Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-4.
Lecture-11.2.
Effect of X (-Z) in time domain and aliasing.

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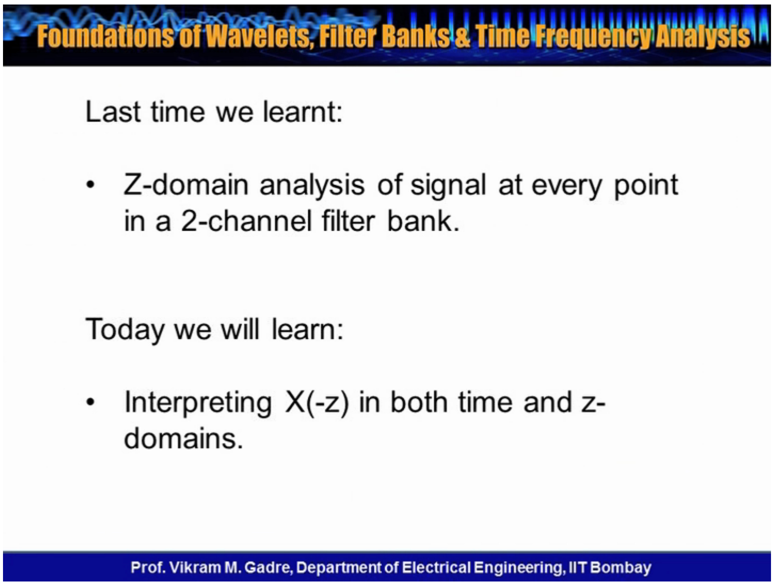

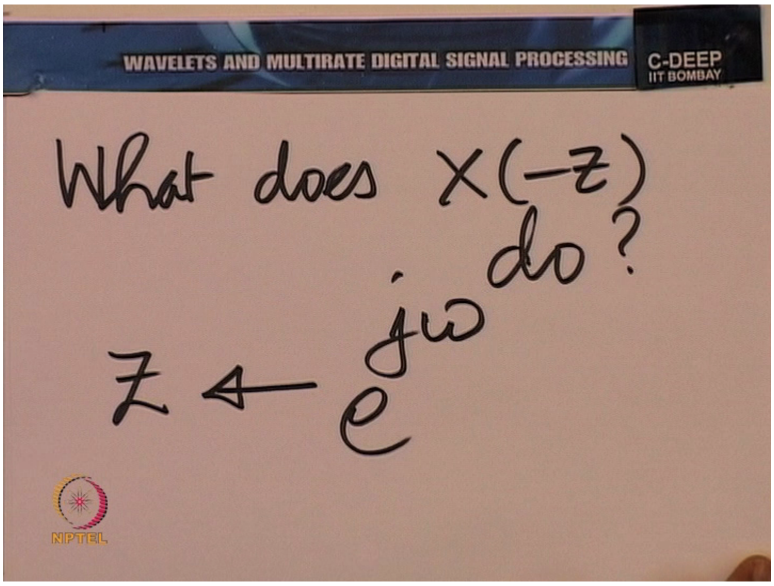

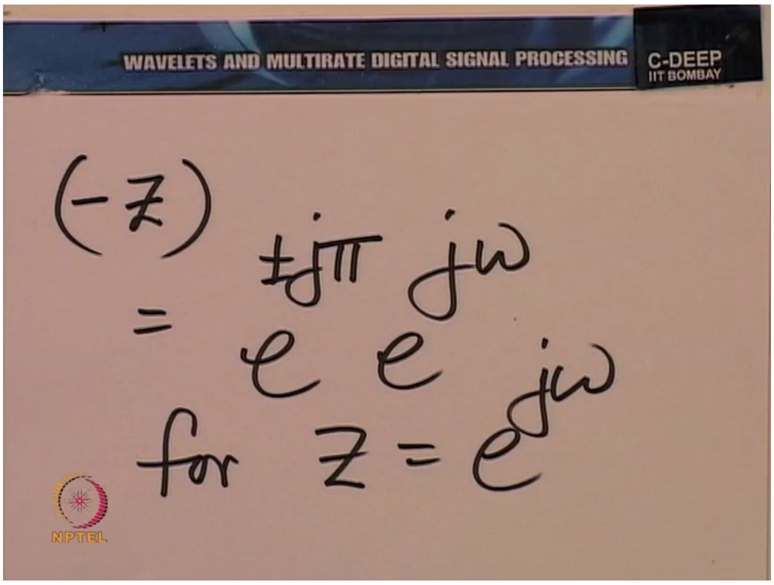

What does X of - Z do? And to answer this question we put Z equal to e raised to the power J Omega as we always do to interpret things in the frequency domain. Now when you replace Z by - Z, you are replacing the race to the power J Omega by, well, - Z is e raised to the power less - J Pi times he raised to the power J Omega for Z equal to e raised to the power J Omega. Recall that e raised to the power + - J pie is -1.

So now we have an interpretation for multiplication by -1 in the Z domain. Essentially we are replacing e raised to the power J Omega by e raised to the power J Omega + - pie. In other words we are shifting on the omega axis by either pie or - pie. Now it should be noted that shifting by pie and shifting by - pie are the same thing that is because there is periodicity on the small omega or the normalised angular frequency axis, periodicity with a period of 2 pie.

So there is no problem if we replace + pie by - pie, shifting by + pie and shifting by - pie are the same thing. With that little remark and observation, what we have done in replacing Z by - Z as far as the normalised angular frequency is concerned is to shift by pie, whether + pie or - pie. And we would best understand what the shift implies if we took an example spectrum. So let us take an example spectrum to complete this discussion.

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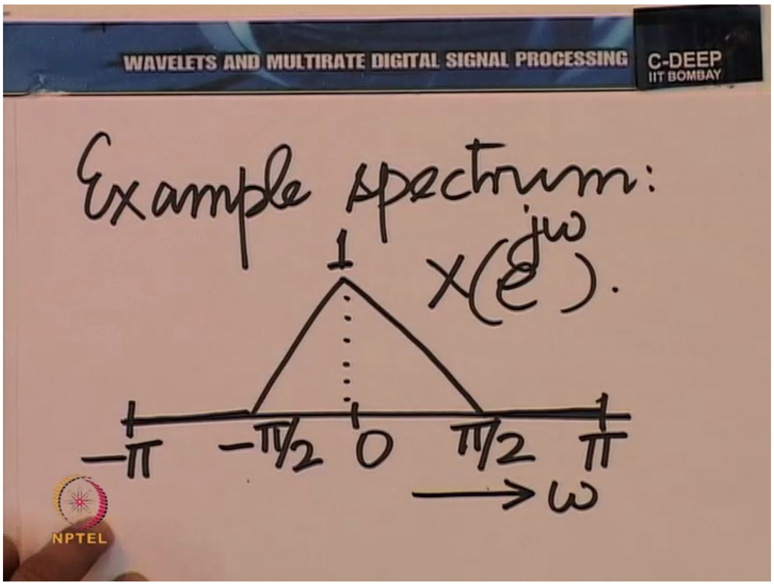

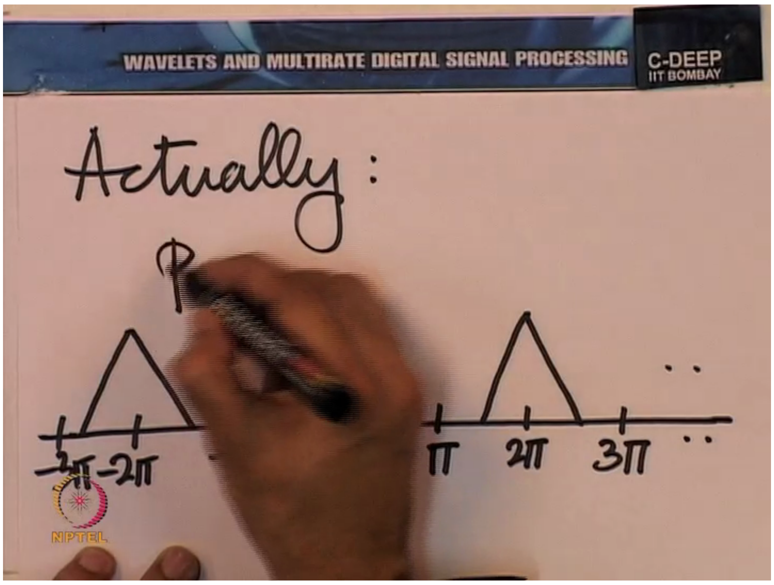

Let us take this spectrum. Consider an idealised spectrum like this essentially with lines lying between - pie by 2 and + pie by 2 and 0 outside. So this is the spectrum of some sequence, let that sequence be X of N. So this is the normalised angular frequency Omega, this is capital X of strictly e raised to the power J Omega but as you know in discrete time processing we sometimes abuse notation a little bit and we write just capital X of omega instead of capital X of e raised to the power J Omega which is more correct.

But anyway I must emphasise that as far as we are concerned from the context, capital X of omega and capital X of e raised to the power J Omega essentially means the same thing. Now with that remark what happens when you shift by pie here? So recall that actually what… Actually this is what X E this to the power J Omega is. So we will do a bit of zoom here, alright, periodically repeated with a period of 2 pie. This is what X of e raised to the power J Omega actually is. So you can visualize, if you shift this either forward or backward by pie, it is going to give you the same thing and what is going to appear after shifting by pie is the following.

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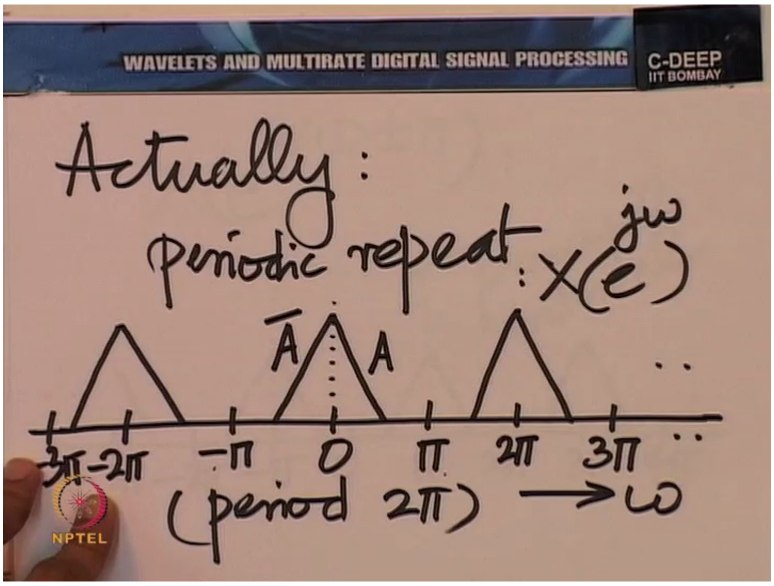

It would therefore appear like this, again we shall do a bit of zooming here and so on here and so on there. So now your triangles are around pie and then 3 pie and so on and here around - pie and then - 3 pie and so on. So now again if you take the principal interval which is here, this is what we get. Now you know we must be a little more explicit in our relationships here. So in X of e raised to the power J Omega, if you look at the principal interval, - pie to + pie, we notice that there are 2 segments here for the positive side of the Omega axis and the negative side of the omega axis.

Let us call the segment for the positive side A and the segment for the negative side A bar and in fact there is a reason for this. You remember that the discrete time Fourier transform has conjugate symmetry for a real sequence. So it is that reminiscence that we are writing A A bar. I agree that if the sequence is complex, this conjugate symmetry would not be there. But very often we deal with real sequences and therefore it is useful put the symbols A and A bar there just for emphasis.

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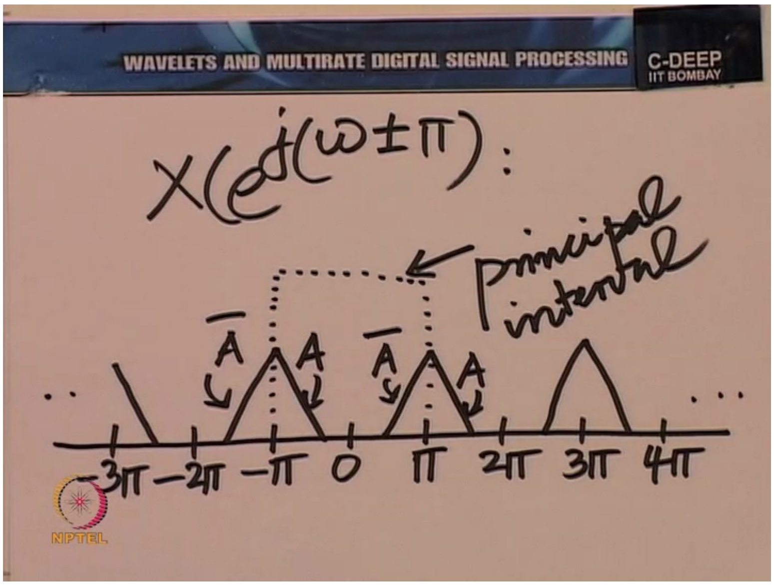

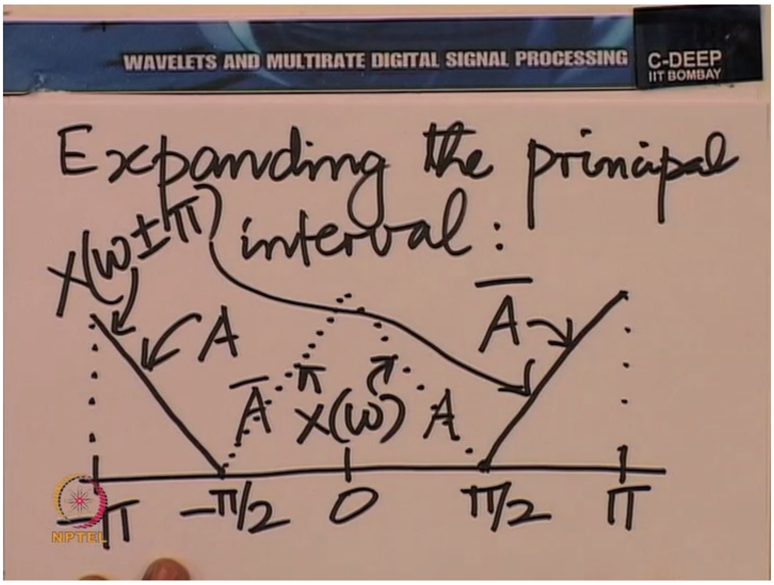

Anyway now what we will do is in X of e raised to the power J Omega + - pie, we shall mark the As and A bars carefully. So here again in the principal interval, this was actually A and this was A bar, is not it. And therefore this was A and this A bar. And now what we need to do is to expand the principal interval for our convenience. So expanding the principal interval, this is what we get. We get an A bar here and we get an A there. Now you know just for convenience I shall show in dotted on the same diagram what we had just for capital X.

So what I am saying is the solid lines here are X Omega + - pie and I shall show in dotted X omega, just to be clear. Again, forgive the abuse of notations where we have replaced e raised to the power J Omega by just Omega for clarity here, all right. So you know in X of omega you had A here and A bar there and now you have A bar here and A there. So what happened is in effect, these so-called negative frequencies between - pie by 2 and 0 have now appeared between pie by 2 and pie. So 2 things have happened, the order of the frequencies has been reversed.

So for example in principle, a frequency let us say of - pie by 4 here is more than a frequency of let us say -, you know here you would have 0, pie by 8, pie by 4, 3 pie by 8 and so on and the same on this side. So if you take a - pie by 8 frequency here and then a - pie by 4 frequency there, the - pie by 4 frequency appears as a smaller frequency here. One can see that because the order of frequencies from 0 to pie or 0 to - pie by 2 has been now reordered between pie and pie by 2.

So increasing magnitude of frequency here becomes decreasing magnitude of frequency there. Secondly, the frequencies themselves change. So you know actually this is exactly what happens in what is called aliasing. Many of us will recall from the theory of sampling, an aliasing, this is something very fundamental. In a course on discrete time signal processing or in the process of sampling a continuous time signal to get a discrete sequence, these things happen.

So if we do not sample at inadequate rate, we get a problem for aliasing. What is aliasing, aliasing is a phenomenon where a sine wave assumes a false identity. And there are 2 things that happens in aliasing, one of them is of course that the frequency of the sine wave appears to change, the other is that when you increase the frequency of the sine wave, the apparent time wave or the sine wave produced by impersonation if you would like to call it that seems to have its frequency reducing.

So an increasing frequency, an increasing actual frequency seems to appear as an apparently reducing frequency. These 2 things together they together make up the concept of aliasing and what we are seeing here is just an instance of aliasing. Why are we seeing aliasing? It is not very difficult to understand. What we have done is to downsample. You know in the process of downsampling we have in fact introduced the possibility of aliasing and this X of - Z term is bringing before you the consequences that would be there if aliasing were allowed to remain.

Think about it, X of - Z is therefore called the aliasing term because it tells you the contribution of possible aliasing, in the 2 band perfect reconstruction filter bank which we shall soon see, this aliasing should be absent. By perfect reconstruction I mean, if Y is to be an exact replica of X, as is the case for example in the Haar system, when you have chosen the filters properly. So you analyse or you decompose and then you synthesise or reconstruct and if the reconstruction is perfect, there should be no aliasing.

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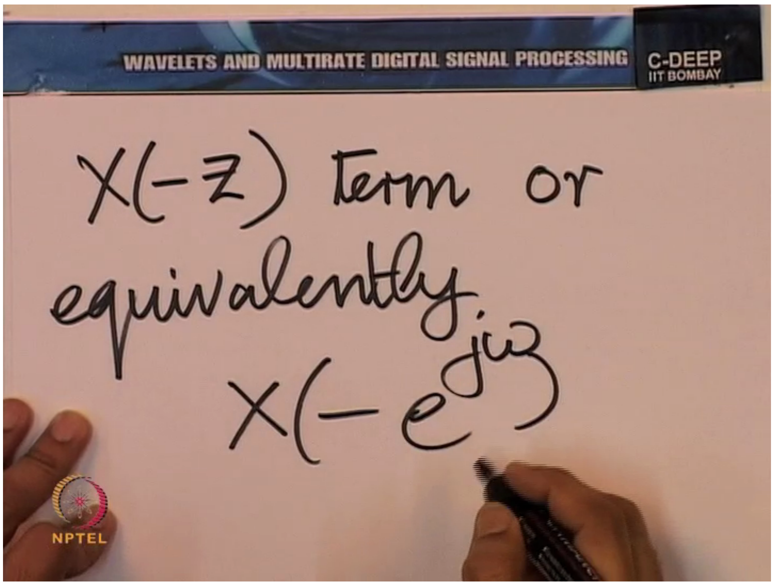

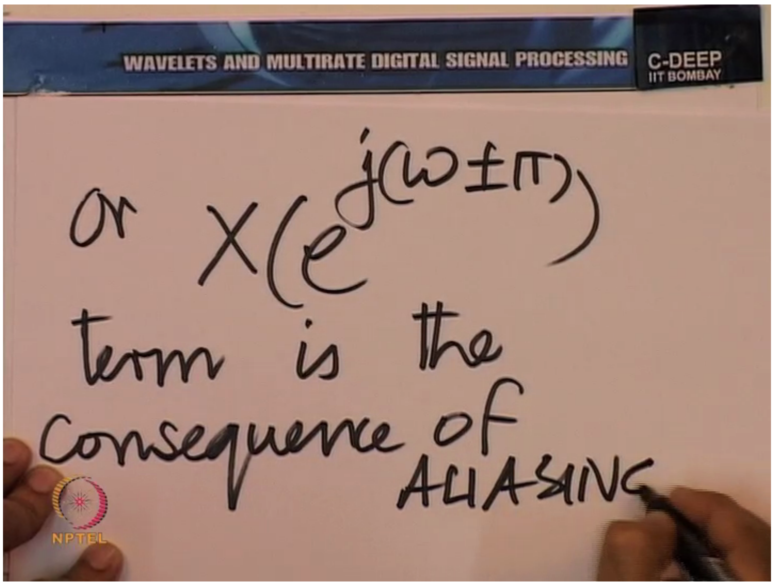

So this X of - Z term is a troublemaker term in general, troublemaker in this sense. Let us make a note of this. So the X of - Z term or equivalently the X of - e raised to the power J Omega or X E to the power J Omega + - pie term is a consequence of aliasing.