Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Demonstration: DWT of images.

Hi guys, I am Hrishitosh Bisht. In the past few lectures, we have studied about filter banks. In this video, we will learn about one such application of filter banks in the area of image processing. In particular, we will be performing discrete wavelet transform on images. Before even the proceed further, it is important to note down one subtle point. The filter banks that we have studied so far are one-dimensional in nature and the object that we want analyse that is images are of two-dimensional nature. So we would need to develop some technique so that we can apply our one-dimensional filter banks to these two-dimensional images.

For this purpose, what we will do is, we can apply its one-dimensional filter banks first along the rows of the images and then along the columns of the images or vice versa as well. For the sake of convenience, let us start with an image at a scale say J+1.

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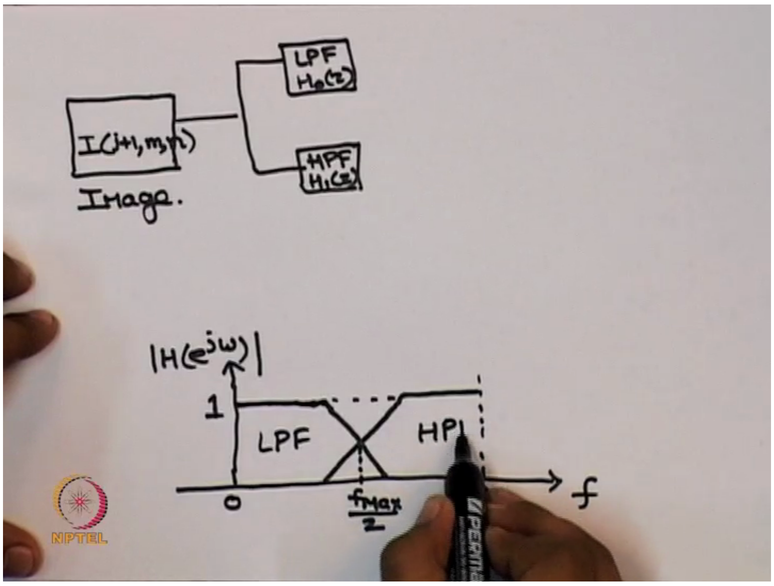

So let this be a much and let us the present this image by I with scale J+1 M and N, where M denotes the rows of the image and N denotes the column of the image. To start with, we can say that this image I is an original image itself and we will later see that this input I can be any signal and at any scale, J. So suppose, I J+1 MN is the starting signal and it is passed through two filters, one a low pass filter and a high pass filter with transfer functions of H0Z and H1Z respectively.

Let us also assume without the loss of generality that they are applied along the columns of the image first. The frequency response of the filters can be shown as suppose that this is our frequency axis F and this is the magnitude. If this is the frequency response of a low pass filter, then the frequency response of the high pass filter should be like this where this amplitude is 1. That is what I mean to say is that this overall filters or this overall structure should be like that it should be able to pass all the frequencies and if this is my maximum frequency F max then this point is my frequency F max upon 2.

Therefore we can say that the bandwidth of the signal is half. Since we are splitting the signal, this is a low pass version and this is my high pass filter version and since I'm splitting the signal into these 2 bands and the bandwidth of the signal is half, in each of these sub bands, this low pass band and this high pass band we will get redundant samples. And according to the Nyquist criteria, we will not lose any information even if we remove half of the samples from our signal.

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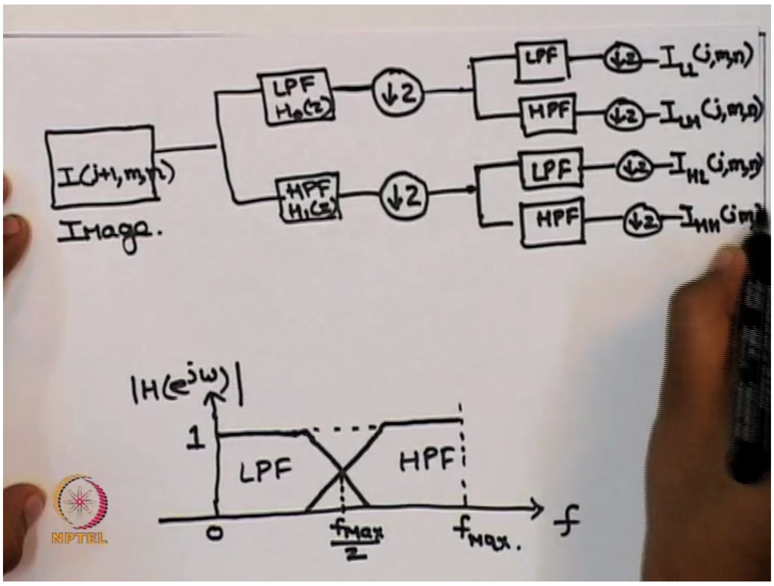

So to achieve this, we apply a down sample by 2 operation at both of these outputs of the low pass filter and the high pass filter. Once we have done this, we can then apply these one-dimensional filters and these down sampler operations now along the rows. And after this output and after this output that we have obtained, we again apply low pass filtering operation and a high pass filtering operation followed by subsampling by 2 and again followed by subsampling by 2 and at the lower output of the high pass version we again pass it through a low pass filter followed by downsampling by 2, a high pass filter followed by downsampling by 2.

But the difference now lies is that, these filters and these downsampling operations are now acting on the rows of the image I J+1. At the output of these filters, we obtain sub bands as ILL, JMN, I LH, JMN, I HL, JMN and IHH, JMN. We known that high pass filter extracts the edges and low pass filter does the approximation.

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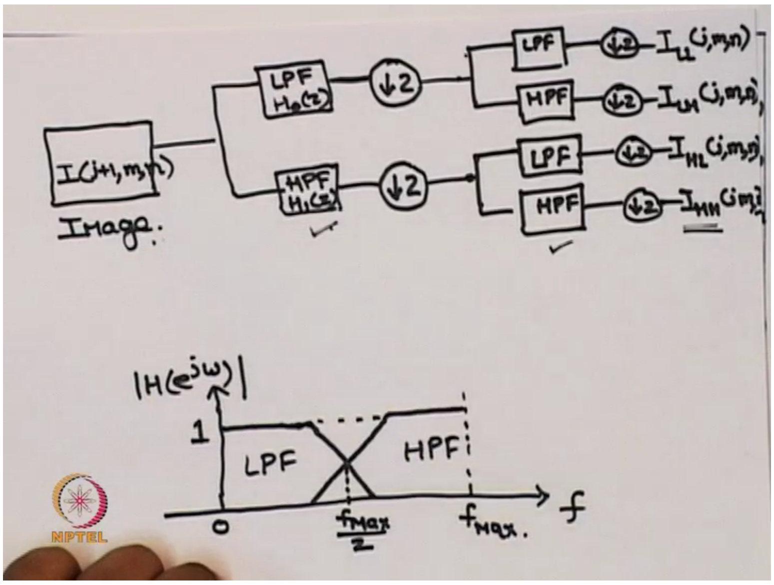

So if we look at the output ILL JMN, it has passed through 2 low pass filters. So this output ILL will give an approximation to the input image that we have taken over here. If we consider this output, ILH we will see that it has passed through a high pass filter which acts on the rows of the input image. So what this ILH output will do is that it will extract the horizontal features of this input image. Similarly, if we will look at the output I HL, the input image has passed through the high pass filter which acts along the columns of this image.

So, I HL will extract the vertical features of the image. If we will see the output IHH, we will observe that it has passed through a high pass filtering operation along the columns of the image and a high pass filtering operation along the rows of the image. So the output IHH will emphasise the edges along the diagonals of the image. That we have taken over here as an input.

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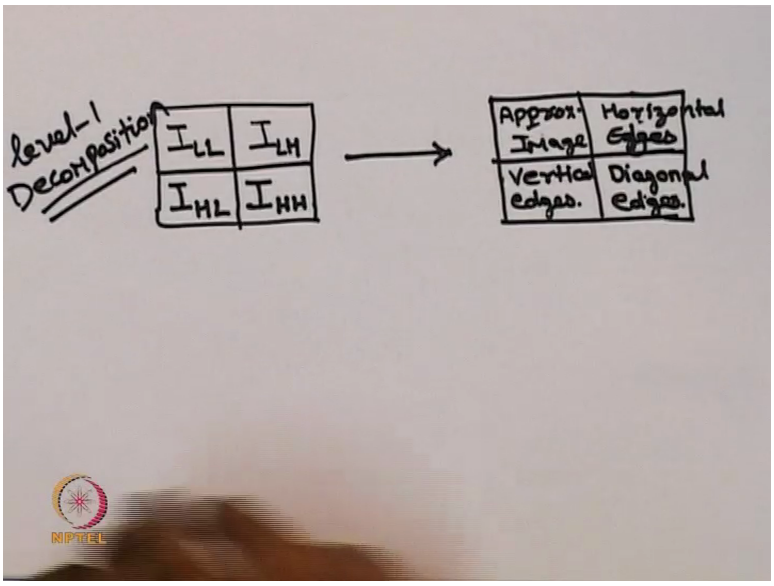

Pictorially, this can be represented as, this is our output that we have got as ILL. This is the subband where our output ILH will occur is our subband where the output IHL will occur, this is the subband that the output IHH will occur. This represents that in this part of the image or in this subband of the image we get an approximation version of the image.

Whereas this part of the image where we have output as I LH we get or we extract horizontal edges. At this subband where we have an output IHL we extract vertical edges and in this quadrant where we have an output IHH, we extracts diagonal edges. Such a type of representation is referred to as level I decomposition.

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If we will do simulation of such a thing using any software like Matlab, what we will obtain is a figure like this. So this was our original image which is Lena and its size is 512 x 512 and after performing the same operation as explained before, we have decomposed this original image into 4 subbands.

This is my 1st subband which we have called as LL subband. This is my other subband called LH subband. This subband is HL subband. This subband is HH subband. So as it can be seen clear from this output that maximum energy that we have captured or the maximum information that we have captured after 1st level of decomposition is contained in the low low subband.

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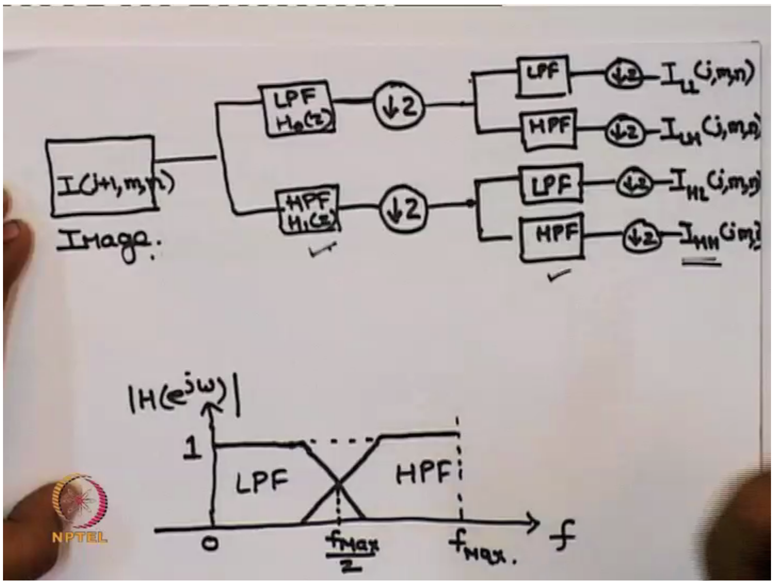

Now, it is important to observe here that the outputs subimages are at one resolution less than the image that we have started with. That is, we have in effect lost the resolution by a factor of 2 along the rows of the image and also along the columns of the image.

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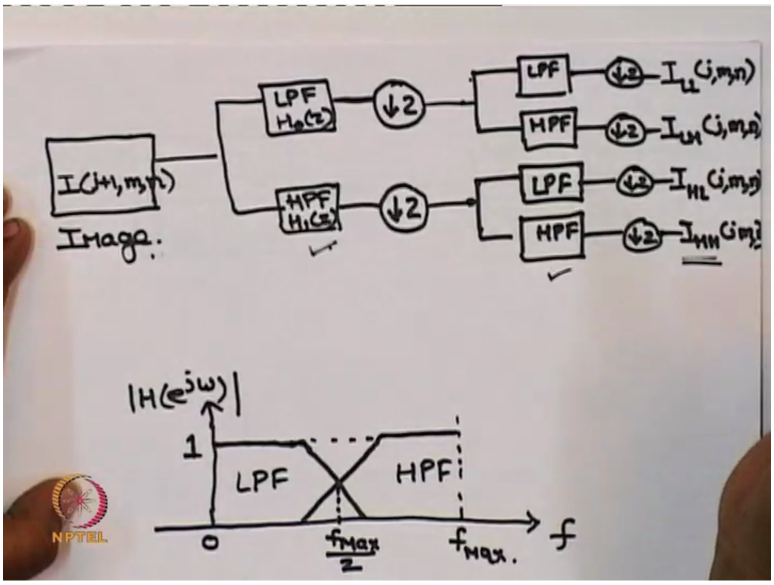

Since we have done subsampling by a factor of 2 both along the rows and along the columns, it means that in effect, we have done subsampling by a factor of 4.

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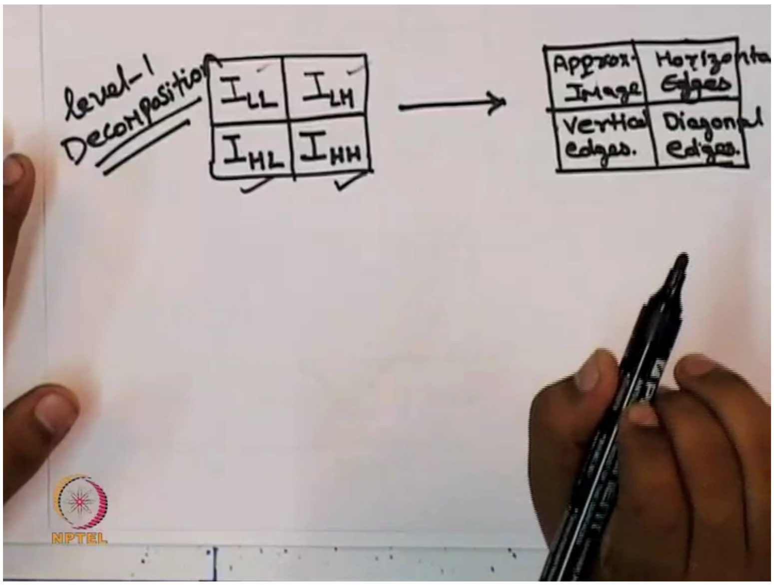

Therefore each of these subbands that is LL subband, LH subband, HL subband and HH subband contains only 1/4th of the total number of samples in our image originally. Now, in principle any one of these subbands or all of these subbands can be analysed or partitioned further.

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But it can be seen from the simulation here that as maximum energy or maximum information content is there in the LL subband so we would like to decompose this band further.

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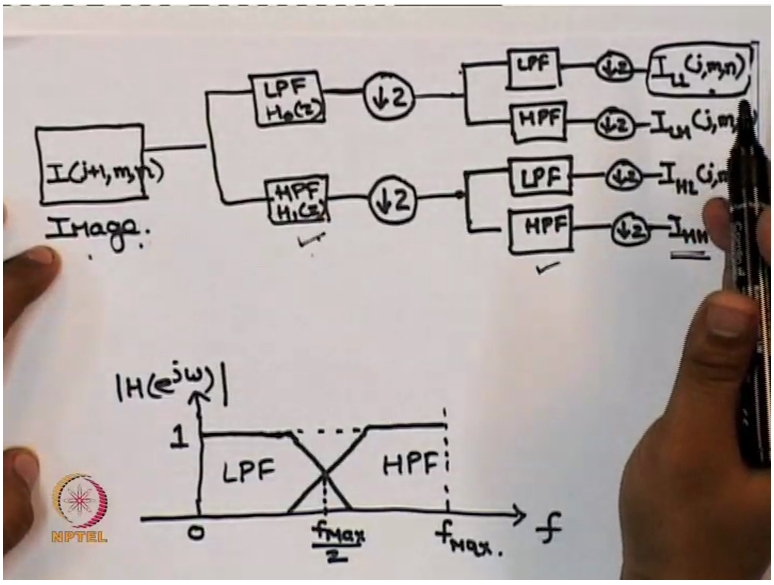

So if we perform the same operation as we have done for level 1 decomposition over here and take the output that we have obtained as ILL JMN and now consider this output as an input over here and perform the same operation that we have carried before we will get level 2 decomposition.

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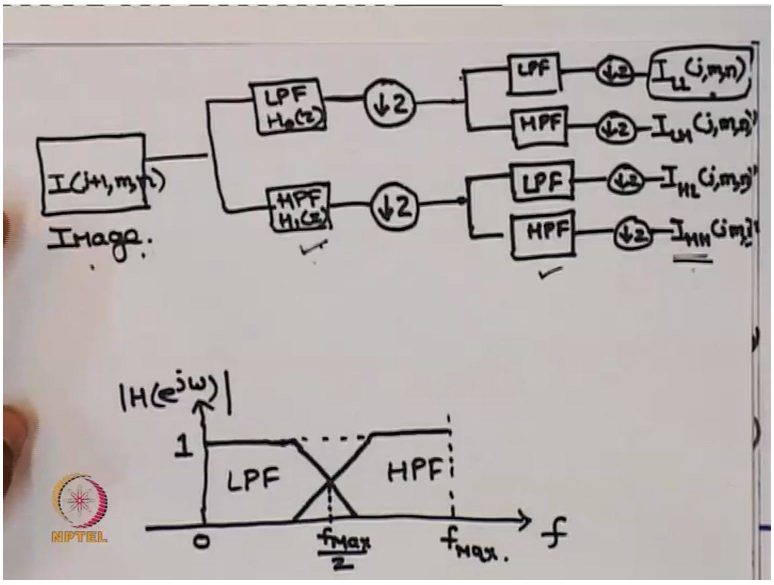

Therefore we repeat the whole process again with LL image as the input and this image now we have to be cautious that now this image is at scale J. Now when this will be at the input, so after doing all these operations again, at the output we will obtain same I LL but at scale J - 1. Here we will get output ILH but now at a scale J - 1 and over here we will get output IHL but scale J - 1.

And over here this will also get output IHH but at scale J - 1. So we have lost the resolution by a factor of 2 along both the rows and columns of the image that that the output ILL is displaying. So to see this further, since this was our level I decomposition and we are performing the same filtering and downsampling operations on this LL subband image, then at level II we will obtain picture something like this.

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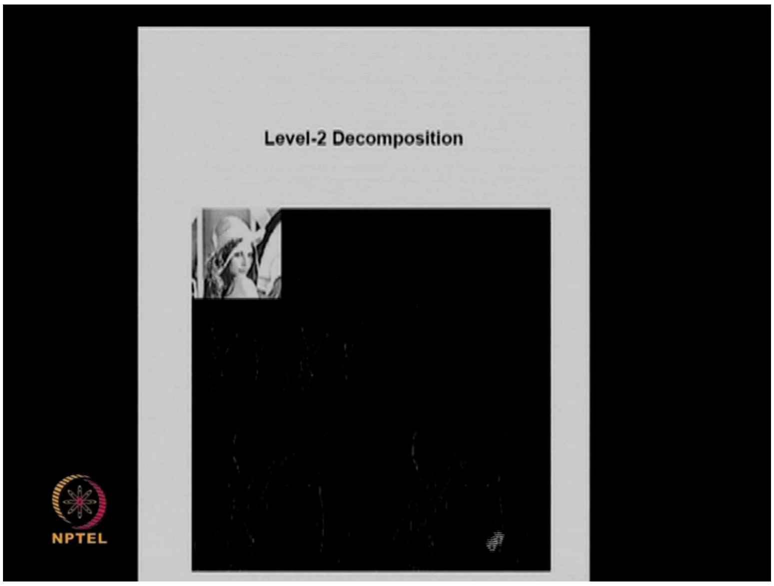

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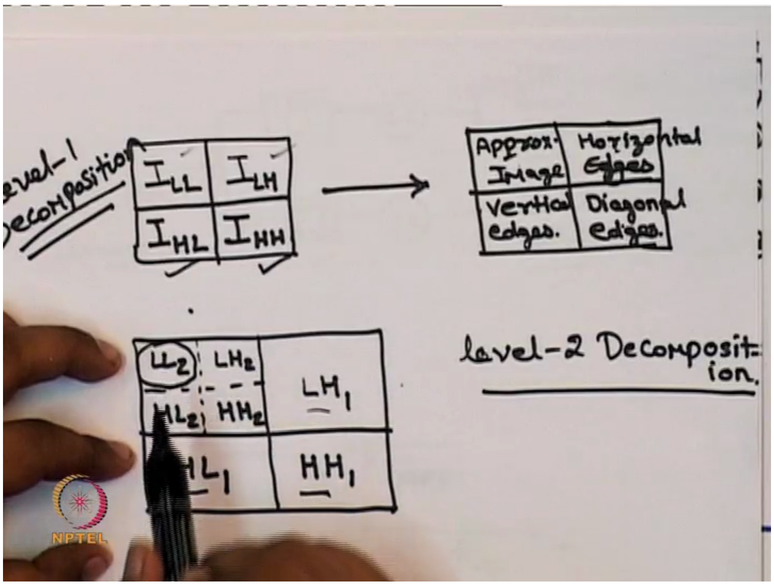

Pictorially, this can be depicted as this. So this was our LH subband. Let us say, this is LH 1. This is our HL subband. Again, let us say this is 1 and this was our HH subband with subscript 1. Now we have taken this LL subband and decomposed it further into 4 parts. So this will be our LL subband but we show this as LL 2.

Over here, we will have LH 2. Here we have HL 2 and this is our HH 2. This kind of decomposition is called level 2 decomposition . It has 7 subbands out of which 4 subbands corresponds level 2 and those 4 subbands are LL2, LH2, HL2 and HH2 and 3 subbands correspond to level I which are LH1, HL1 and HH1.

If we want, we can repeat this process again on this LL2 part. As we have discussed before the maximum information after 2nd level of decomposition would be contained in this LL2 part. If we perform another decomposition on this LL2 part then we will say that it is a level 3 decomposition. So to summarise what we have done so far is, we have taken our original image which we have considered to be at the largest resolution and therefore is also considered to be at the highest scale and decomposed it into 4 parts.

Into LL band, HL band, LH band and HH band. Then again we repeat the process to the LL subband content and decompose it into further 4 parts and we can proceed like this further. Now, after decomposing each image, at each level of decomposition, we are only keeping one fourth number of samples of the total image. So in a sense what we're doing is we are going from a finer resolution to a coarser resolution.

So what we have achieved from this? The basic objective of wavelet analysis is to extract the localisation. Here, by localisation we mean spatial and frequency localisation. So by doing such a decomposition, either we do a level 1 decomposition or a level 2 decomposition or any other level of decomposition. What we have achieved is we have found out at what spatial locations we have high frequencies and that what spatial locations, we have low frequencies. Thank you.