Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-3.
Lecture-9.2.
Fourier transform of Scaling Function.

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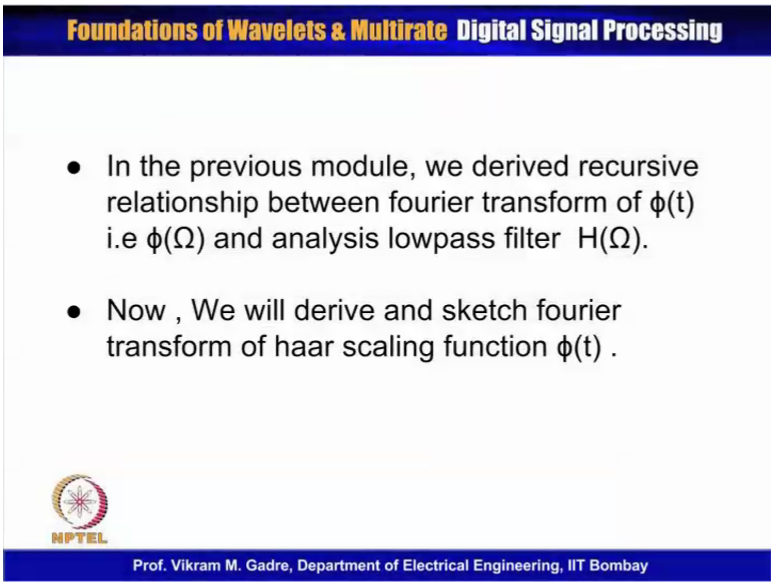

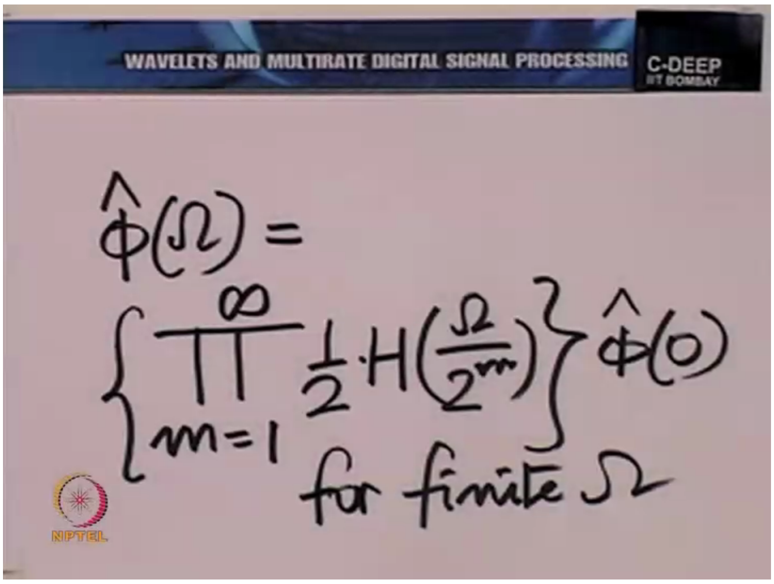

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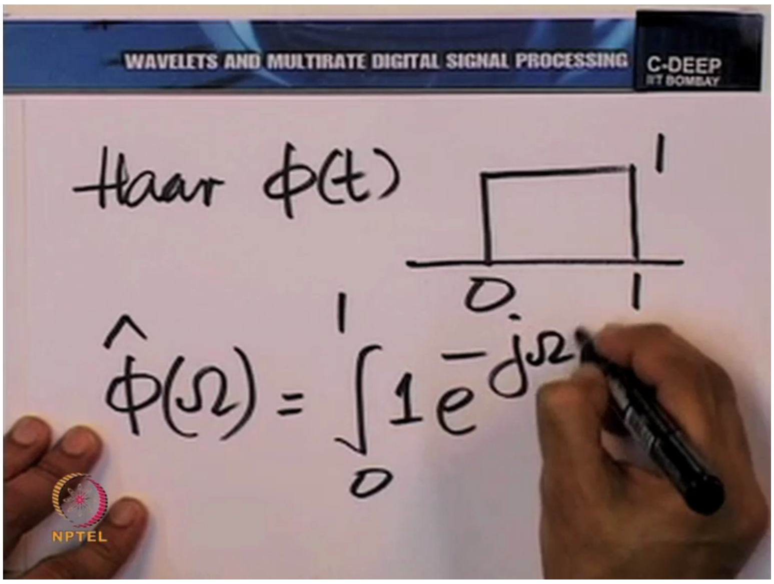

You see if you look at the Fourier Transform of the Haar scaling function for example. Let us look at it let us look at the Fourier transform of phi T in the Haar context. Accentuate is this and the Fourier transform is easy to calculate.

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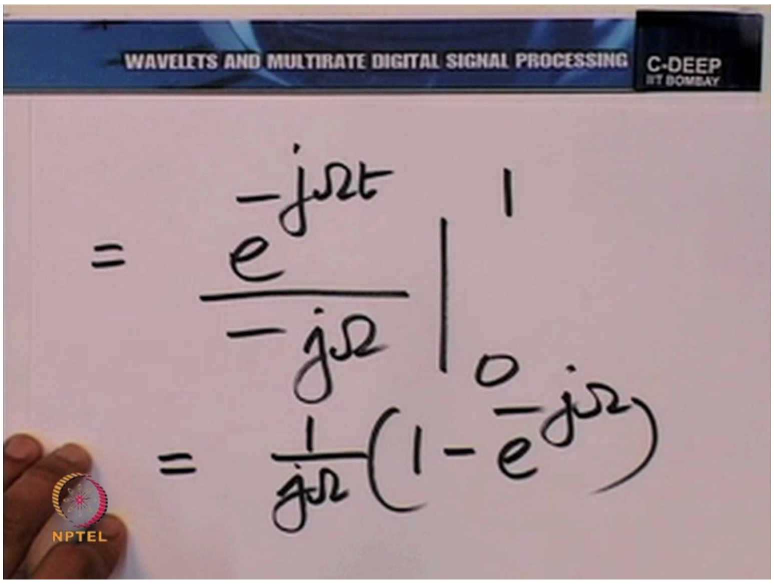

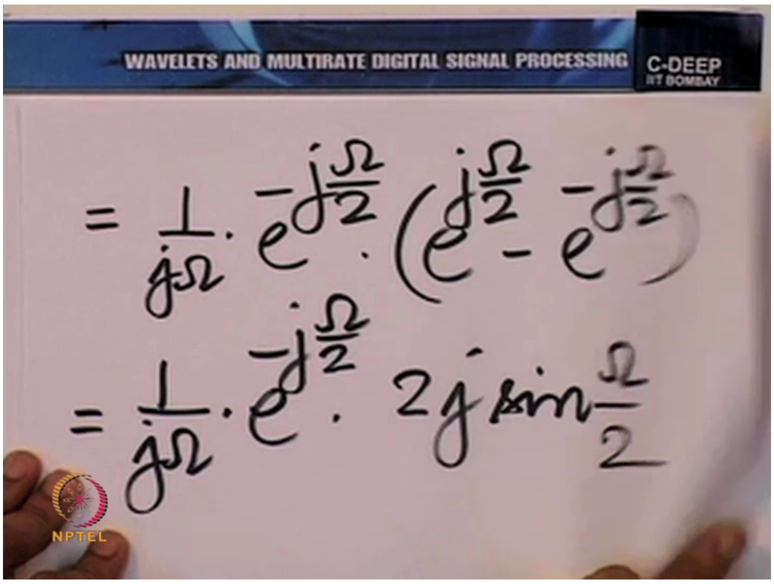

Now we can simplify this using the standard trick of taking out e raised to power – j omega by 2 term and then we noted that we have a sign hidden there.

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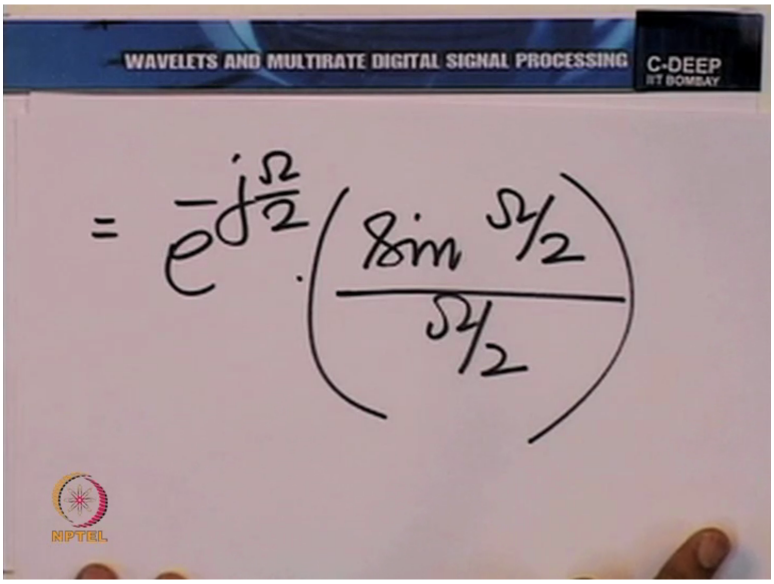

And doing away with the Js we have. Now as you can see, this Fourier transform goes towards 0 as capital omega goes towards infinity. So the Fourier transform vanishes as capital of omega goes towards infinity. And in fact we can even sketch the magnitude to get a feel of this.

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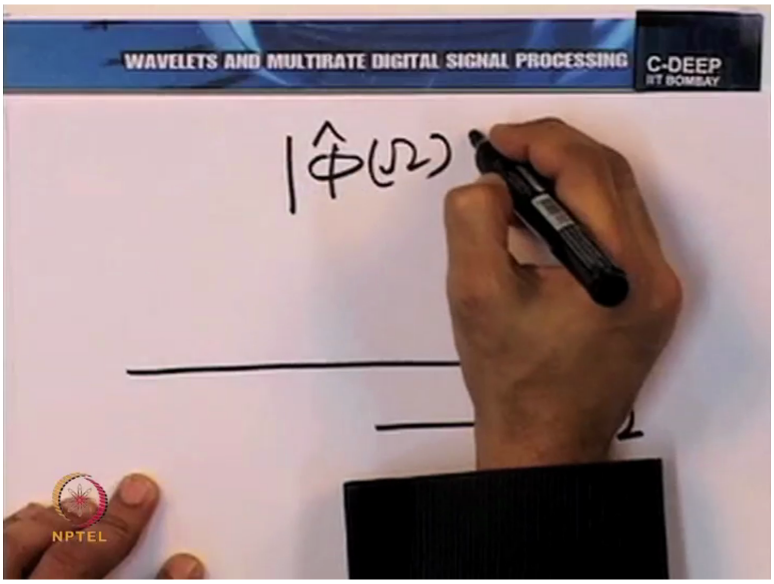

The magnitude of phi cap omega looks like this.

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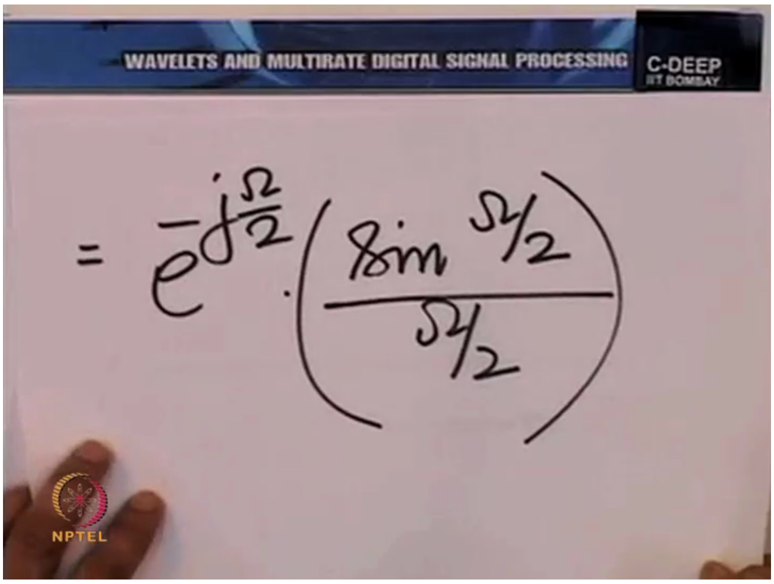

You see, at omega equal to 0, you will notice that one can use Lobita's rule and show that this is equal to 1 in magnitude.

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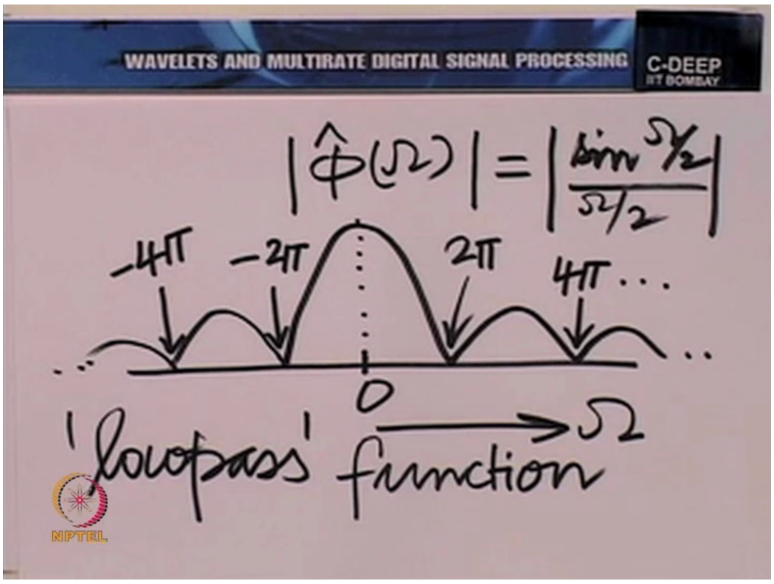

So it has a pattern like this. Of course we know where this comes. This will come at omega by 2 equal to pi or at to 2 pi. This would come at 4 pi and so on. This is how the magnitude looks. Now you know, this very clearly shows that psi has a concentration are around 0 frequency. So this is interesting. We begin from the low pass filter, we construct a dilation equation, a recursive dilation equation starting with a low pass filter and we get a low pass function. Phi T is essentially a low pass function. A low pass function means it is predominant in the frequency domain around capital omega equal to 0.

Low pass function of course is an informal term. You may always argue that after all it does have bands at higher frequencies too. But the point is, its prominent bands are around 0 and the farther you go away from 0, the more the spectral magnitude drops off. In that sense, it is low pass. In fact just as we try to build the idea of ideality in a filter bank, you also would like to build the idea of ideality in this phi T. The ideal phi T is actually the ideal low pass function. So low pass function which is like a brick wall.

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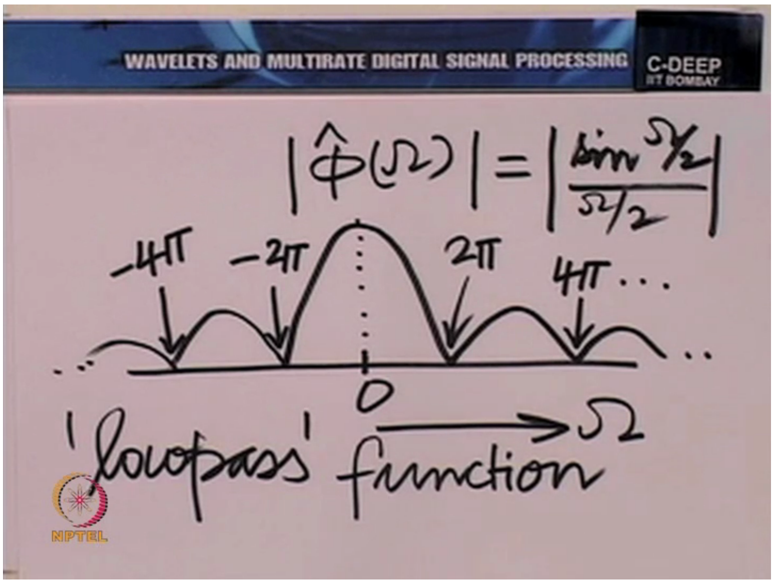

So in some sense, phi T in the spectral domain or phi cap omega for that matter is moving towards the ideal. You know this is where we are moving and where we are is somewhere here, very far away from it of course. Not surprising, after all that is what we saw in the Haar multiresolution analysis, ideal here, actual there. Now once again, there is the same conflict that drives the engineer, the scientist, or the mathematician. We know that we want to go towards the brick wall ideal.

But we also know the brick wall ideal is unattainable for various reasons. The reasons are similar to what I talked about last time for the unattainablity of the idealism in a filter bank. So we will not need to repeat them once again here. However, what we will now do is to see what is the relationship whether ideal or practical, what is the relationship between phi cap omega and the discrete time Fourier transform of the filter bank low pass filter in terms of construction?

So in other words, we have written down a dilation equation in the frequency domain but we need to translate that dilation equation into a constructive step. How do we construct phi T given the low pass filter impulse response? In a way, if we do that, we have answered the question, how is the design of a filter bank related to the design of a multiresolution analysis? So let us do that. Towards our objective, let me put before you once again that infinite product here.

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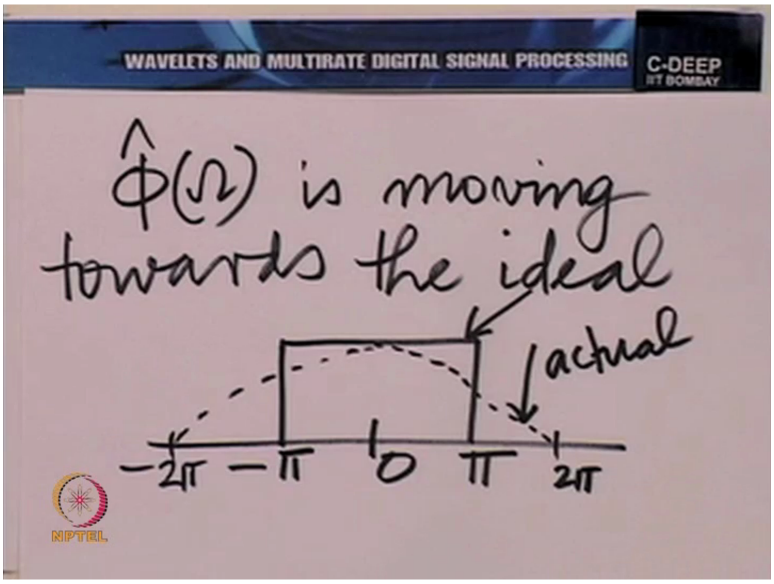

So you know, now you also understand why phi cap zero should not be 0. Phi cap zero must be nonzero. In fact, phi cap zero is very often the maximum value of the magnitude of phi cap omega because of the low pass character. So we have seen this lowpass character in the Haar context. Now, we shall assume it to be true of most multiresolution analysis and proceed from there. So this is just a constant, you know a nonzero constant to be wary but what we need to identify is this.