Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-3.
Lecture-10.2.
Upsampling by a general factor M-a Z-domain analysis.

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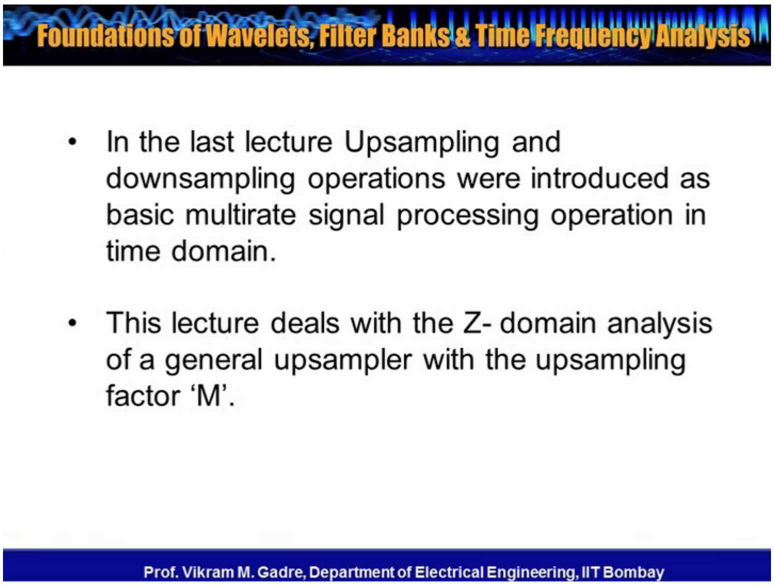

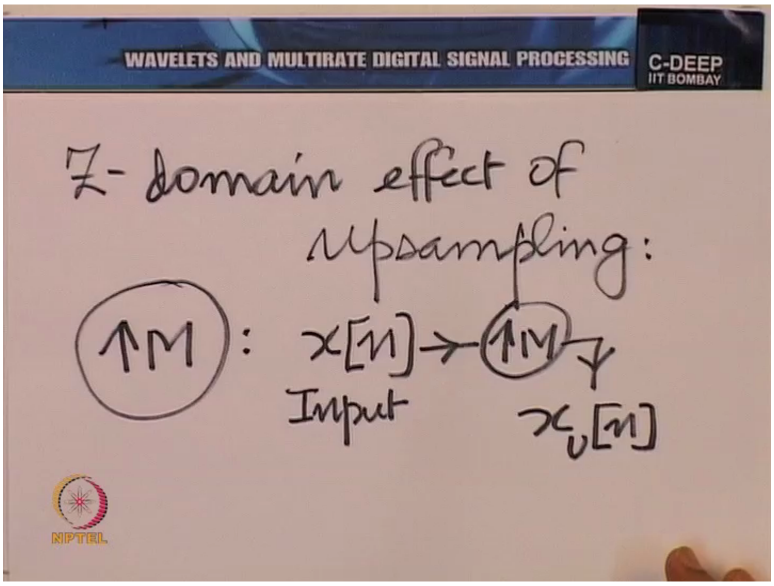

Now you know it is easier to understand the effect of upsampling, in fact by a general sample of M in the Z domain, so let us begin with that. Let us consider a general factor of M, upsampling by M, most general. What do we do in the upsample by M, that is easy. You see the original sequence is X of n, so this is the input. After upsampling by M, let the sequences become XU of n. now we can write down XU of n explicitly in terms of X of n. In fact instead of doing that, let us straightaway go into the Z domain.

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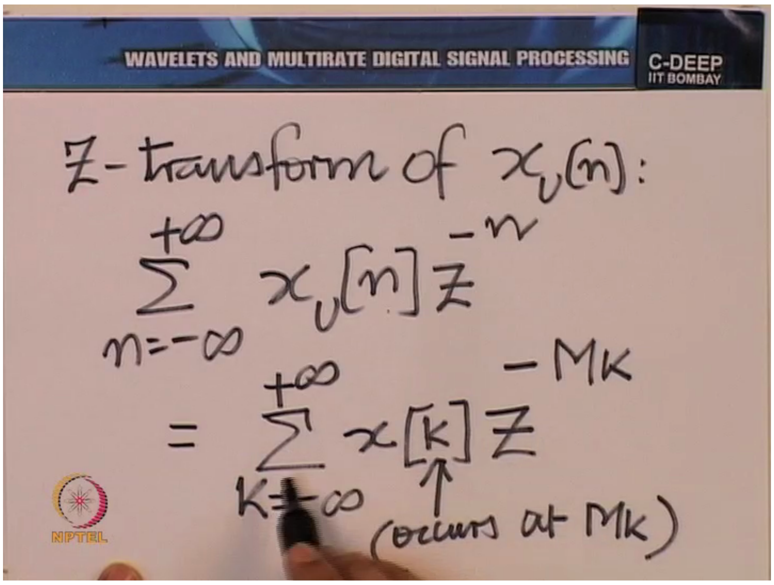

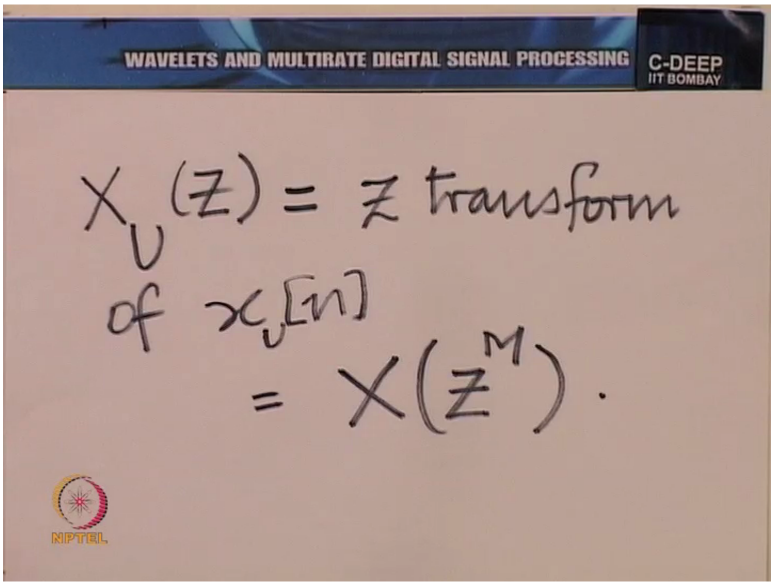

Let us consider the Z transform of XU n. That is easy, summation n going from - infinity to + infinity XUN Z raised to the power - n. And the only catch is among all these ns, only those ns have nonzero sample which are multiples of capital M. And therefore we can rewrite this summation here as summation again, let us say K going from - to + infinity, X of K, now remember X of K occurs at the point M times K in XU n. And therefore here we have Z raised to the power - M times K coming up.

Now this is a familiar expression, this is very like the Z transform of X, except that you replace Z by Z raised to the power capital M. So in fact there we have a very simple relationship for XUZ to XZ. Indeed, the Z transform of the sequence XUN, namely XU Z, capital XUZ is the Z transform of the sequence X with Z replaced by Z raised to the power M, simple. Now in fact this also brings us before us very clearly the invertibility.

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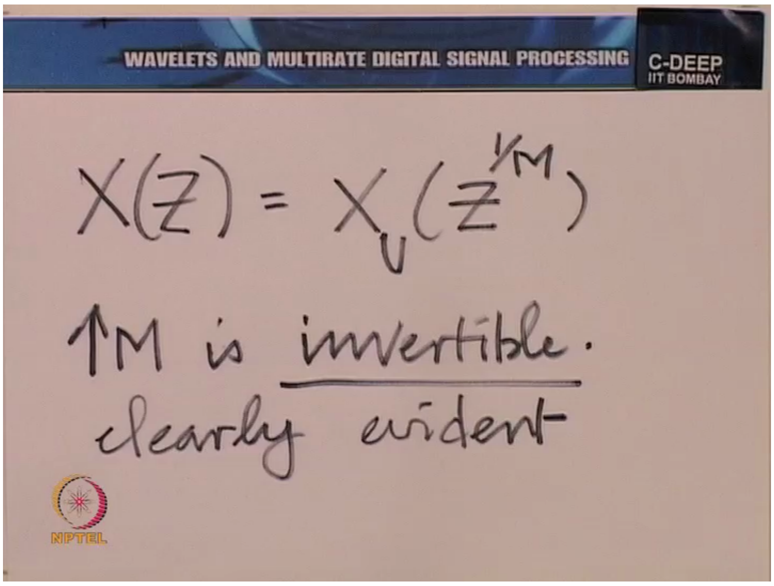

So just as you see X of, XU of Z is X of Z raised to the power M, you can also say capital X of Z is XU Z raised to the power 1 by M. So what we are saying in effect is that this operation of upsampling by M, you know, this operation is invertible, clearly shown here, clearly evident. Whether we look at it in the time domain or whether we look at it in the Z domain, this property is evident, the invertibility.

Very soon we shall look at the downsampling operation and there we shall also see the non-invertibility evident in the Z domain, perhaps not as evident but evident all the same. So let us now go to the downsampling. Now, as I said downsampling is a concatenation of 2 processes. In one process we kill some samples, in the 2nd process we compress the so-called partially killed sequence. This process of killing some samples is what requires little bit of effort to express in the Z domain.

And here we shall not attempt straightaway to jump to generalisation to any capital M, we shall instead take capital M equal to 2, I listed the idea and then allow for the generalisation to any other capital M. So then let us proceed then with downsampling with a factor of 2 now 1st. So here we are, when we wish to downsample by a factor of 2, what we are saying in effect is we are killing every other sample. And how do we kill, one way to kill his to multiply by a killing sequence so to speak.

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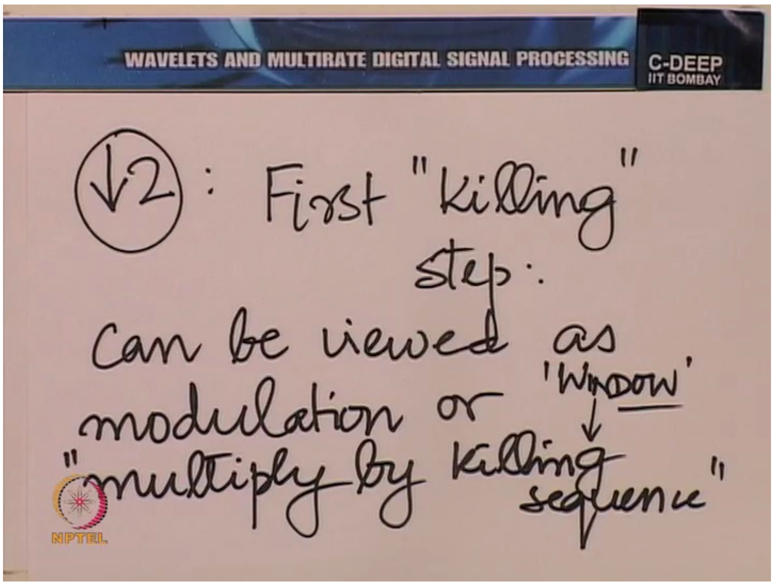

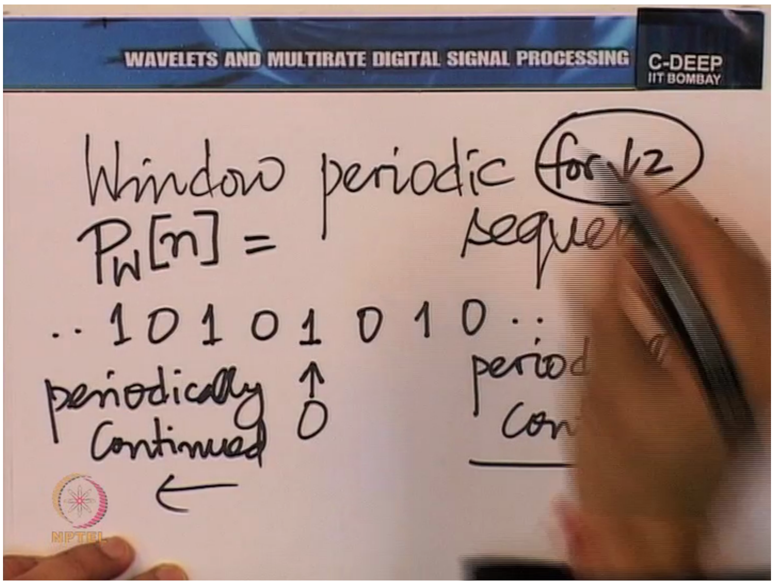

So when we consider downsampling by a factor of 2, the 1st killing step can be viewed as a modulation step or multiplication by another sequence, by killing sequence let us call it. Instead of giving such a violent name, maybe we should call this a window sequence, that is a milder name. You know, what does a window do? A window allows you to see a certain part of the scene outside. In a certain sense when we retain some samples and destroy all others that is exactly what we are doing.

We are allowing those samples to be retained to be passed through the window and those that are destroyed to be stopped. So if you wish we can call this sequence that keeps some and removes the other as a window, but a window periodic sequence. You know, I use the word window periodic sequence because the window sequence name has also been used in the design of finite impulse response filters.

And the window periodic sequence has the following pattern, at the point 0, it takes the value 1, at the point one, it has the value 0 and then periodically thus. And the same on the negative side, periodically continued on both sides. So you know you can think of it as a periodic window where whenever there is a 1, you are allowing the scene to pass and wherever there is a 0, you are stopping the scene.

Let us call this periodic window sequence PWn where P is to denote periodic and the W subscript is to denote the window behaviour. So this is a window periodic sequence for downsampling by 2.