Foundations of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-4
Lecture -12.1.
Revisiting Aliasing And The Idea Of Perfect Reconstruction.

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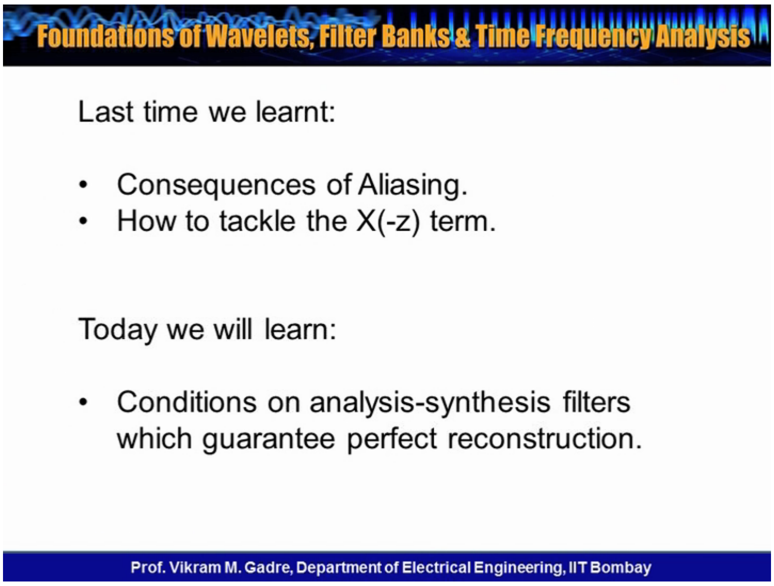

A very warm welcome to the lecture on the subject of wavelets and multirate digital signal processing. Let us spend a minute on what we had done in the previous lecture. We have looked at the 2 band filter bank in the previous lecture. And we have written down a set of conditions based on what happens when you go past a downsampler and an upsampler in the Z domain. So let us put down the conditions once again or let us in effect put down the relation between the input and the output in the Z domain, where it is valid to use the Z domain, which is true in many circumstances.

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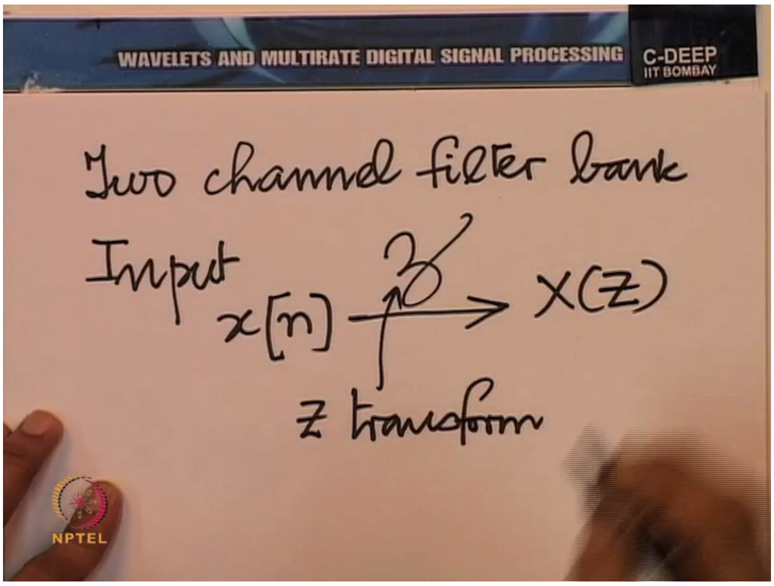

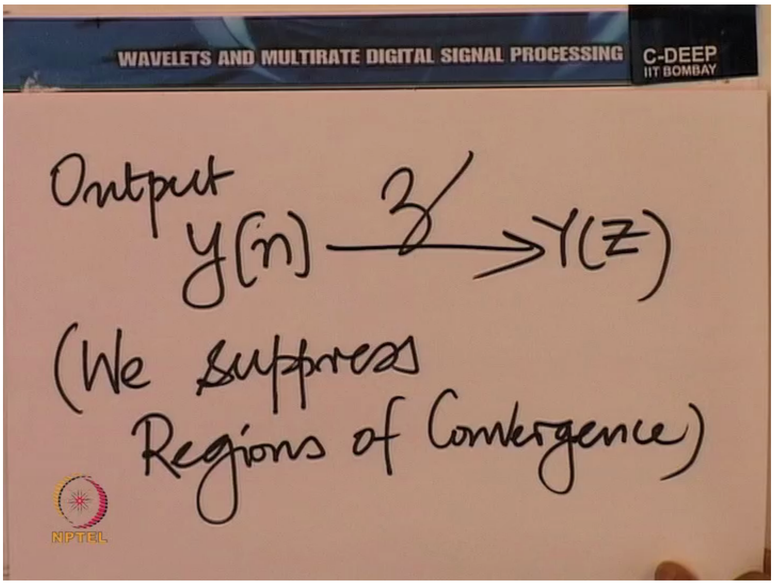

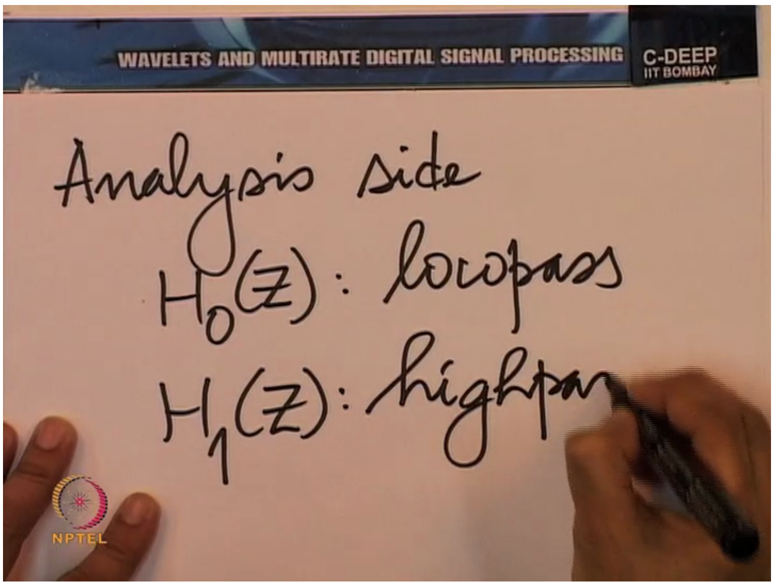

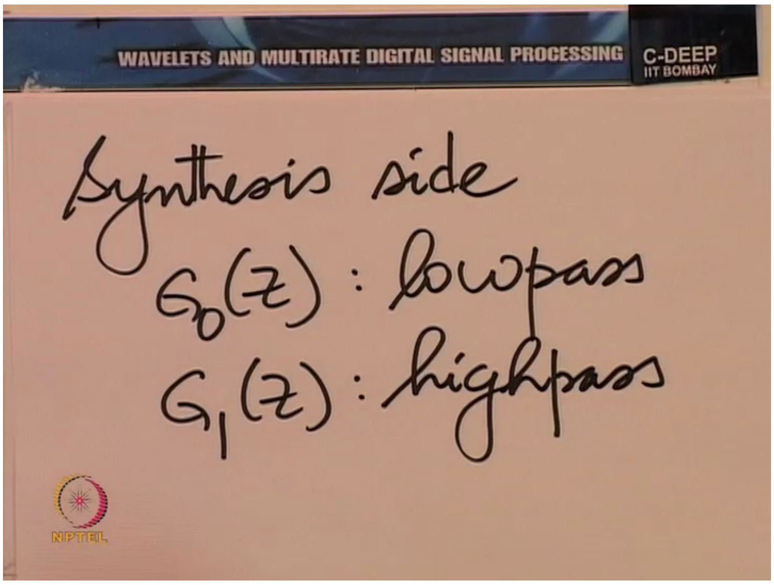

So let us summarise what we had derived the last time. We said to a 2 channel filter bank input xn, it is Z transform, now I will use this script Z to denote the Z transform capital xZ, output yn with Z transform capital YZ. Now incidentally we suppress the region of convergence. So we do not explicitly mention the region of convergence. Analysis side H0Z, the so-called lowpass filter and H1Z, the high pass filter, synthesis side G0Z lowpass filter, G1 Z high pass filter. So this is the circumstance, of course you know whether downsamplers and upsamplers are.

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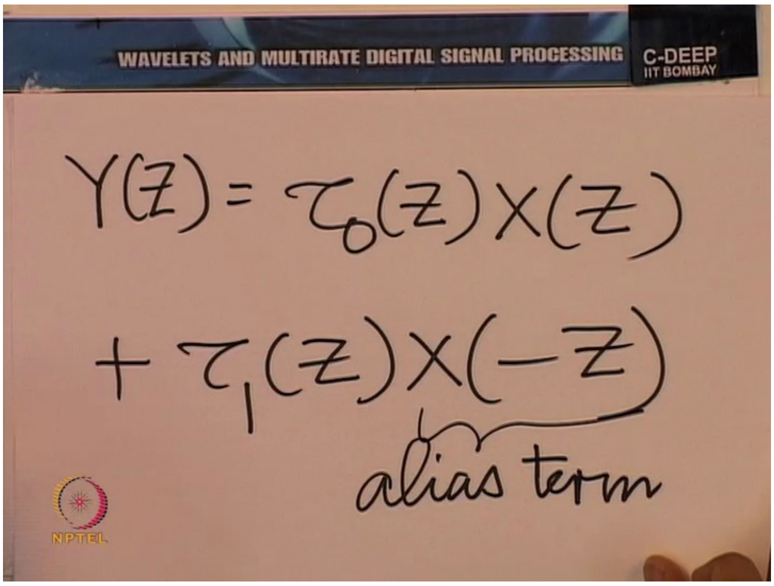

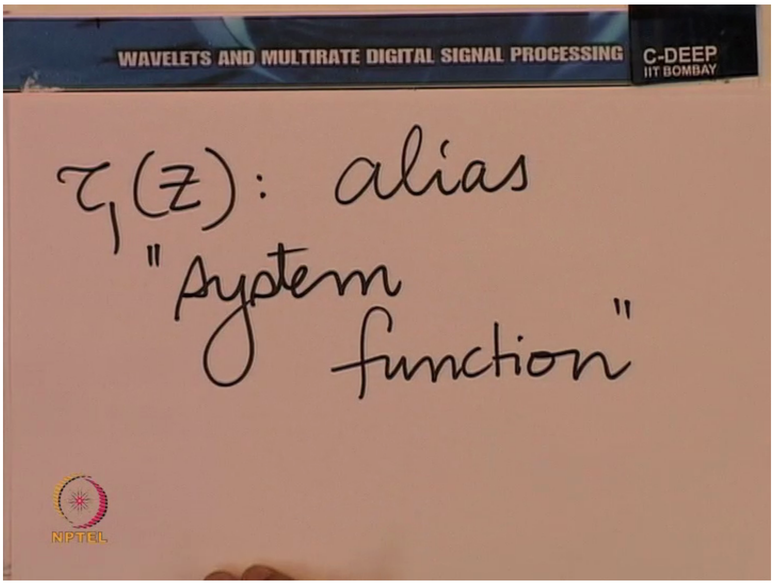

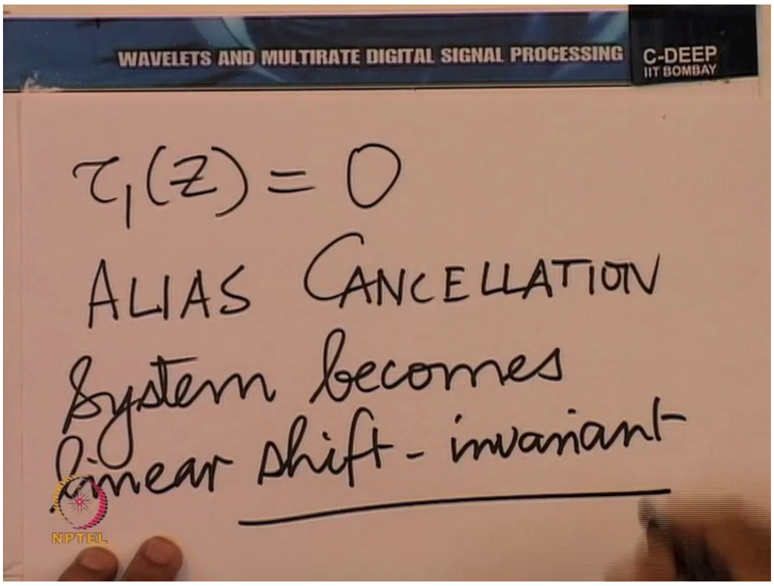

The relation between yZ and xZ is as follows. We have yZ is Tao 0Z xZ + Tao 1Z x of - Z and recall that we have called this the alias term. And therefore Tao 1Z was called the alias system function. I put system function in inverted commas, I must emphasise here. You know when there is an alias term, the word system function is actually a misnomer. One should not use the term system function because the system is not linear and shift invariant. On the other hand if Tao 1Z is 0, the system becomes linear and shift invariant.

So this is something we must now take note of. Tao 1Z equal to 0 is essentially what is called the condition for Alias cancellation. And we note the system becomes linear and shift invariant. Now we have looked at one possibility under which Tao 1Z could be 0. And we have said the most general possibility can be accommodated by what is called a cancelling term RZ. Let us put that down again.

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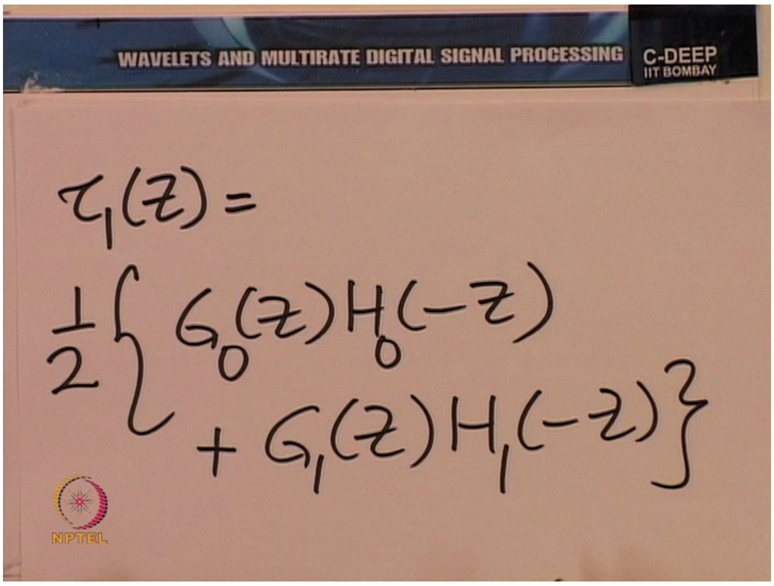

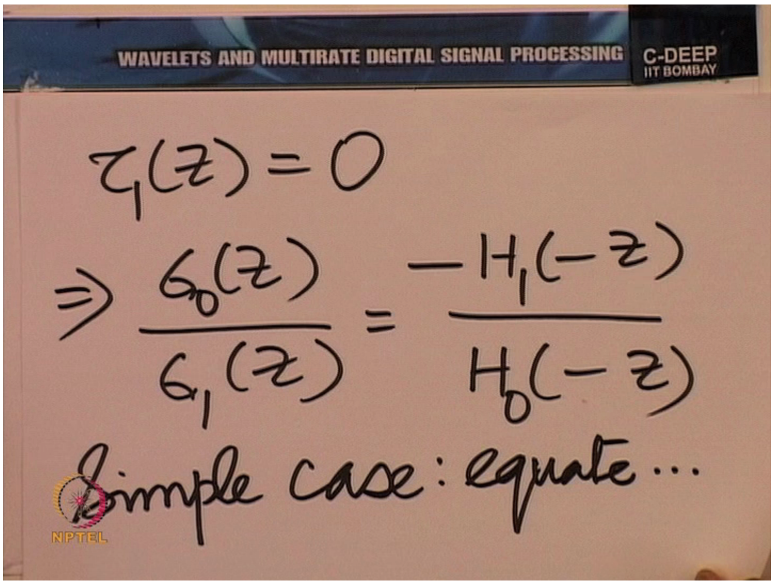

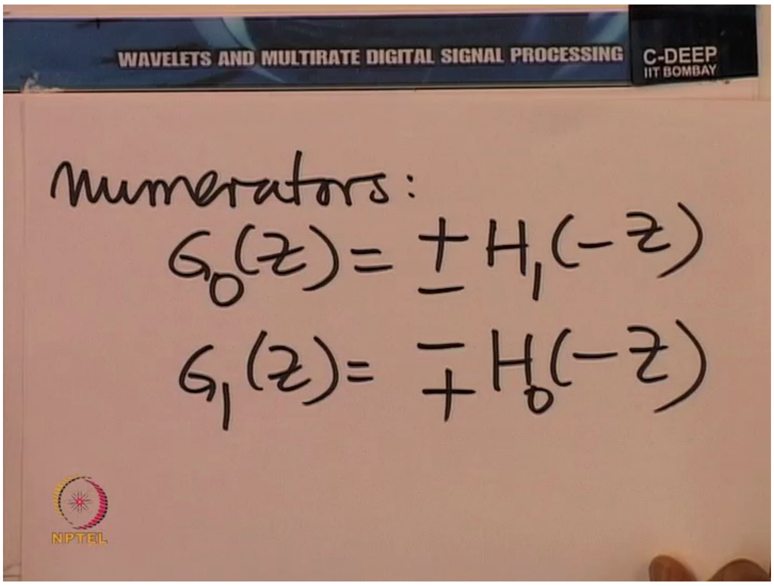

We said Tao 1Z is essentially the function or the expression half G0Z H0 - Z + G1 Z H1 - Z. And Tao 1Z equal to 0 means essentially that G0Z by G1 Z must be - H1 of - Z divided by H0 of - Z. And the simple case is equate the numerator and denominator. So equate, if you equate the numerator, you get G0Z is + - H1 - Z and G1Z is correspondingly - respectively + H0 - Z. We have also interpreted these expressions in the ideal case.

If H0Z was the ideal lowpass filter, then this would become an ideal high pass filter as we expected. And if H1 is the ideal high pass filter, this would become the ideal lowpass filter all with a cut-off of pie by 2. So in that sense we have a neat interpretation for the ideal case, even if the filter is not ideal, we have a reasonable interpretation, in the sense that we could always take this to be a nonideal lowpass filter with a cut-off of pie by 2. And this would again become a reasonably close filter high pass with cut-off of pie by 2 and vice versa for this.

If this is a high pass filter with cut-off pie by 2 and this becomes a reasonable lowpass filter with cut-off pie by 2. Whatever it be, let us now of course consider the 2^{nd} condition. You see this is Alias cancellation, with Alias cancellation there assuming that there is linear linearity and shift invariant in the system, there is a linear shift invariant system there. What is the system function then? If Tao 1Z is equal to 0, then you have yZ is equal to Tao 0Z xZ.

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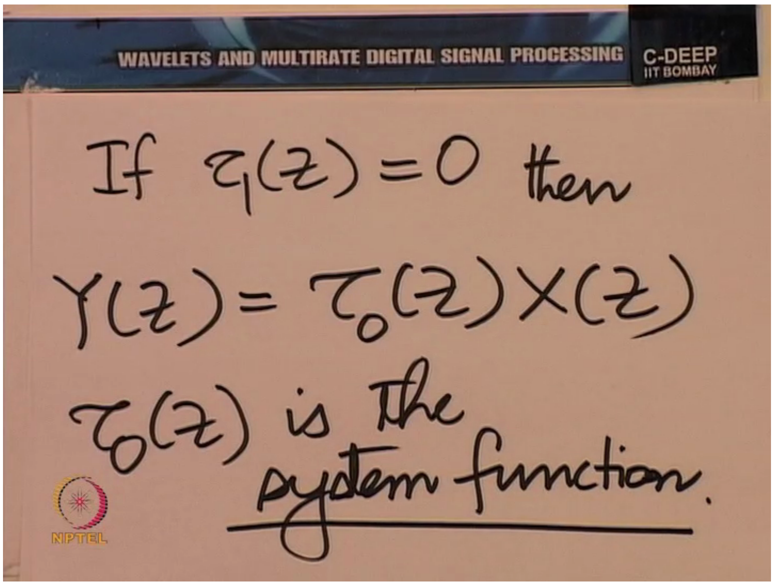

And we are saying effectively that Tao 0Z is the system function in the true sense. So this is an LSI system with the system function Tao 0Z. Now you see one of the things that one needs to worry about is what should Tao 0Z be. Tao 0Z is also modifying experience for the input.

And ultimately we want the composition and reconstruction. So in reconstruction we want xZ to be almost the same as y yZ. If not quite, if not identical, at least there might be tolerable changes, what are these tolerable changes that we can allow? Or more appropriately, what can be and what should be tolerate here, that is what we need to think about.

What can be and what should be tolerate. Well, if you are talking about time systems, then we have to tolerate a delay. 1st let me explain this intuitively and then let me put it down mathematically. The filter does some processing, it takes some time to process at the analysis side and at the synthesis side. Now you need finite time to process at the analysis side, you need finite time to reconstruct at the synthesis side. So if you want to know other change between the input and the output, at least you have to accept the change of a delay to allow for some time to process.

So if we do not want any other change, at least we should allow for it of the form of Z to the power - D in Tao 0 Z. The other thing that we do not mind allowing is an overall multiplicative constant. After all we do not mind if the whole input sequence is multiplied by some constant C, because we can always multiply by 1 by C at the output, it is a simple operation to do, simple amplifier or attenuator, that is not difficult to do. So we do not mind if the whole LSI system that we has here after Alias cancellation, multiplies by a constant and delays by D.

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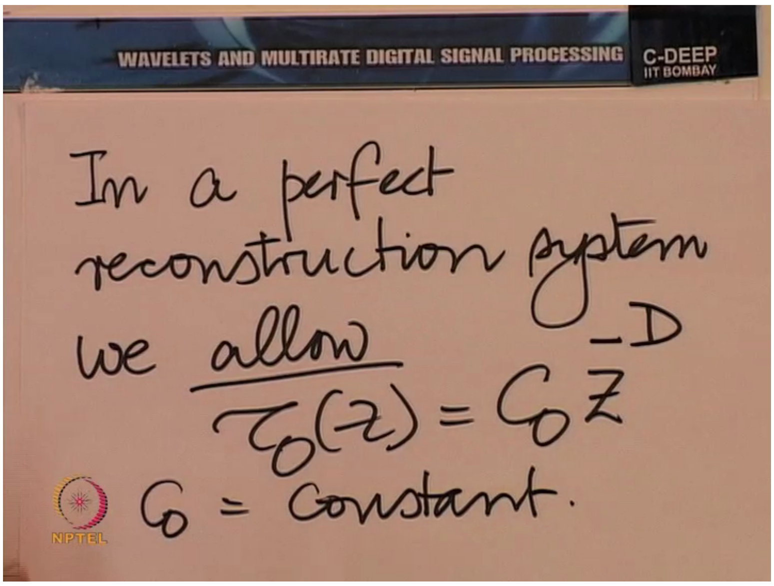

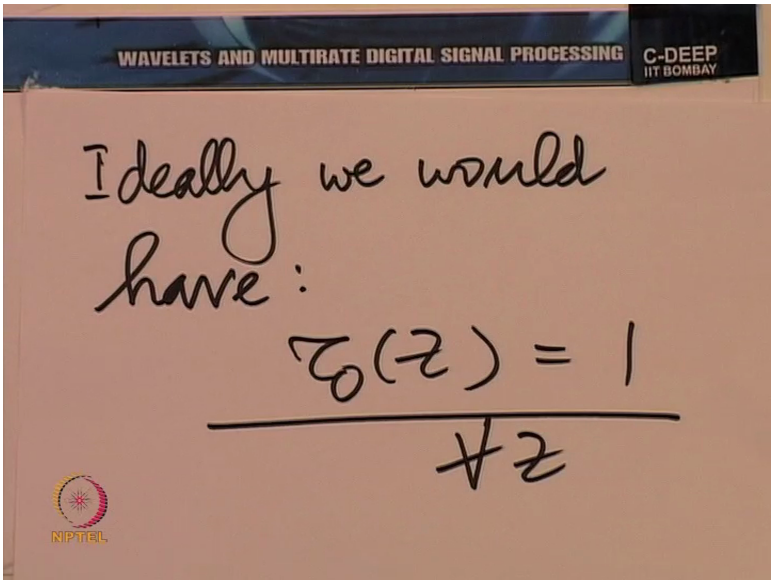

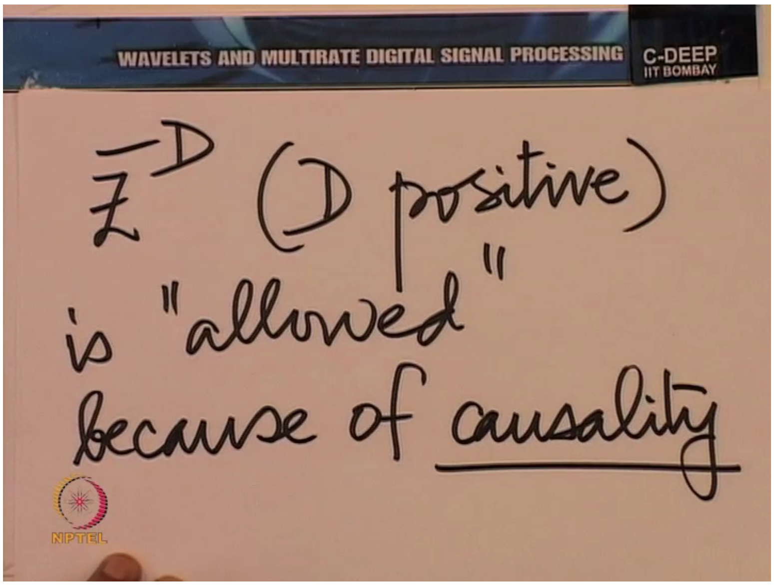

And that is exactly what we shall now put down mathematically. So we are saying, in a perfect reconstruction system, we allow, we should say we allow Tao 0Z to be of the form some constant times Z raised to the power - D, is a constant. Ideally we would have liked Tao 0Z equal to1 for all Z, essentially an identity system. Now that would as I explained before make the system noncausal. So this allowance of Z raised to the power - D, of course D is positive here, is allowed because of causality.

Now we will take the example again as I said of the Haar MRA and the filter bank correspondence to the Haar MRA. Once again we will see if we understand the Haar case, we understand a lot of things at once. So let us put down the filters for the Haar case.