Foundations of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-4
Lecture -12.3.
Introduction to Daubeshies family of MRA.

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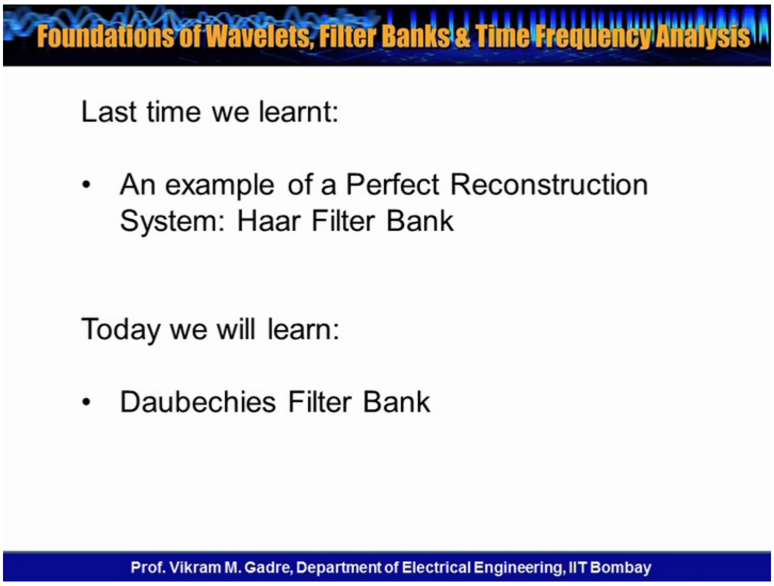

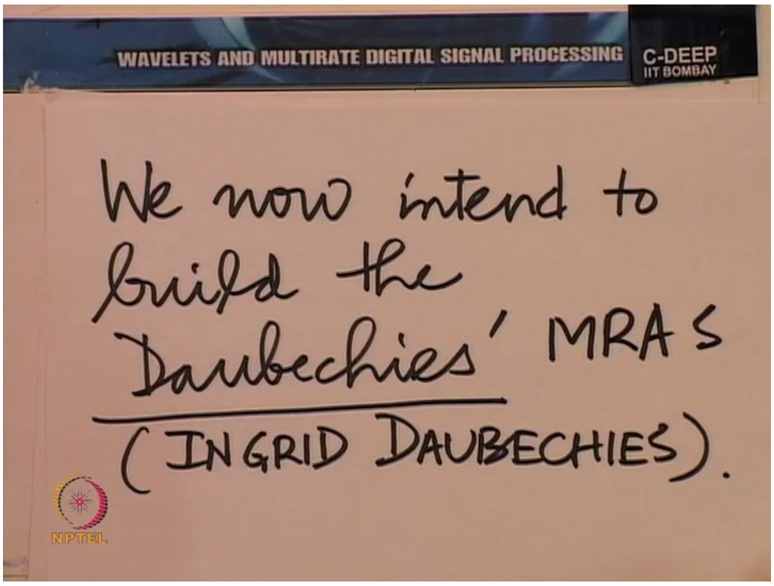

So I will just write that down. We now intend to build what is called the Daubeshies family. You know this name is actually the name of a mathematician scientist, whatever you might want to call her and the full name is this. I believe this is currently pronounced as Dobash but I could be wrong, I think we could just probably say Daubeshies and be content. So anyway we now intend to build the Daubeshies filter bank, the Daubeshies MRAs.

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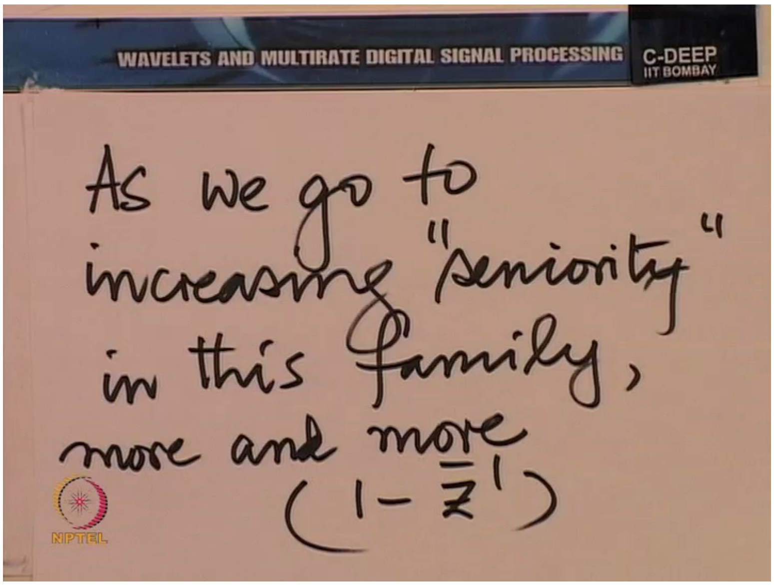

And the feature of these Daubeshies MRAs is that as we go to the senior members of the family, as we go to increasing seniority there are more and more 1 - Z inverse terms. So on the high pass branch we are effectively cancelling or killing higher and higher order polynomials. So the other way of looking at it is that if you are cancelling or killing them on the high pass branch, they must go to the lowpass branch. So we are retaining more smoothness from the lowpass branch, that is another way of looking at it.

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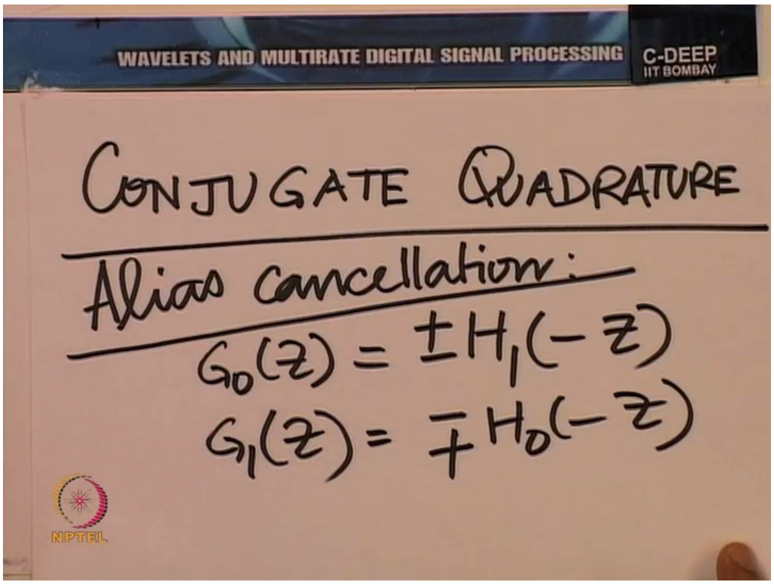

And addition to doing this, we also want the same kind of analysis and synthesis filter. So all this together leads us to a special class of filter banks which we shall now put down explicitly. And the filter banks are called conjugate quadrature filter banks. So we are looking at 1 class of what are called conjugate quadrature filter banks. Now the describing equations of these conjugate quadrature filter banks are very simple, we start from the Alias Cancellation condition. The Alias Cancellation condition says G0 Z needs to be essentially H1 of - Z.

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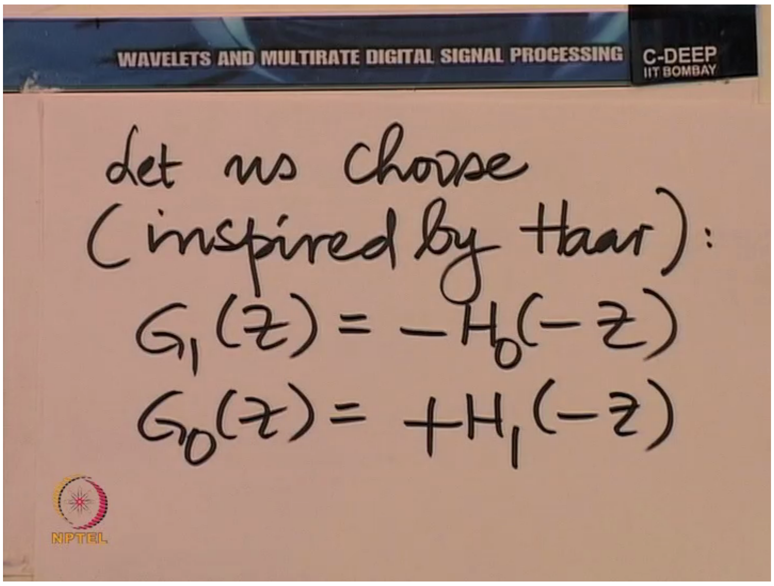

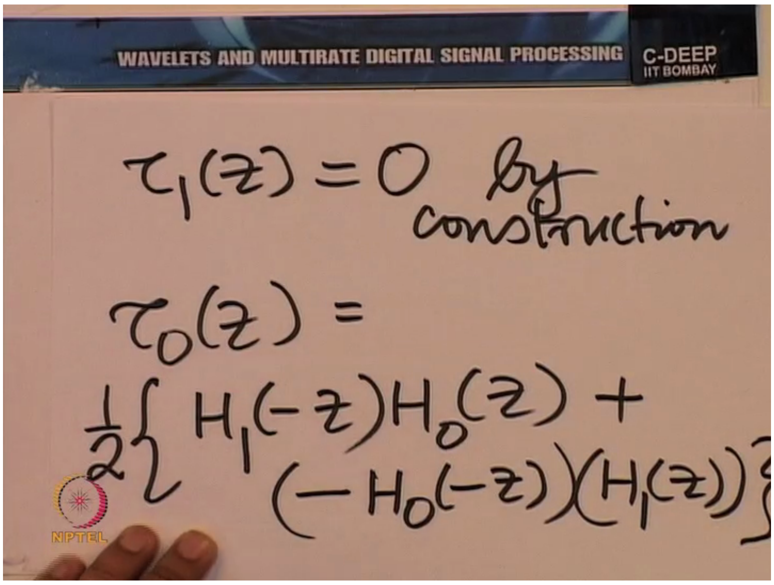

And we have the freedom to put + or - here. Similarly G1 Z needs to be correspondingly - or + H0 - Z. Now taking inspiration from the Haar, let us take the following choice. G1 Z is - H0 - Z and therefore G0 Z is then + H1 of - Z. Now you know we keep away the factor of 2 for the moment, because after all that factor can be absorbed in the C0 that we have allowed. So with this substitution what do we get? The Tao 0Z, of course Tao 1Z is identically 0 by construction, but Tao 0Z takes the following form then.

Tao 0Z takes the form half G0Z which is essentially H1 - Z times H0Z + - H0 - Z times H1Z. So we have an interesting situation here. We have this H0Z H1 - Z product. You know H1 Z is essentially a high pass filter, H1 - Z therefore essentially becomes or aspires to become a lowpass filter with a cut-off of pie by 2. So here you essentially have a cascade of 2 lowpass filters with a cut-off pie by 2 and correspondingly this becomes a cascade of high pass filters with cut of pie by 2.

And you are effectively saying that the overall system function with this cascade of lowpass filters of cut of pie by 2 and this pair of high pass filters with cut of pie by 2 must go towards the perfect reconstruction situation, that is the interpretation of this equation here. So we now focus on H1 Z.

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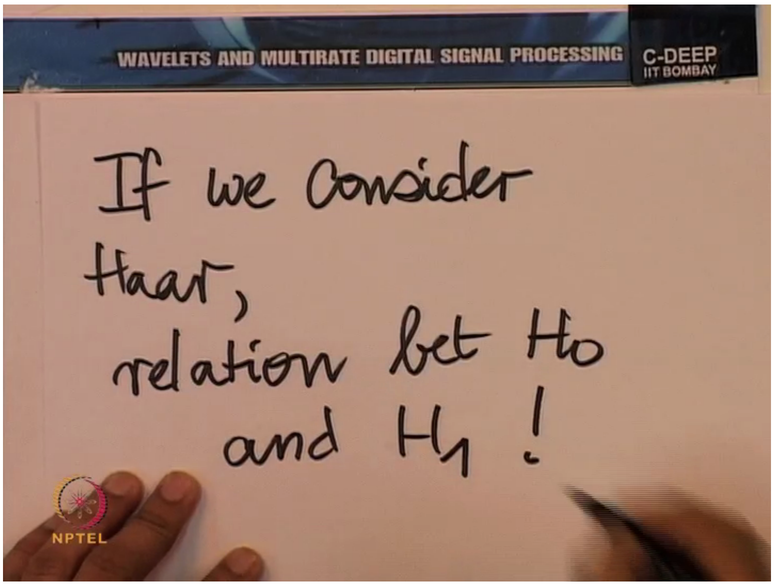

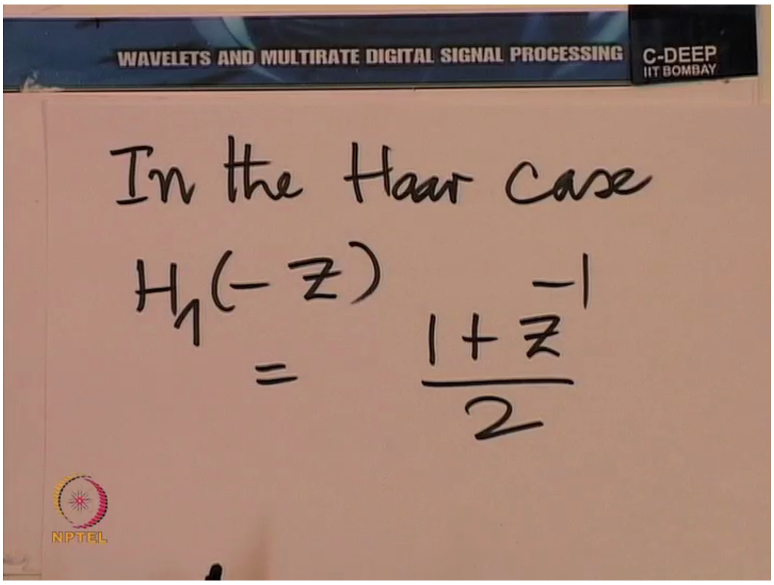

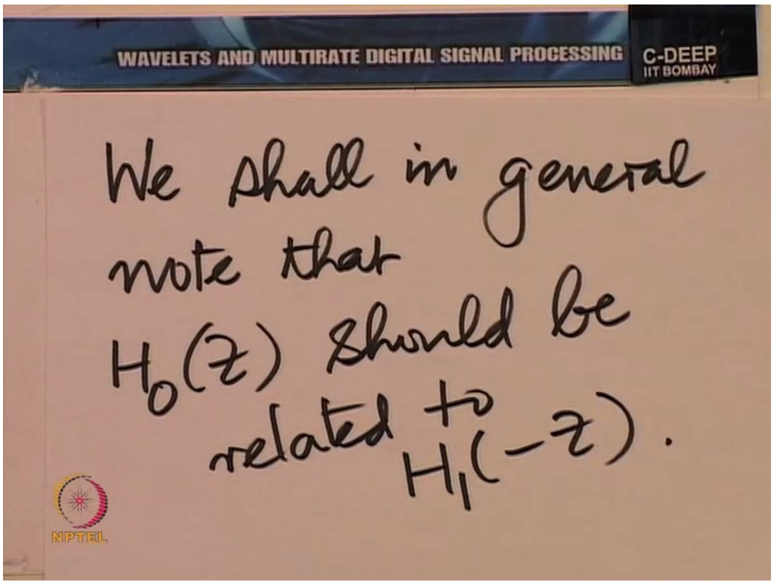

You see if we look at the Haar once again, there is a relation between H0 and H1, in fact what we are trying to say is we of course need to relate the synthesis to the analysis for the purpose of Alias Cancellation. But now we have a perfect reconstruction requirement, so we want this Tao 0Z to essentially go to a delay and a multiplying constant. Now that means you need a relation between H1 and H0. And one simple thing to do is to make H0Z related to H1 - Z, that is what we have actually done in the Haar. If you look at it carefully in the Haar case, H1 - Z is essentially 1 + Z inverse by2.

So you know H1 - Z and H0Z are very closely related. Now let us generalise this. In fact the only catch is you know we will later on need to make a little adjustment here. So at this moment even if we simply accept H1 - Z to be equal to, in the Haar case this is equal to H0Z but we may need to make a little adjustment here. So what we will do is we shall in general note that H1 or rather H0 should be related to H1 - Z.

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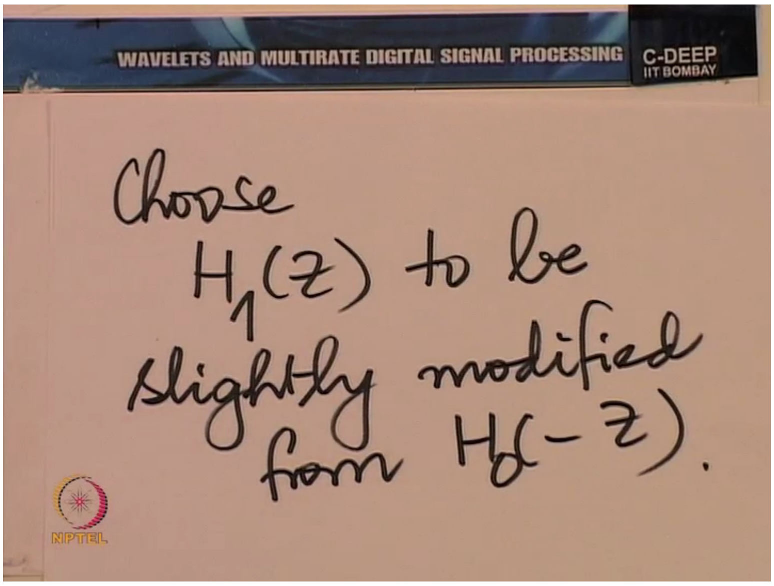

So in the Haar case they are equal but in general we ask for a relation, very close relationship. The other way of saying it is that if you replace Z by - Z in H0, you should get the H1. So what we are saying is choose H1 to be derived from or to be slightly modified from H0 of - Z. Modified in what way? So in fact you know here again there is a little bit of an issue. What I am now going to do is to put down a choice for H1 by knowledge or by exposure to the filter banks that I have and justify it later. So you will have to bear with me for a little while, not too long, maybe just about a lecture.

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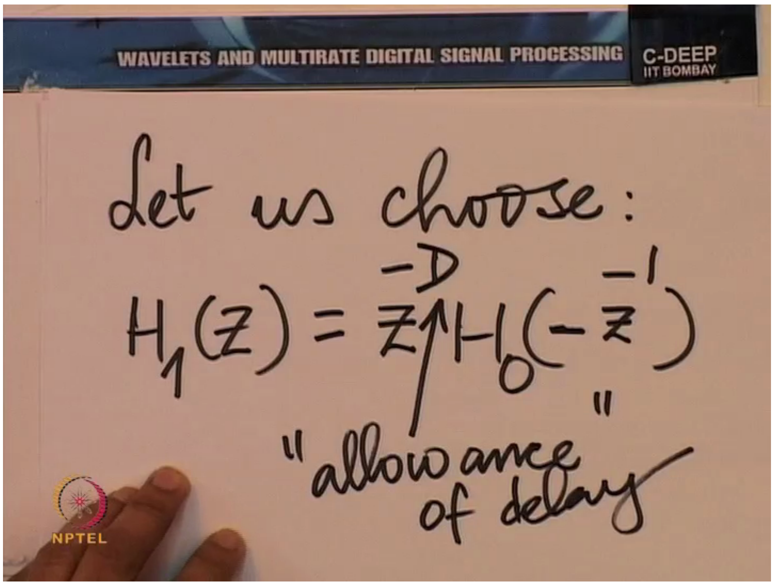

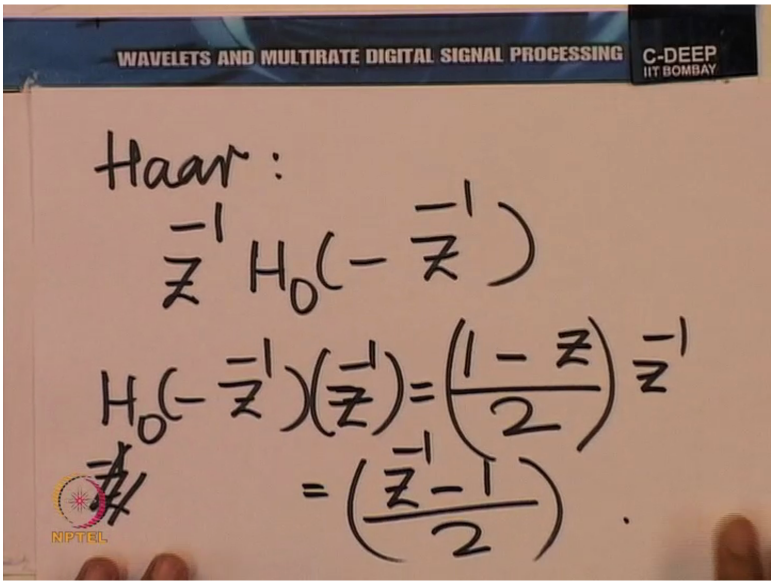

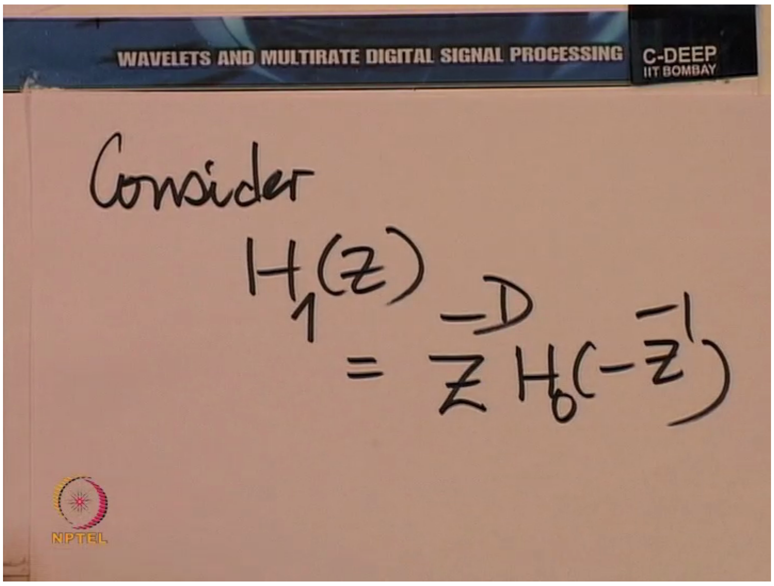

I shall put down the choice here, I shall partially justify the choice in this lecture and completely justify the choice in the next lecture where we once again look at the whole system in toto. So let us choose H1 Z not to be H0 of - Z but H0 of - Z inverse. And we will also allow for the possibility of a Z raised to the power D here. So we will say - D, we will allow for this possibility. If we do that, in fact we can verify for the case of Haar, it is very easy.

For the Haar case let us take Z inverse times H0 - Z inverse and indeed H0 - Z inverse for the Haar case is essentially 1 - Z by 2. And then if I take Z inverse times H0, I multiply this by Z inverse, multiply both sides by Z inverse, I have essentially Z inverse - 1 by 2. So we are doing well. Now let us make this the most general case. So we consider H1 Z to be of this form, Z to the power - D H0 - Z inverse. And write down the Tao 0Z for this.

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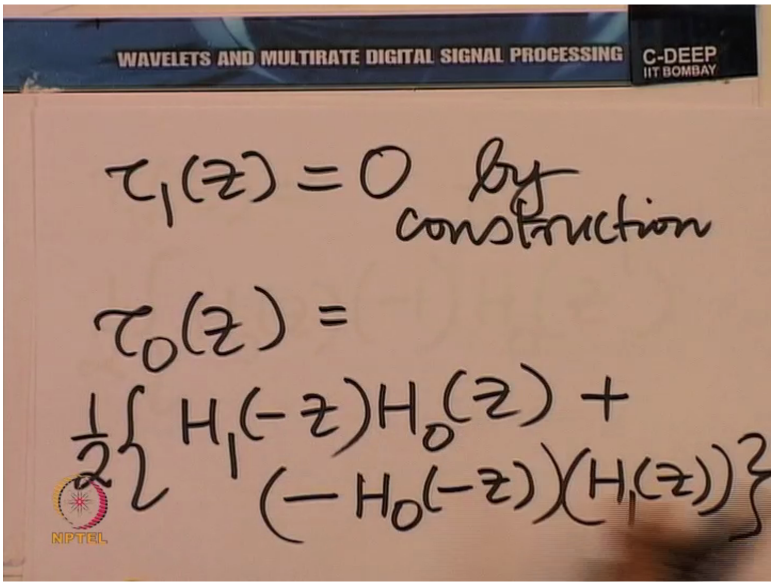

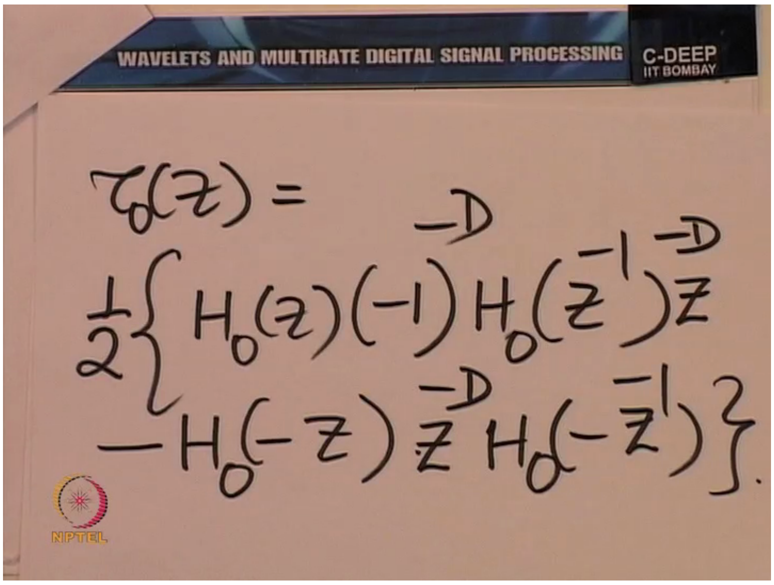

Tao 0Z would become half then H0 Z H1 of - Z, so H1 of - Z becomes -1 raised to the power - D times H0 Z inverse. Now -, you see you have G0, so again now you have G0 and let me put back the expression for you just for convenience, we want Tao 0Z to be this, H1 - Z times H0Z, for which we have this term H0Z into H1 - Z, all right. So you have Z raised to D there and - H0 - Z times H1 Z. And H1 Z we have accepted to be Z raised to the power - D times H0 - Z inverse.

So we have Z raised to the power - D here. Now what we intend to do in the next lecture is essentially to look at this expression. So we have H0Z, H0Z inverse here, H0 - Z, H0 - Z inverse there, the Z raised to the power - D terms there and of course you have a -1 rest to the power - D here and a -1 here, we need to choose these strategically, it is clear. And then we also need to put some conditions on H0 so this indeed becomes a perfect reconstruction situation. We shall complete this in the next lecture and take it further from there. Thank you.