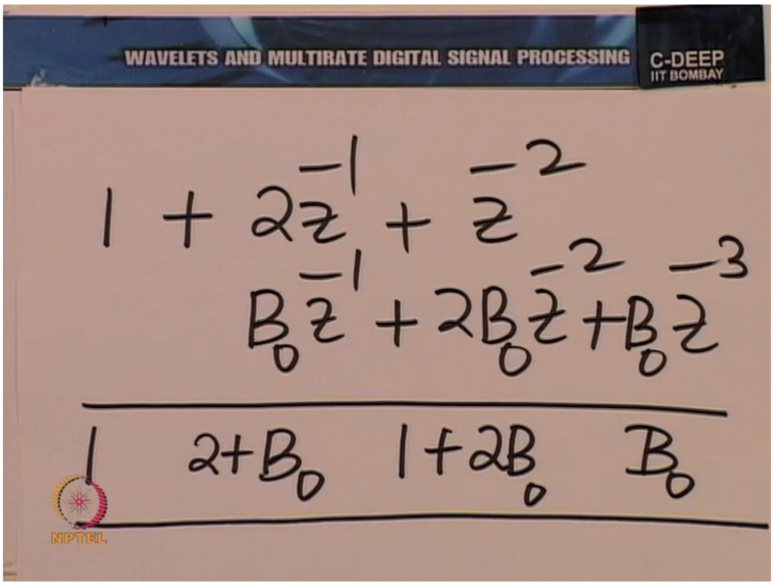

And we can expand this further, that product will be one + 2Z inverse + Z raised to the power - 2+ B0Z inverse times this. So B0Z inverse + 2B0Z raised to the power - to + B0Z raised to the power -3. So in that sense we have the following impulse response for the filter. 1, 2+ B01 + 2 B0 and B0 here, this is the impulse response. Now, we have setup the lowpass filter for the 2nd member in the Dobash family, where do we go from here?

We shall use the constraints that we just derived, namely that the dot product of this impulse response with its shifts, by even shifts must be 0. And we shall see the constraints that emerge on the free parameters. In the next lecture therefore we shall constrain the value of B0, make a choice of B0 and derived precisely impulse response of the Dobash 2nd member. And thereby also establish a general procedure for building up the Dobash family lowpass filters.

Concurrently we shall explain how this family evolves and recall again the significance of going from one member to the other. With that then we shall conclude the lecture today, thank you.