Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-4.
Lecture-11.3.
Consequences of aliasing and simple approach to avoid it.

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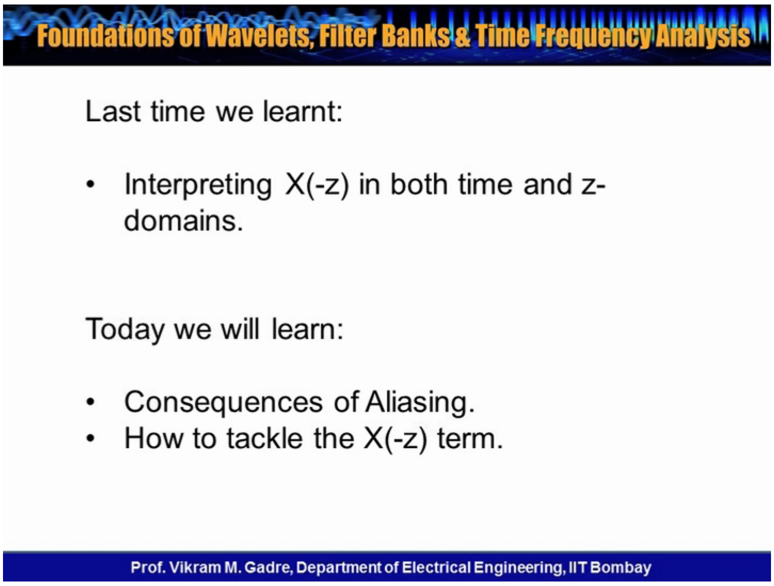

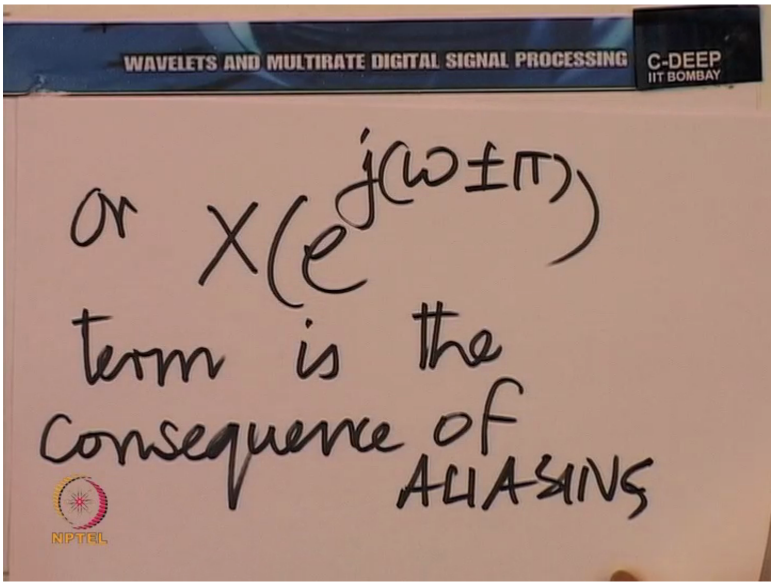

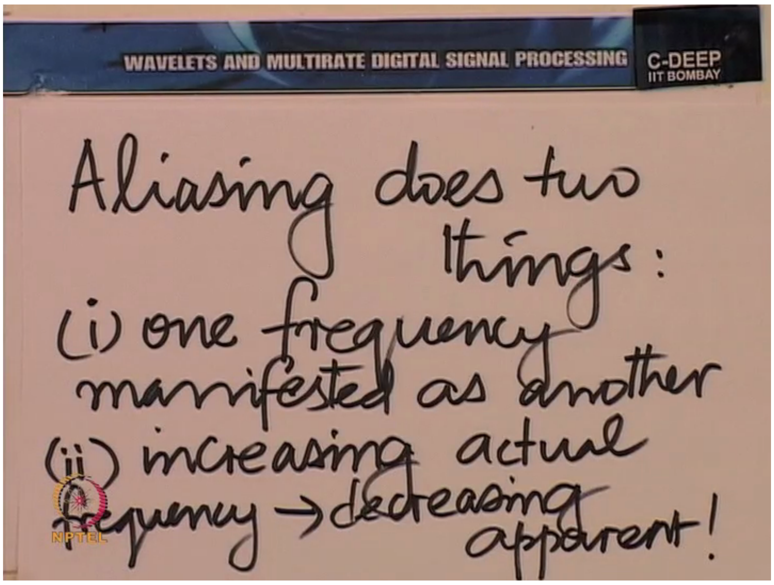

What is the consequence of aliasing, we should put it down very clearly. Aliasing does 2 things, 1 one frequency manifested as another, 2 increasing actual frequency leads to decreasing apparent frequency. So in fact just to emphasise let me put back before you and stress this point.

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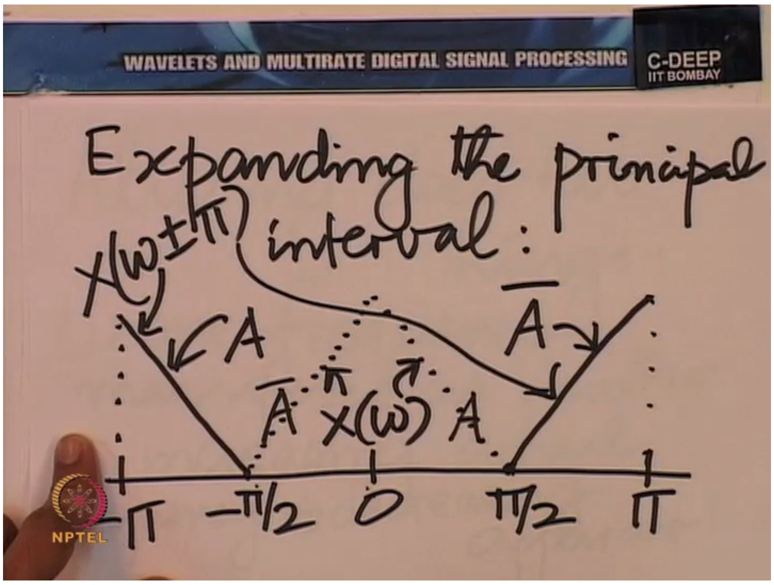

If we want $Y(Z)$ to reconstruct $X(Z)$ we must first do away with ALIASING!

WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING C-DEEP IIT BOMBAY

$$\frac{G_1(Z)}{G_0(Z)} = -\frac{H_0(-Z)}{H_1(-Z)}.$$

WAVELETS AND MULTIRATE DIGITAL SIGNAL PROCESSING C-DEEP IIT BOMBAY

Very simple choice:

$$G_1(Z) = \pm H_0(-Z)$$

$$G_0(Z) = \mp H_1(-Z)$$

As you increase the frequency from 0 to pie by 2 here, the frequency appears to decrease some pie to pie by 2 there. Now the 1st thing we have to do if we want perfect reconstruction is to do away with aliasing. So we put down a condition for aliasing cancellation. If we want YZ to reconstruct XZ, we must 1st do away with aliasing. And how can we possibly do away with aliasing, that essentially means we want tao 1Z to be equal to 0 or in other words we want G0Z H0 - Z + G1 Z H1 - Z equal to 0.

And in fact if we wish to explicitly express the synthesis filters in terms of the analysis filters, we could do that as well. Let us rearrange this equation to get G1 Z by G0Z is - H0 - Z by H1 - Z. And of course a very simple choice is G1 Z equal to the numerator, let us say + or - H0 - Z and correspondingly G0Z is - respectively + H1 - Z. This is a very simple choice of synthesis filter from the analysis filters which can give us alias cancellation.

Now in fact we can even spend a minute in interpreting what we have just said here in this very simple choice. I must of course emphasise, this is a simple choice but definitely not the only choice. In general you should, you should note that there would be a factor cancelled in the numerator and denominator. So more general choice would be as follows.

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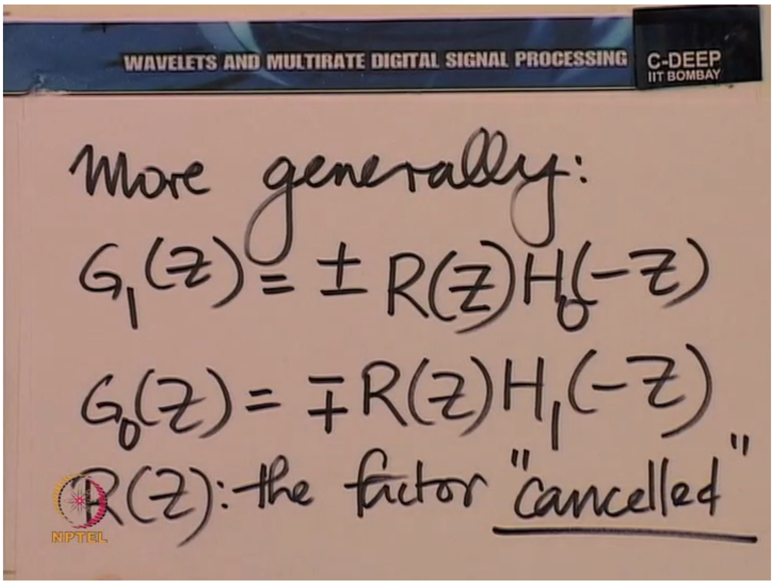

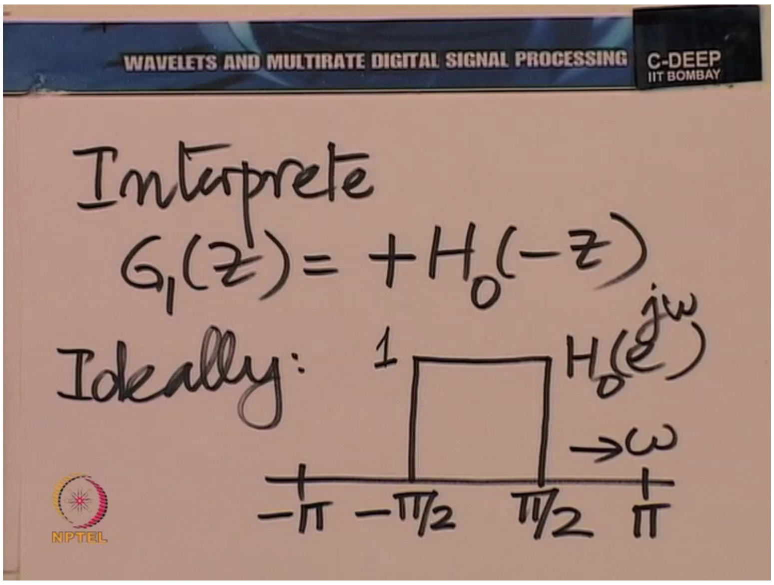

G1 Z is of the form + - some factor, let us say R of Z times H0 - Z. And G0Z is respectively - or + RZ H1 - Z. So RZ is you know in some sense the factor cancelled, you know when you divide G0Z by G1 Z, some factor has been cancelled and what is left is H0 and H1, that is the way you should look at it, that is the more general situation. Anyway now let us interpret what we have written here, namely G1 Z, this is a very simple choice of G1 is H0 of - Z and G0 is + or -, you know you just interchange + or - here H1 of - Z.

So let us take the upper one, G1 Z is H0 - Z, the other one follows similarly. You see, ideally this is what the frequency response of H0 should be, it should be an ideal lowpass filter with a cut-off of pie by 2. So this is what the frequency response should look like, this is 1, the height here. So what would H0 e raised to the power omega J Omega + - pie look like? In other words, what is H0 of - Z then?

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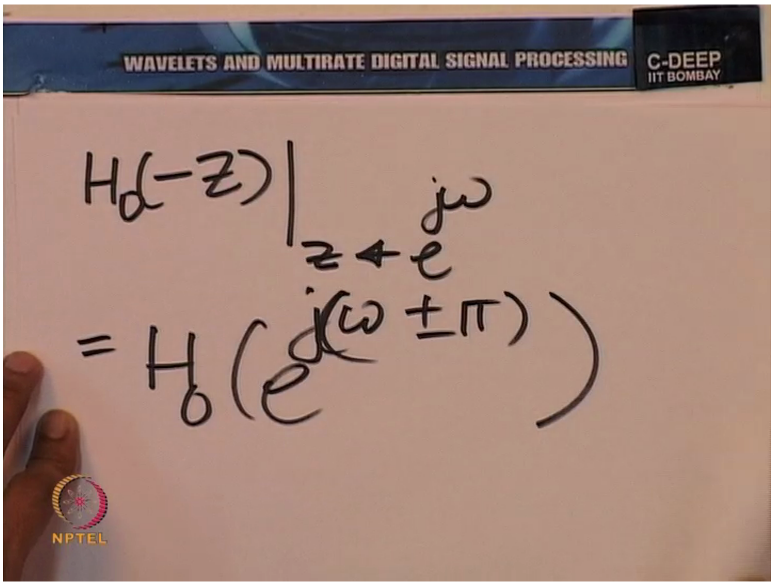

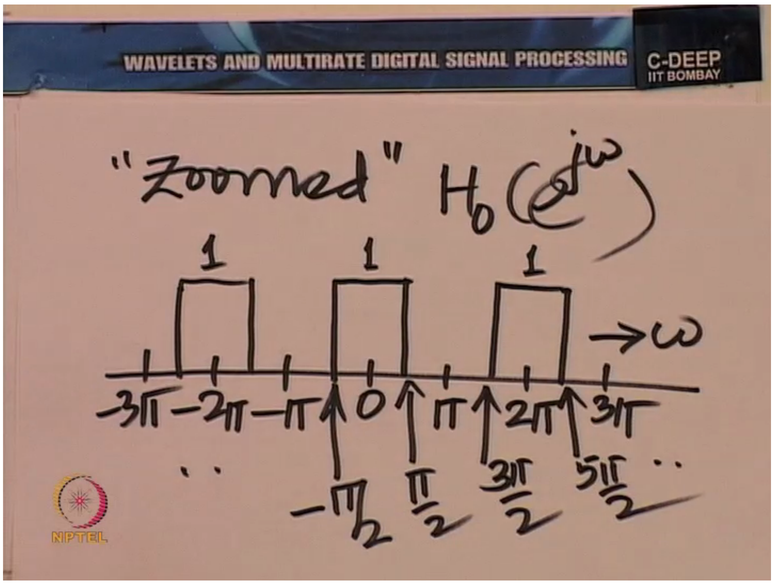

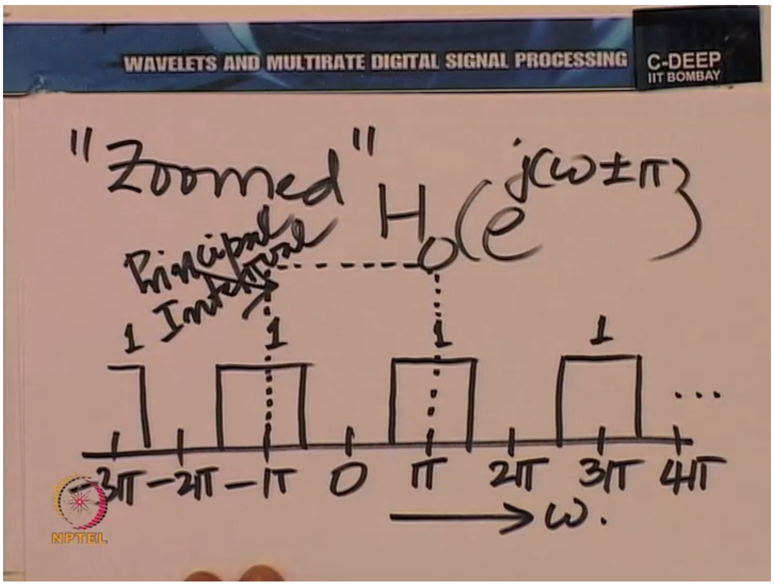

H0 of - Z with Z replaced by e raised to the power J Omega is essentially of course as you know H0 e raised to the power J Omega + - pie. And that is as you can very easily infer looks like this. Again let us use strategy of zooming and then contracting back again. So I have pie, - pie there, 2 pie, 3 pie and so on. This is what H0 look like, taking note of the periodicity, remember. This is - pie by 2 there, this is pie by 2, this would therefore be 3 pie by 2 and so on.

So the zoomed H0 e raised to the power J Omega + - pie would look like this, it would have an appearance like this and so on here. These pass bands so to speak are now going to lie at the odd multiples of pie and you can continue that drawing. Now once again we confine to the principal interval, the principal interval is here. And when we do so and then zoom back to emphasise only the principal interval, what do we get?

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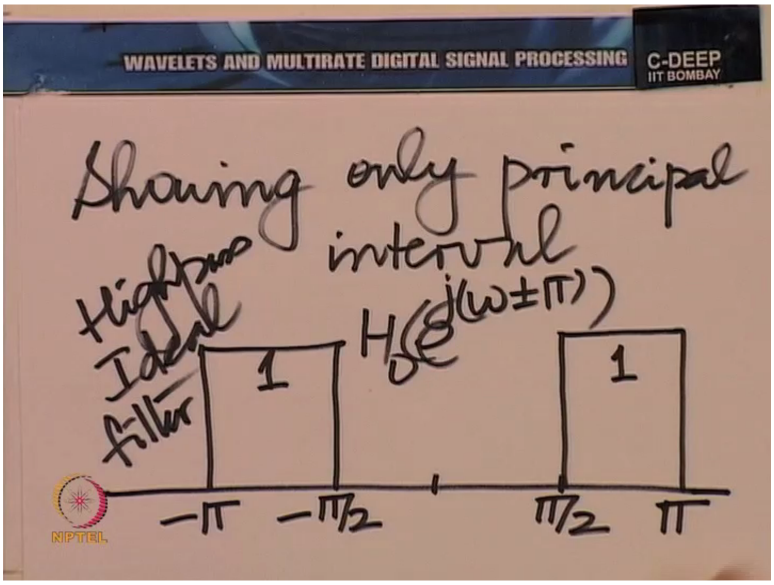

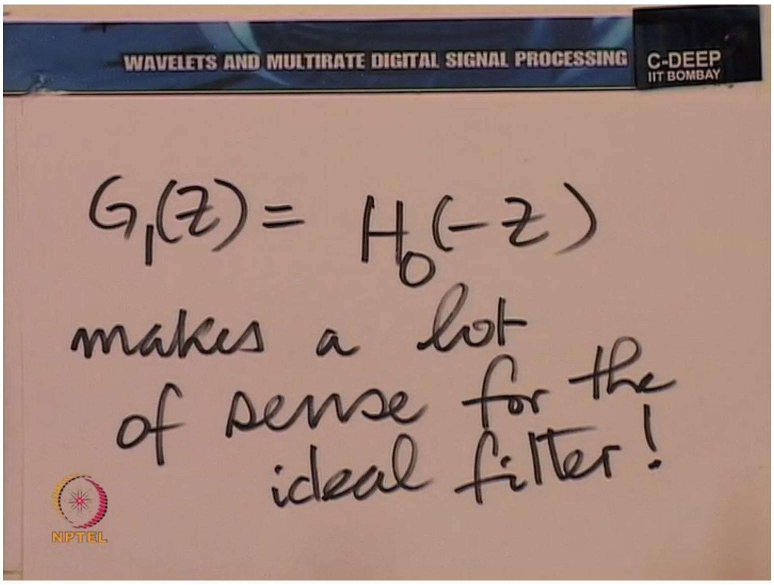

We get this, lo and behold, from a lowpass filter with a cut-off of pie by 2, we have a high pass ideal filter of course again with a cut-off of pie by 2. So this falls into place very well, in fact the equation G1 Z is H0 - Z makes a lot of sense. All that we are saying is that on the synthesis side if you had an ideal lowpass filter with a cut-off of pie by 2 at H0, you should put an ideal high pass filter with cut-off of pie by 2 at the point G1. And you know if you look back at the other one of the 2, namely G0, you would make a similar inference.

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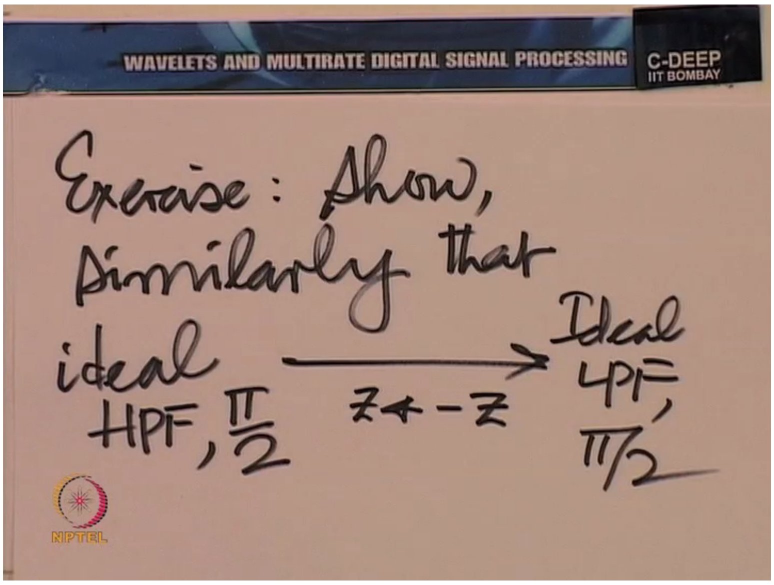

G0Z is H1 of - Z with a - sign. So if you look at the magnitude sense, again a high pass filter would become a lowpass filter, in fact I leave this to you as an exercise. So exercise, show similarly that an ideal high pass filter HPF with a cut-off of pie by 2 on replacing Z by - Z becomes an ideal lowpass filter with a cut-off of pie by 2. And that also makes a lot of sense for the other of the 2 requirements for alias cancellation.

So today we have looked that the condition for alias cancellation and we have interpreted the simplest of the conditions. Now if you do not stick to the simple condition and if you have the factor of RZ, all that we need to do is to say that we are also modifying that filter a little, beyond just this lowpass to high pass and high pass lowpass conversion, that RZ factor would carry out that modification. So we have taken one out of 2 steps today in building a perfect reconstruction 2 band filter bank.

We have taken the step of alias Cancellation. In the next lecture we shall take the 2^{nd} step, namely perfect reconstruction. So we have done away with tao Z, I mean tao1 Z, but now we need to see what to do with tao 0Z which we shall do in the next lecture. Thank you.