Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-3.
Lecture-10.1.
Introduction to upsampling and downsampling as Multirate operations.

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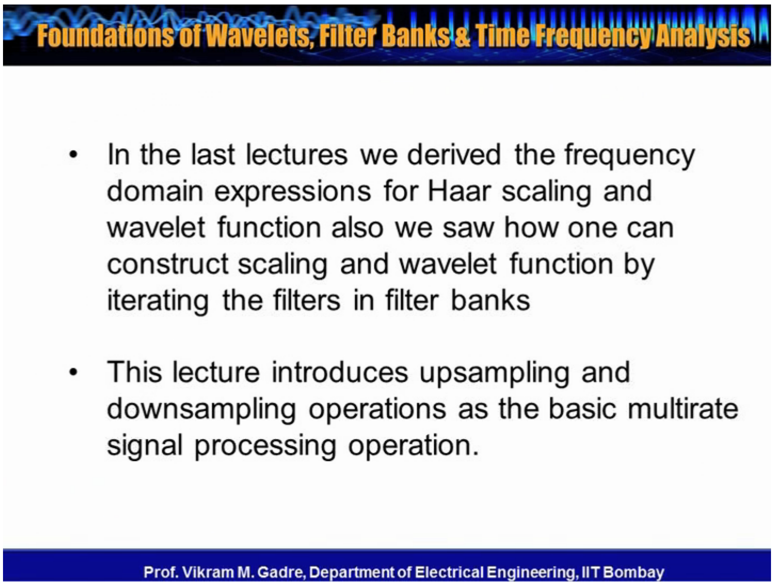

A warm welcome to the lecture on the subject of wavelets and multirate digital signal processing. Recall that in the previous lecture we had established the intimate connection between the scaling function, the wavelet function I mean the continuous time scaling function, the wavelet functions and the 2 channel filter bank about which we had been discussing in a few lectures before that. Towards the end of the previous lecture we were convinced that there is an intimate connection between designing a 2 band filter bank and designing a multiresolution analysis.

In fact the connection is so strong that when one knows the impulse response of the lowpass filter in the filter banks that had been looking at, one can by iterative convolution obtain the scaling function and having obtained the scaling function one can obtain the wavelet function. So it is a very powerful reason to study the 2 channel filter bank in great depths and that is exactly what we intend to do today. Let me put before you the theme of the lecture today.

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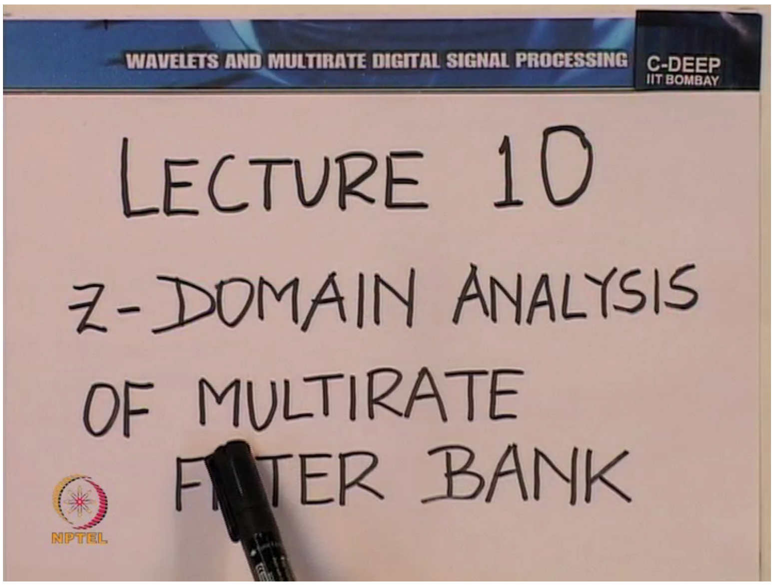

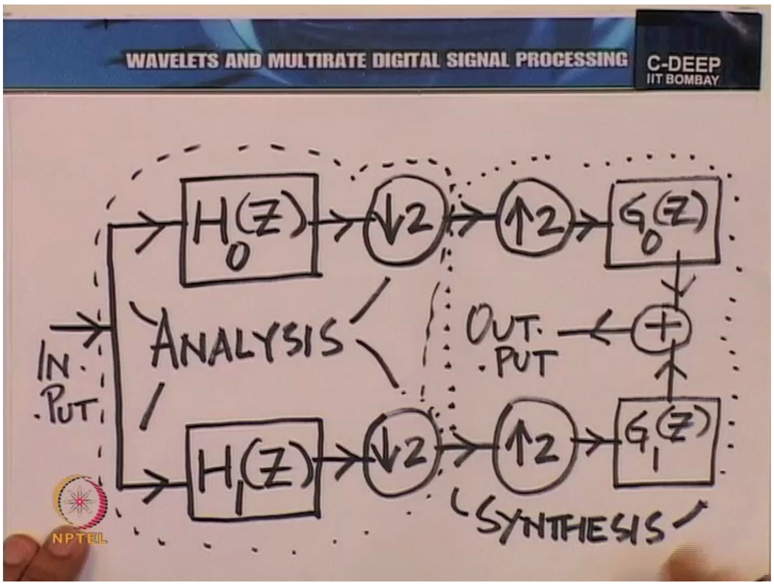

So today we intend to talk about the Z domain analysis of the multirate filter bank and to do that we shall begin by considering the basic operations of the multirate system, namely downsampling and upsampling. The rest of it is known to us from a basic exposure to discrete time systems but how to deal with multirate operations is what we need to understand much more carefully in the next few minutes. So let us once again draw the structure of the 2 channel filter bank which we had been looking at in some of the previous lectures.

Recall that the 2 channel filter bank has an analysis component and a synthesis component. We had given the following nomenclature for the analysis and the synthesis side, I shall just recapitulate that. So on the analysis side we have 2 filters, the lowpass analysis filter and the high pass analysis filter followed by downsampling operations. On the synthesis side we have 1st the upsampling operation followed by the synthesis lowpass filter and the synthesis high pass filter.

And you will recall that the outputs of these 2 were added and this constituted the overall output. So we have the input here and the output here. To recall, this is going towards the ideal discrete time lowpass filter with a cut-off of pie by 2 on the normalised angular frequency axis and so is this. This is going towards the discrete time ideal high pass filter with a cut-off of pie by 2 again and so is this.

This is what we call the analysis side of the filter bank upto here and this is what we call the synthesis side, when I mark this, I mean all this, so we should put a dotted boundary around, so this is all the synthesis side and this is the analysis side. Anyway, with this little recall what would it in the previous lecture, let us identify what it is in this filter bank which is unusual, which is beyond what one normally is exposed to in a basic course in discrete time systems.

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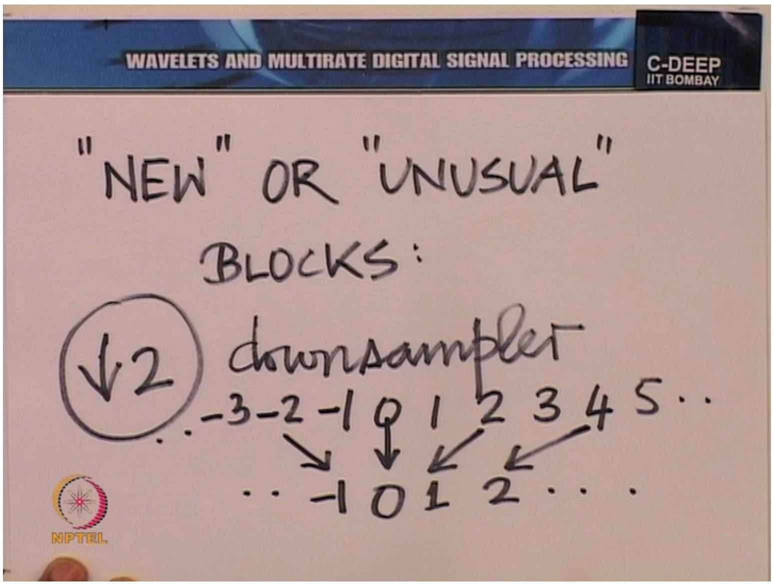

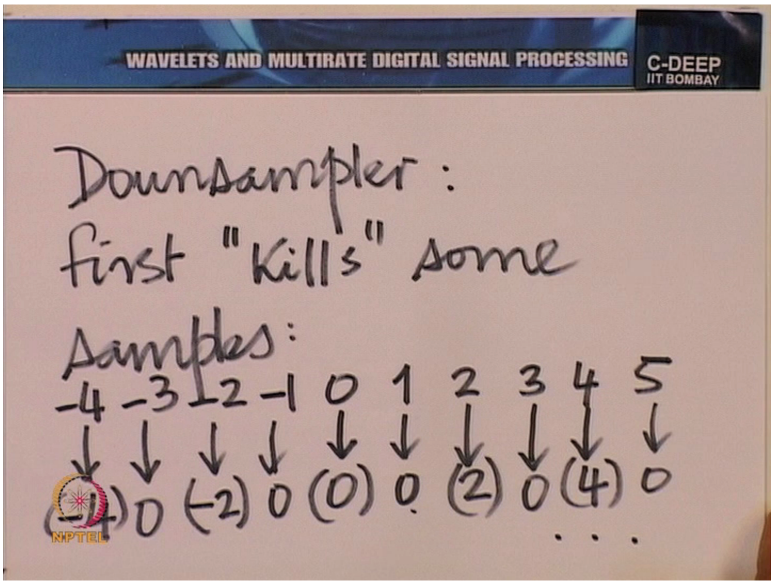

If you look at it carefully, there are 2 unusual blocks here and they are the downsampler and the upsampler. So the unusual or the new blocks, the downsampler and you know what the downsampler did, when you had a sequence indexed by the integer, so let us say it is an indexed by 0, 1, 2, 3, 4, 5 and so on and of course - 1, -2, -3 and so on on this side. The downsampler, let us the 0 sample go to the point 0, the 2 sample to the point 1, the 4 sample to the point 2 and so on so forth.

And on this side -2 sample comes to the point -1 and so on so forth. So in a certain sense, the downsampler is both, remover of some samples and a compressor of the remaining samples. There are 2 steps in downsampling if you think about it and we shall in fact analyse the downsampler treating it as a cascade of 2 steps. So to state it clearly, downsampler 1st kills some samples. So for example if I go back to the same indexing 0, 1, 2, 3, 4, 5 and so on here and -1, -2, -2, -3, -4 and so on there.

Then 0 is as it is, 1 is reduced to 0, 2 is as it is, so when I write the number in brackets, what I mean is this sample is brought forth as it is and when I write 0 here, what I am mean is this sample is made 0. So this is made 0, 4 as it is, 5 is made 0, -1 made 0, -2 as it is, -3 made 0, -4 as it is and so on so forth, this is the 1st step.

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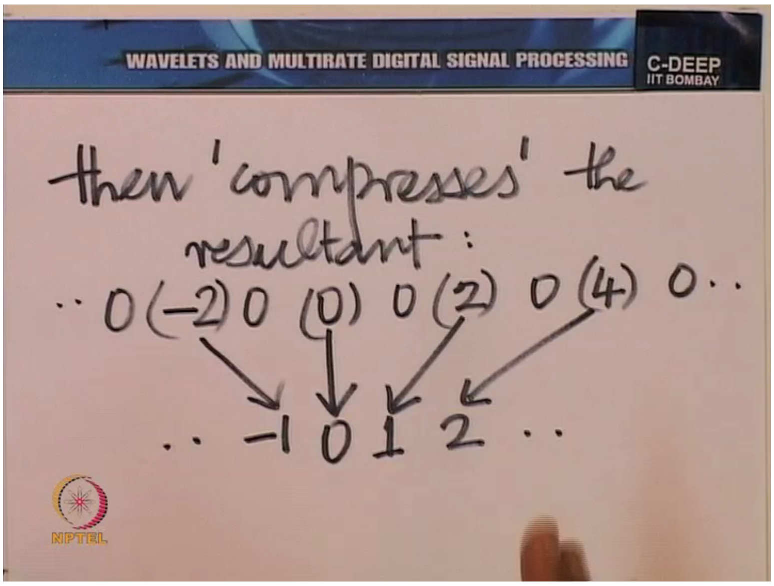

The 2nd step is to compress, so its 1st kills some samples, then compresses the resultant. So you have 0 where it was, then a 0, 2 where it was, then 0, 4 where it was, then 0 and so on so forth here. You have 0 in place of -1, then -2 where it was, 0 in place of -3 and so on so forth here. And the compressor brings 0 to the point 0, this to the point 1, 4 to the point 2 and so on so forth. And here it brings -2 to the point -1 and so on so forth. So there is a so-called compression taking place, removal of the unwanted zeros.

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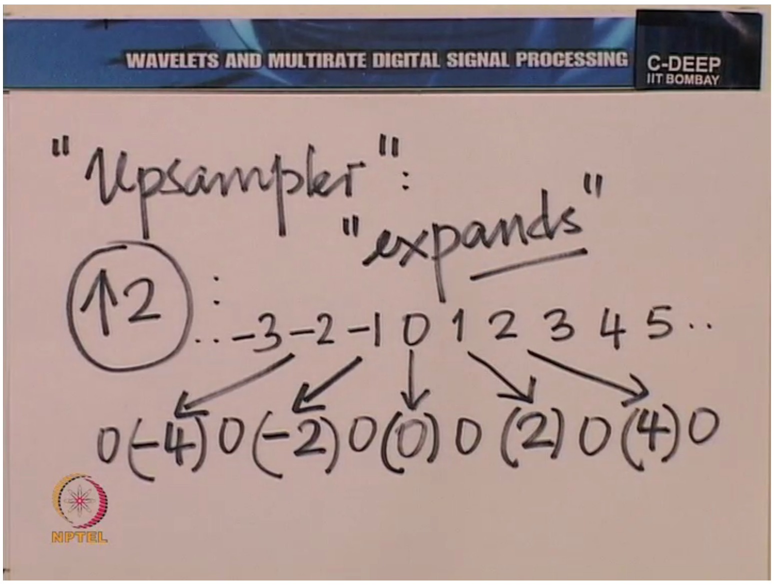

Now when we do a Z domain analysis of this, we shall do it precisely in these 2 steps. Let us before we proceed to the Z domain analysis, look at the 2nd unusual block here. The 2nd unusual block is the upsampler and we denoted upsampler by up arrow followed by 2. And you know what the upsample does. If you have a sequence indexed by 0, 1, 2, 3, 4, 5 and so on so forth and of course to continue -1, -2, -3 and so on so forth on this side. The 0 sample goes where it is 0, the 1 sample goes to 2, the 2 sample goes to 4, the -1 sample goes to -2 and -2 to -4 and so on so forth.

So what will do is we will put these in brackets now and we insert zeros in between. So in other words we expand in some sense, an upsampler also expands in some sense. So some people like to call it an expander because it in some sense outdoes this last step that a downsampler does. You know, if you think about it, this last step which a downsampler does, namely compresses by removing zeros is outdone by the upsampler. We should make a note of that, it is important.

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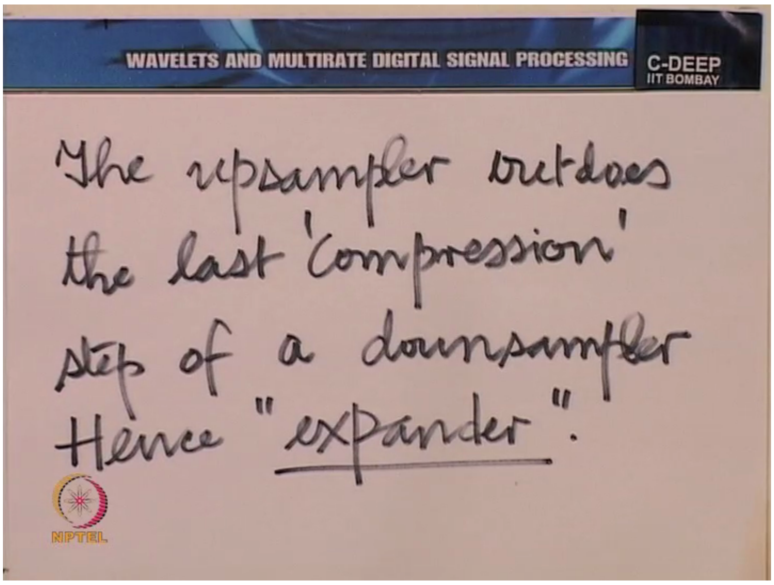

The upsampler outdoes the last compression step of a downsampler, hence it is also called an expander. Now I must emphasise that the upsampling operation is invertible. So even the original sequence is unaffected by the process of upsampling, you can get back the original sequence after upsampling. The same is not true of downsampling. When you downsample, you lose some information forever.

So in a certain sense you know downsampling 1st as I said kills some samples and then also compresses, in some sense covers up that killing part, if you want to call it that, by removing evidence of the zeros that have been created. What the upsampler does is to put back the zeros where they were, as if you are recreating the evidence of killing, in a lighter vein of course. Whatever it be, the upsampler takes you from a lower rate system to a higher rate system and the downsampler takes you from a higher rate system to a lower rate system.

So when you downsample, you are actually changing the effective sampling rate. You know if you think about it, in downsampling what are you doing, you are retaining one out of every 2 samples. And in principle what time you allocate to processing each of these samples after downsampling is up to you. What is practically done in most discrete time systems is when downsampling takes place one allows more time to process samples after downsample.

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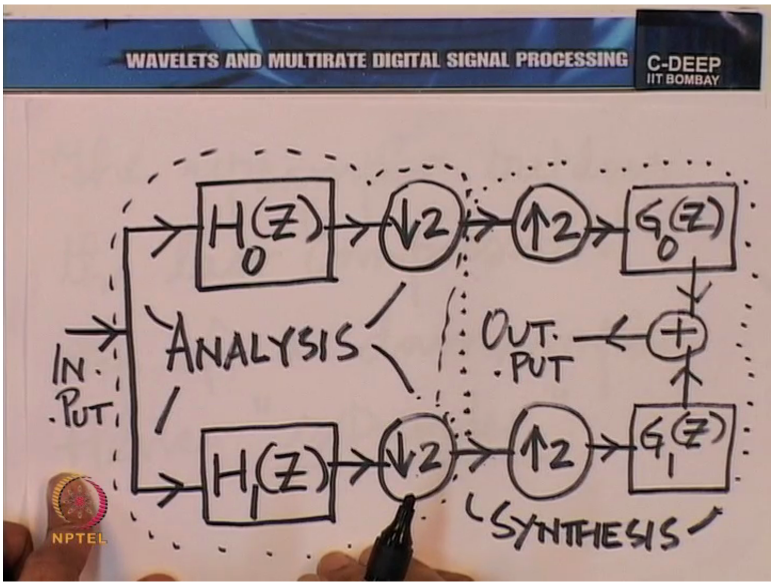

So in a certain sense that downsampling and upsampling operation is taken care of by having different blocking rates at different points in the system. You know, what we are saying in effect is the same system has different effective sampling rates at different points. And to emphasise this point lets go back once again to that 2 channel filter bank that we drew a few minutes before.

You know, if you look back at this 2 channel filter bank here, what we are effectively saying is that there is one sampling rate operating here at the input and the same sampling rate operates here at the output. However although those sampling, the sampling rate here and the sampling rate here which are the same also operate after these filters and before and after these filters, they do not operate here you know.

So at this point, after the downsampler and before the upsampler and after downsampler here and before the upsampler, we have effectively a different sampling rate. In fact the sampling rate here is in principle one half of the sampling rate here and that is what is called the multirate system. Now it is clear why we are talking about the multirate systems, we have 2 different sampling rates in the same system.

No one can generalise this, you can have more than 2 sampling rates in the same system, in fact, you know you might have different kinds of downsampling and upsampling operators. So let us expand our view in that sense now. So in general one could be talking about upsampling and downsampling by a factor of M so to speak. And that is not too difficult to understand, now that we have written what downsampling by 2 means and what upsampling by 2 means.

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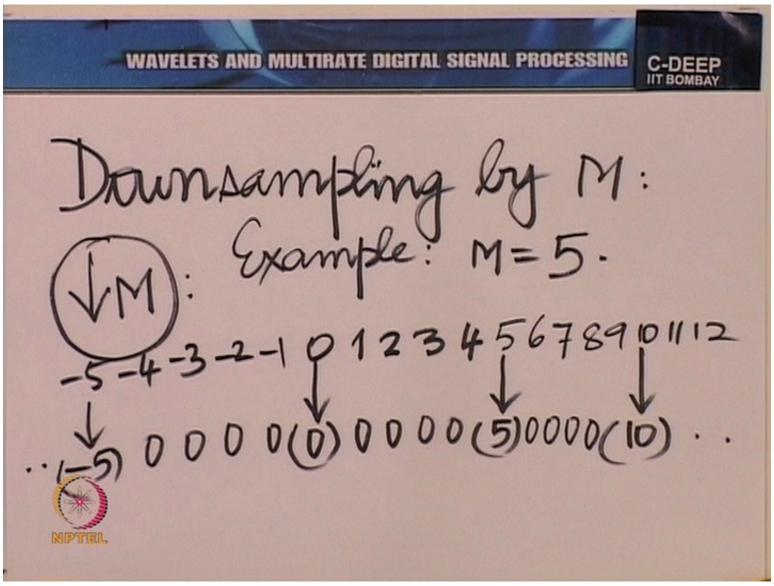

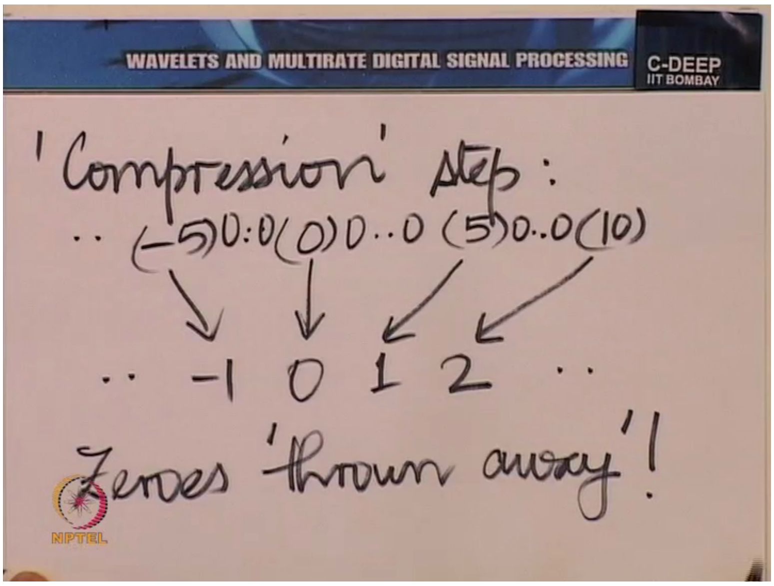

So let us write it down, downsampling by M in general, which of course we denote by down followed by M. So let us again take an example for clarity. Suppose we take M equal to say 5 you know to bring in some variety. So I have these samples here 0, 1, 2, 3, 4. Now what I am writing here please remember are the indices of the sample, not the values of the samples, so 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and so on. And similarly on this side -1, -2, -3, -4, -5 and so on.

What do we do in downsampling by factor of 5, the 1st step is to kill some of the samples, so 0 sample is kept to where it is, the 5 sample is kept to where it is, the 10 sample is kept to where it is and -5 sample is kept to where it is but all the other samples, like 1 sample, 2 sample, 3 sample, 4 sample, the sample number 6, 7, 8, 9 are all reduced to 0. So also -1, -2, -2, -3, -4, all of these are reduced to 0. So this is the clean step so to speak and then the compression step.

The compression step will bring the 0 sample to the position 0 the 5 sample of the position 1, the 10 sample to the position 2, the -5 sample to the position -1 and so on so forth here and remember there are zeros in between here which are simply thrown away, zeros thrown away. So now we understand how to generalise downsampling. In a similar way we can generalise upsampling.

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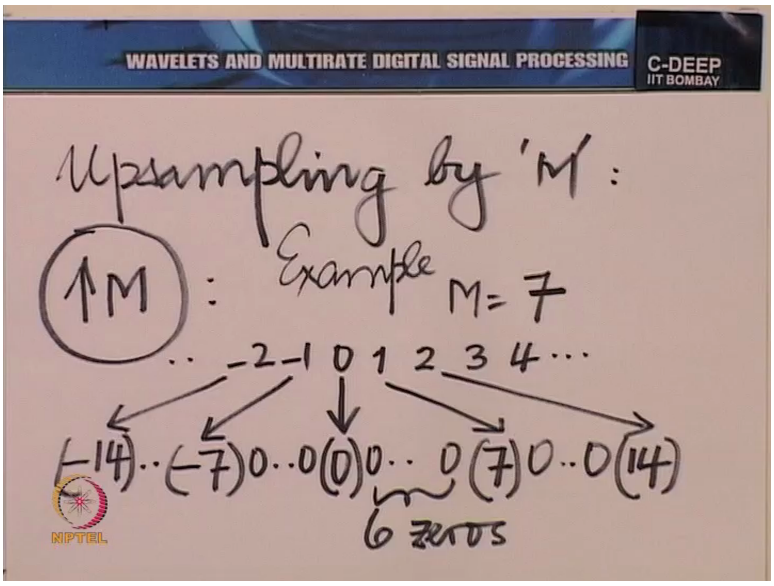

So suppose we have upsampling by M which we denote up arrow followed by M. And once again for variety let us take the example of M equal to 7. So you have the sequence 0 with indices I mean, 0, 1, 2, 3, 4 and so on so forth. And -1, -2, -3 and so on so forth here. When you upsample by 7, the 0 sample goes to where it is to 0 the 1 sample would go to 7, the 2 sample to 14 and so on so forth, the 3 sample to 21, the 4 sample to 28 and the -1 sample would go to -7, -2 to -14 and so on so forth. And of course in between, so I put Brackets around these just for clarity here, in between you would insert zeros.

So how many zeros would insert here, 6 zeros, here too 6 zeros in between, here also 6 zeros and so on. This is upsampling by factor of 7. We understand now how we can generalise both downsampling and upsampling. And now what we need to do is to analyse these operations from a transform perspective. So in the transform domain, which transform, of course we 1st begin with the Z transform because it is easier to understand the effect in the Z domain and then go into the frequency domain by substituting Z equal to e raised to the power J Omega.