Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-3.
Lecture-8.3.
Realizable Two-band Filter Bank.

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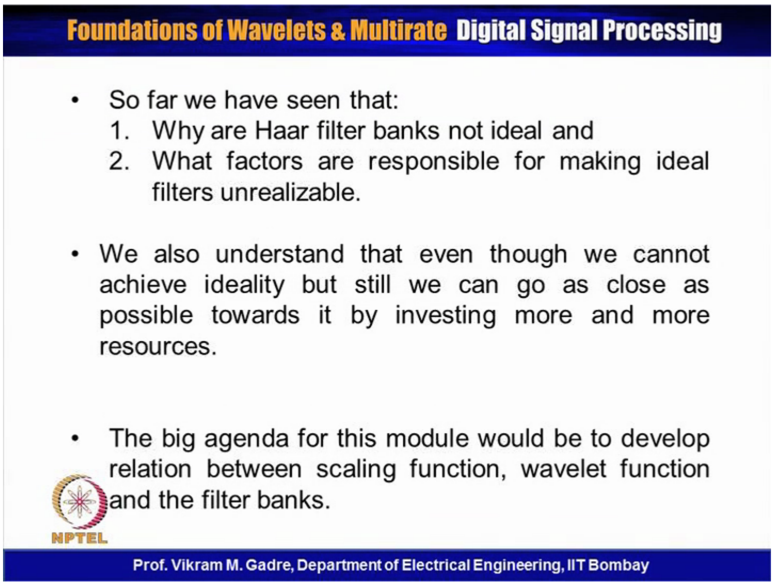

Suppose I do happen to design different two band filter banks. So 1st what would a realisable 2 band look like, we must first put that, we must write that down in terms of a drawing first.

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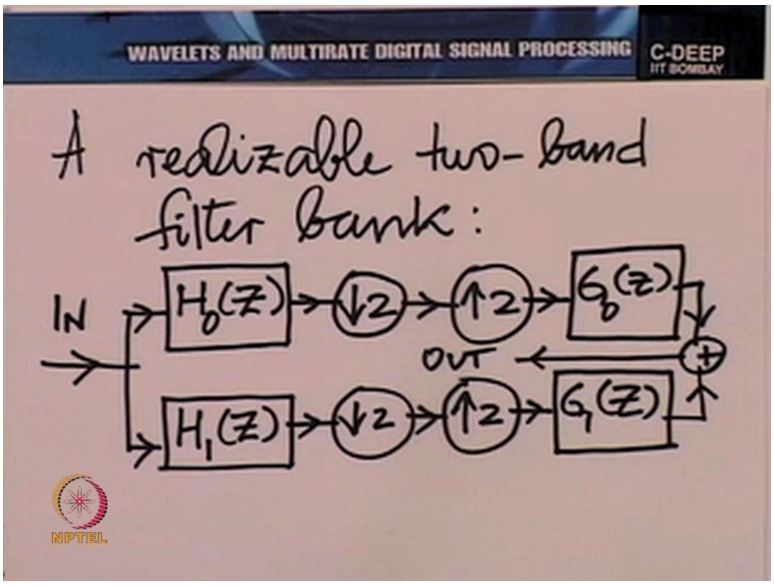

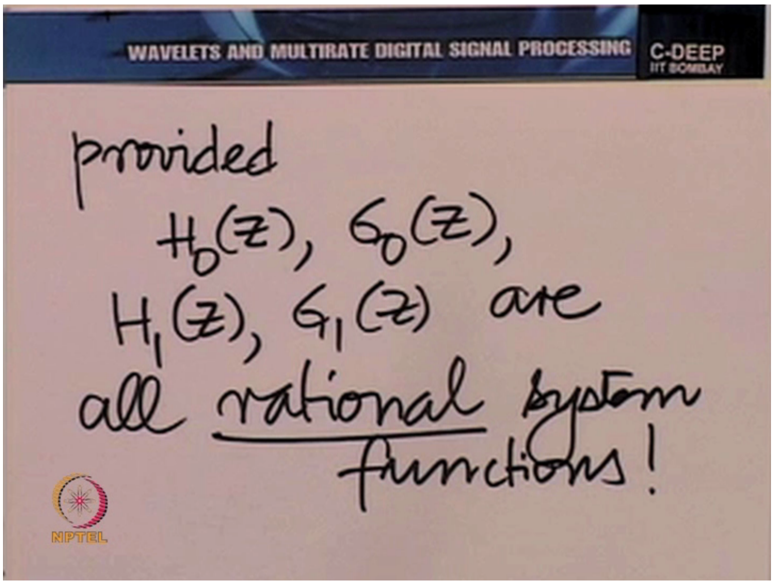

So a realisable 2 band filter bank is like this. So I straightaway write it in terms of system functions here. this is a realisable two band filter bank provided H0Z, G0Z, H1Z, G1Z are all rational functions, rational system functions.

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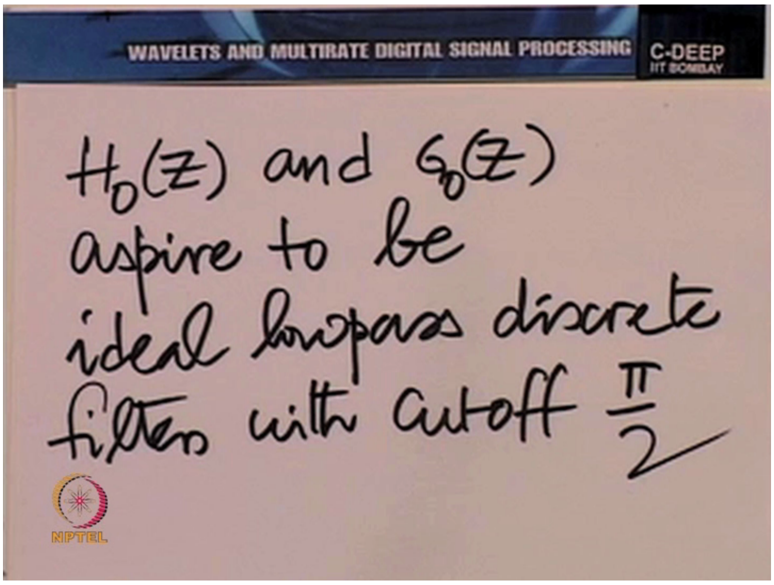

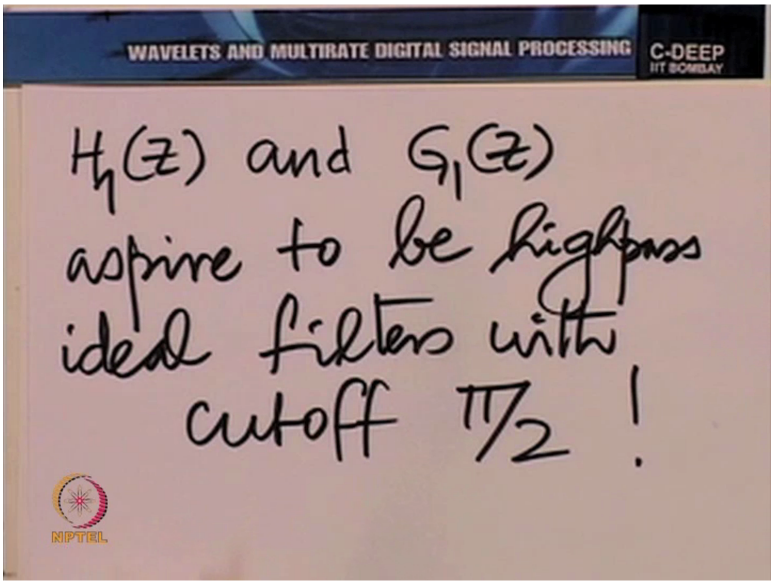

And of course, H0Z and G0Z aspire to be ideal low pass filters with cut-off pi by 2. H1Z and G1Z aspire to be high pass ideal filters with cut-off pi by 2. now, having put down the structure more generally we must now ask what is the connection between the multiway solution analysis and this filter bank that we're trying to design? So to answer that, let us look at the Haar once again. In fact, let us begin with the phi t in the Haar.

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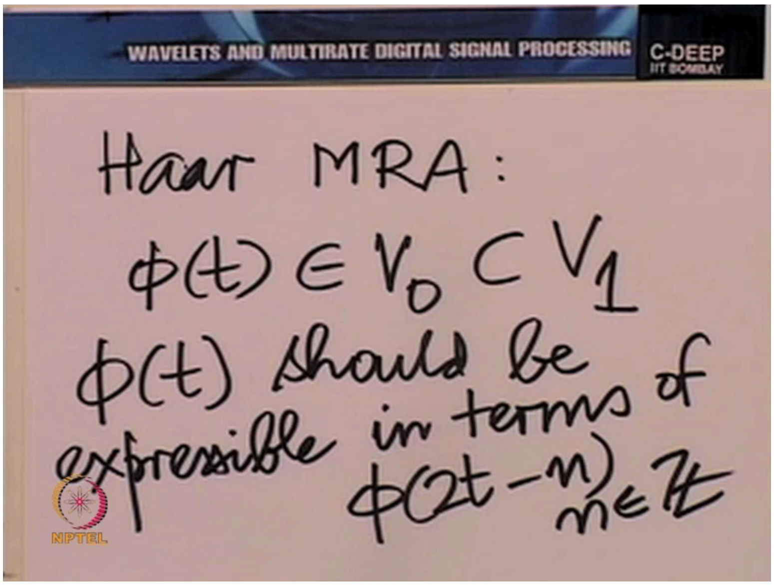

So, for the Haar MRA, we notice something very interesting. Phi t belongs to V0 which is a subspace of V1. So phi t should be expressible in terms of the basis of V1. And what is that basis of V1? It is phi 2t - n for integer n. It should be expressible in these terms. It is very interesting. You know, phi t is an element of V0. V0 is a subspace of V1 and the basis of V1 is again, the dilates of phi t by a factor of 2 and their integer translates. So phi t can be expressed in terms of its own dilates and translates. this leads to what is called as recursive dilation equation on Phi t. And lo and behold, what is that dilation equation? that is also not difficult to determine.

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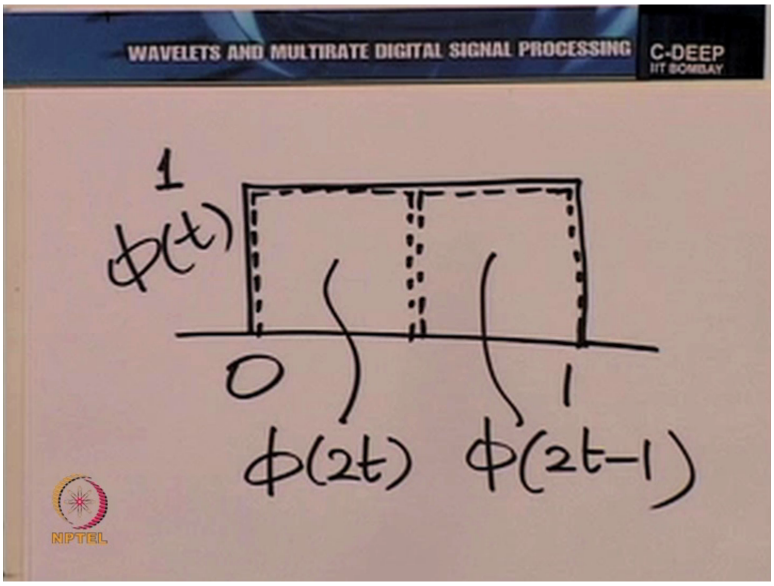

In fact, we can even say graphically. Indeed, you know if you recall, phi t looks like this. I've kind of scaled up the drawing. And all that you need to do to get this recursive equation is to notice that this can be redrawn like this. So, it has 2 components in it. And the 1st component is phi 2t and 2nd, phi 2t -1 and the whole thing is phi t. So we have a beautiful dilation equation which governs phi t.

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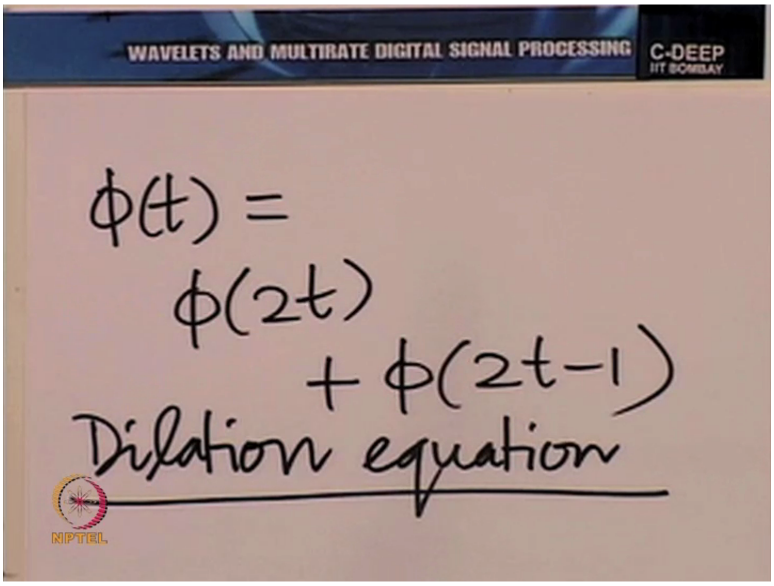

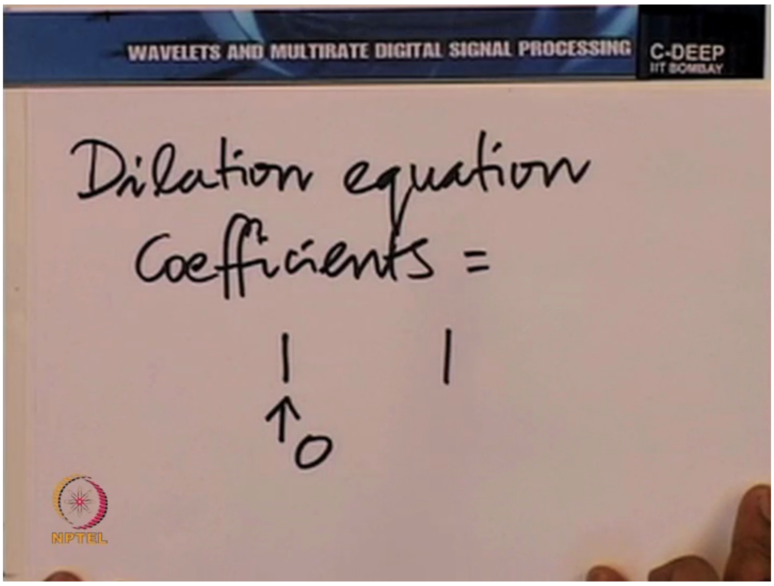

Phi of t is phi of 2t + phi of 2t -1, a beautiful dilation equation which governs phi t. now let us look at the coefficients in that dilation equation. So if we put the dilation equation before you once again the coefficients are 1 and 1. Let me try and call this a sequence. You know so if you agree, I will talk about the sequence which is 1 at n equal to 0 and then one subsequently as shown at n equal to 1, this is a way of denoting a finite link sequence.

the number below the arrow tells us the value at the point of the arrow. And all the other numbers tell the value of the sequence at adjacent points. So this for example means at n equal to 0, the sequence takes the value 1 and at the next point which is of course n equal to 1, the sequence will take the value 1, 2.

So this is the sequence corresponding with the dilation equation coefficients. Let us carry out a similar exercise for the wavelet now. So let us take the Haar wavelet and let us express the Haar wavelet also in terms of the basis of V1. And what is our ground for doing so? Let us put it down clearly.

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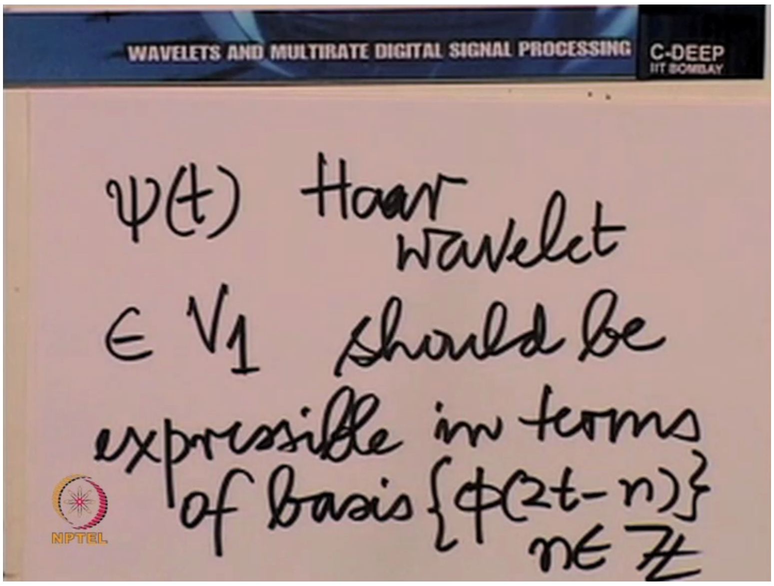

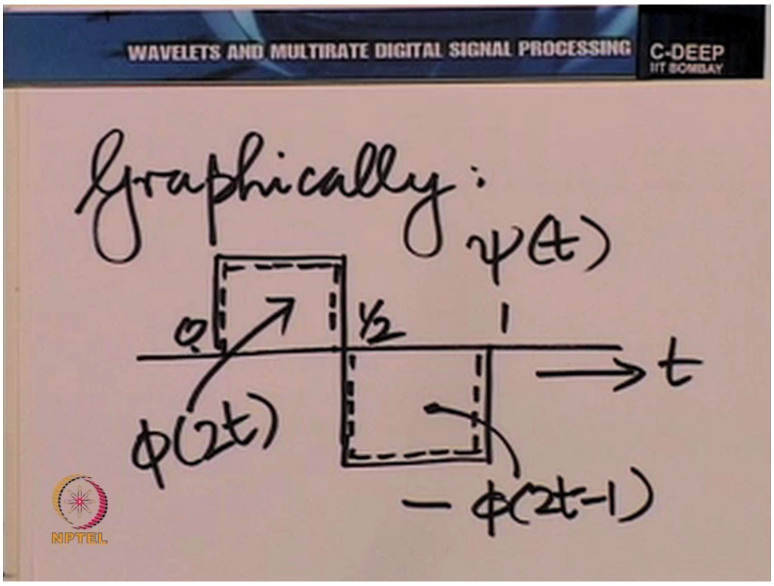

You see, recall that psi t or the Haar wavelet for example also belongs to V1. So it should be expressible in terms of its basis. What is that basis? Phi 2t - n for integer n. And again, if we look at it graphically, it is not difficult to do. Graphically, one can sketch phi t actually and psi t too. So this is psi t. And you can see the phi ts embedded in it. So you have one here and you have one there. This is easily seen to be phi of 2t and this is easily seen to be -phi 2t-1.

And therefore we have a very simple dilation equation for psi t. now here, it is not recursive but it is a dilation equation all the same. So dilation equation for psi t.

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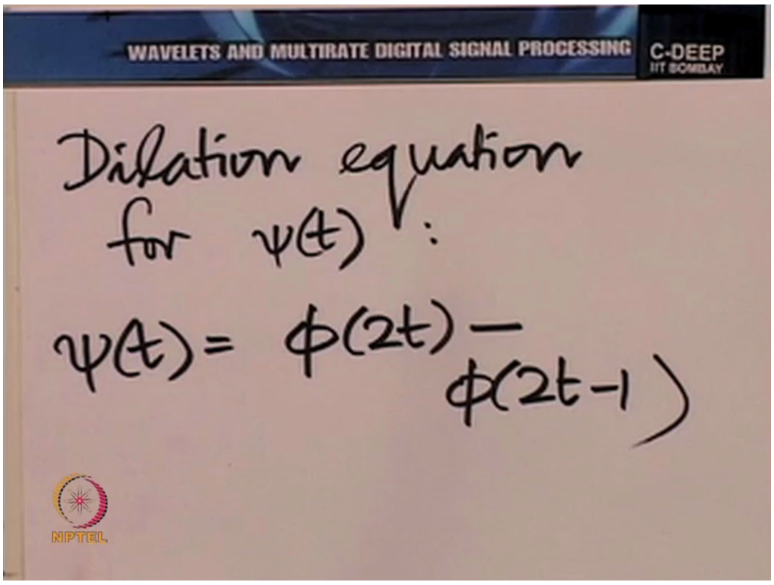

Psi of t is phi 2t - phi 2t - 1. And once again, let us put down the coefficients of this dilation equation as we did previously. So you know, the coefficients again would be again the coefficients involved in expanding in terms of phi 2t - n.

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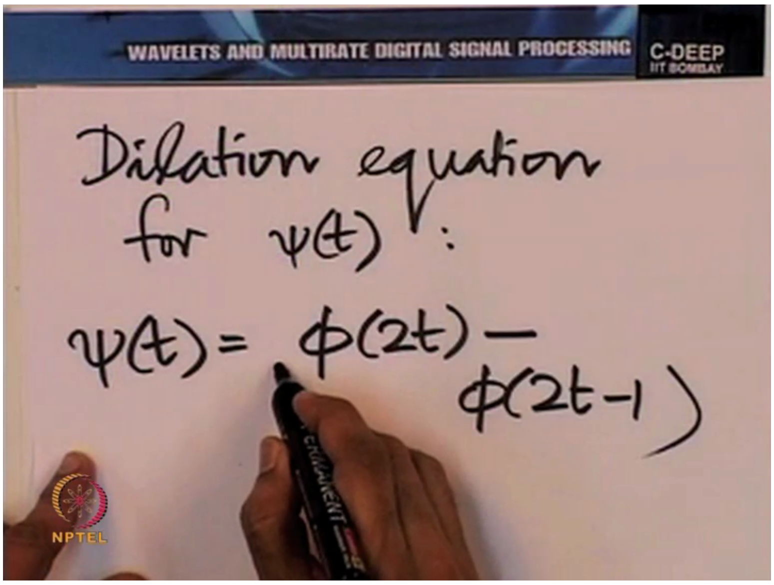

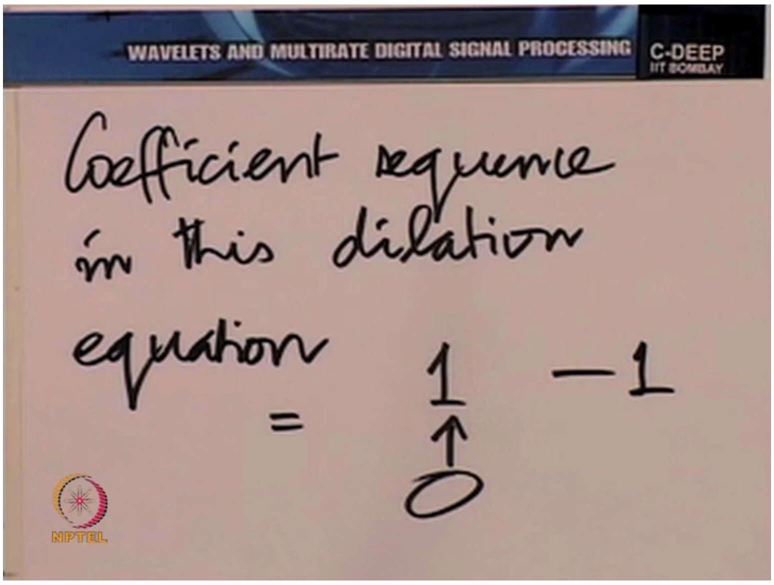

As I as I can see from this dilation equation, the coefficients are 1 and - 1 respectively at 0 and 1. So I put that down. Now things are beginning to make sense and perhaps even ring a bell. Let me put both these coefficient sequences before you once again and then I'm sure it will ring a bell.

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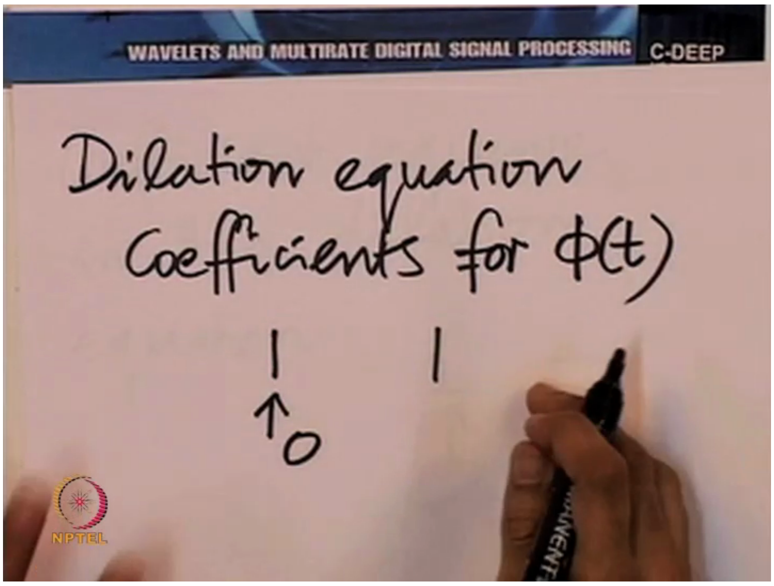

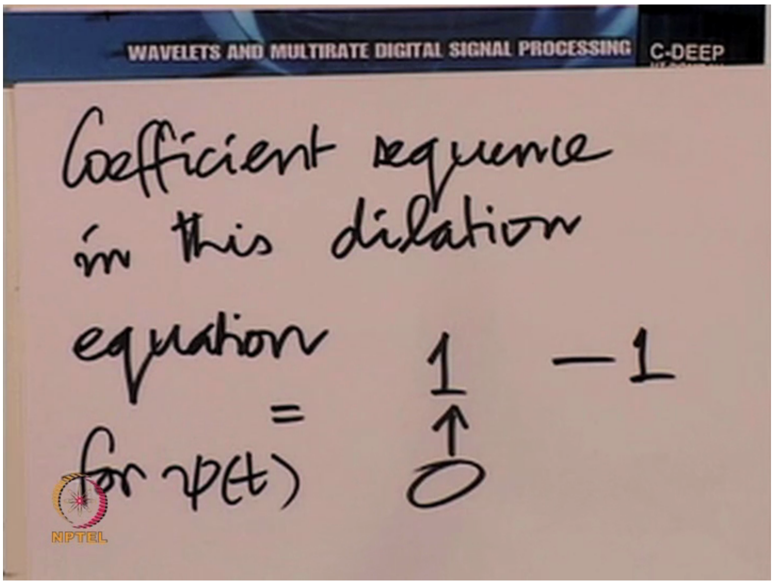

Look at the dilation equation coefficients for the phi t itself. In fact, let me write it down. the dilation equation coefficient for phi t. And let me then put before you the dilation equation coefficients for psi t. So let us write it down in this, for psi t. Does this ring a bell? Yes indeed. If you look at the impulse responses, either on the analysis side or the synthesis side of the low pass filter and the high pass filter these coefficients are essentially those impulse responses. A minor difference, you see on the analysis side, you have a factor of half. For the moment, keep aside that factor of half. Otherwise, these dilation equation coefficients are just those very impulse responses.

Synthesis side, similarly. Perhaps with a + - ambiguity but otherwise the same. So we have a very intimate relation which we have seen here. The coefficient sequences in these dilation equations that no one phi t and psi t are actually the impulse responses of the filters. Now in fact I go one step further. We shall progress to show that if I know these impulse responses, I can go the other way too. So here I have by serendipity so to speak by surprise or chance discovery, with this relationship. Now we will take that serendipity, that discovery further.

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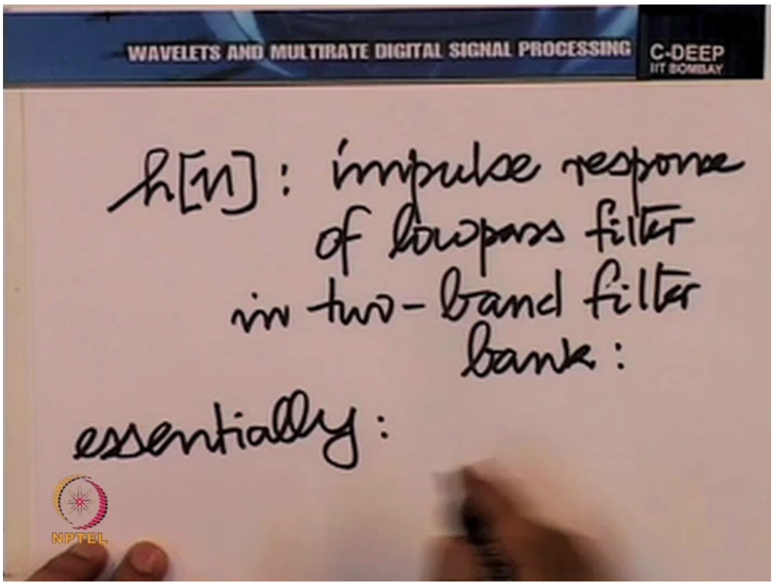

So indeed, let us know at the moment that within a scaling factor, if H of n so to speak is the impulse response of the low pass filter in question in the two band filter bank, then essentially what we have is the following dilation equation. So we will continue the dilation equation.

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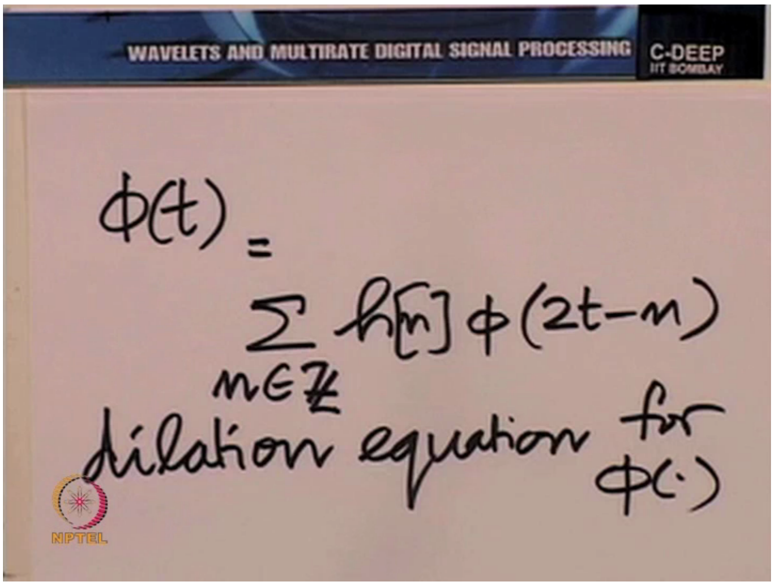

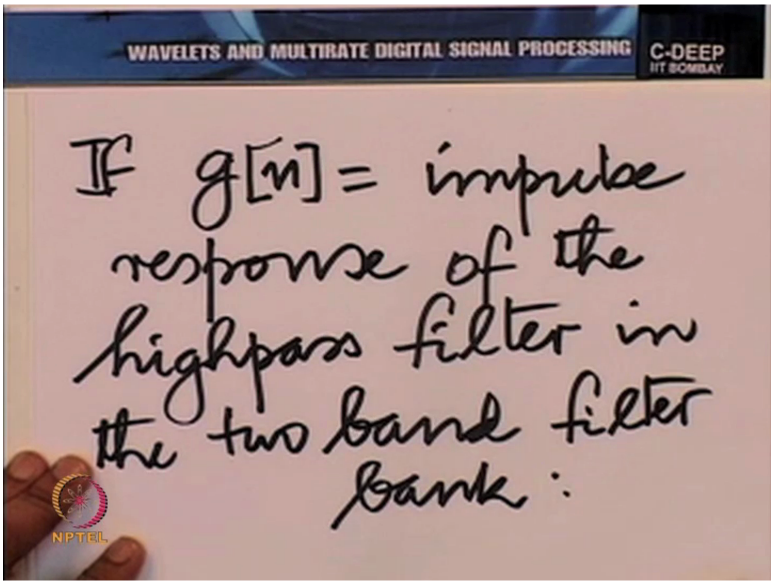

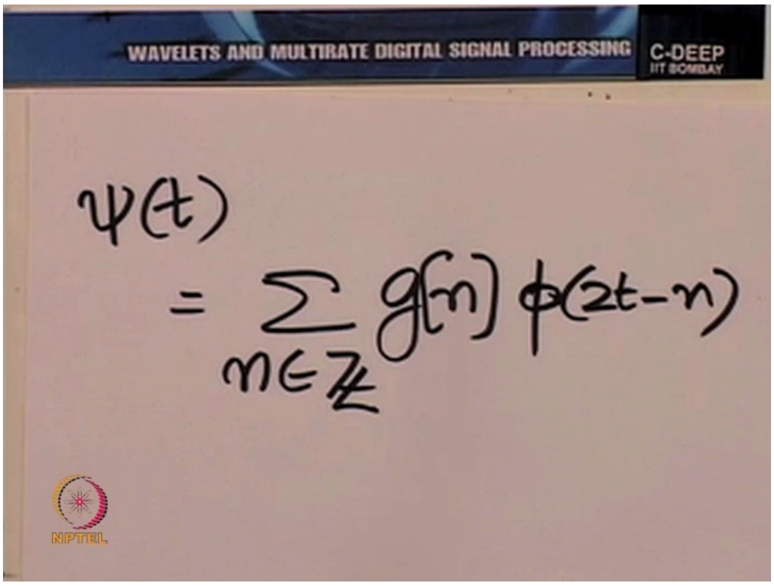

Phi of t is a summation on n. n over the set of integers, Hn phi 2t – n. this is the essential dilation equation for phi. And conversely, if Gn is the impulse response of the high pass filter in the two band filter bank then we have psi t the summation n over the set of integers Gn phi 2t – n. So in the time domain, we have made an intimate relationship. The low pass filter impulse response allows us to expand the essentially phi t in terms of its own dilates and translates. The scaling function in terms of its own dilates and translates.

The high pass filter helps us expand the wavelet in terms of the dilates and translates of the scaling function. Once again, low pass filter impulse response, a recursive expansion of the scaling function in terms of its own dilates and translates, the high pass filter response and expansion of the wavelets in terms of the dilation and translates of the scaling function.

Now, we want to go a step further. We want to show that once you have this dilation equation, we can actually completely characterise phi t and psi t knowing the two band filter bank. I shall in the next couple of minutes only give the strategy for doing so at which will actually do this in the succeeding lectures, the lectures to follow. Let me put before you the strategy that we are going to follow. And for that purpose, let me put down the equations before you once again. So let us recapitulate the 2 equations we have written.

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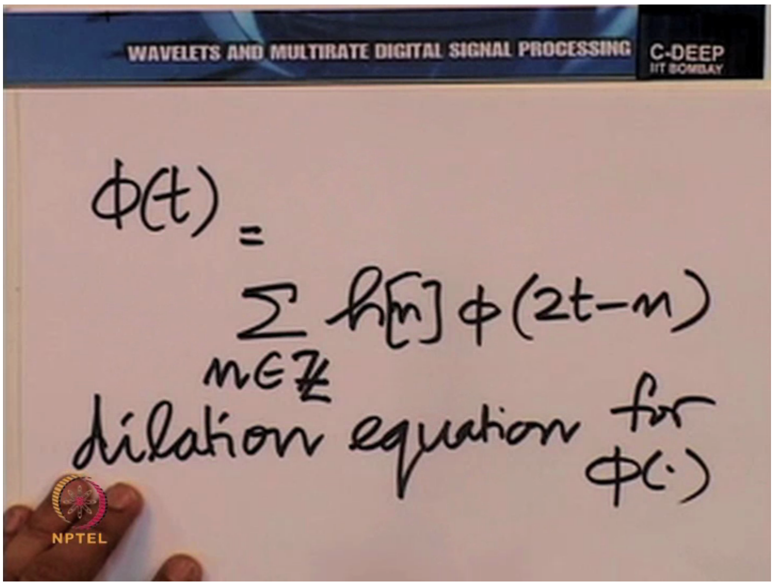

We have this dilation equation relating phi t to its own dilates and translates. What we shall do with the next lecture is to take Fourier transforms on both sides and noting that we can express the Fourier transforms of phi t or rather phi 2t - n in terms of that of phi t we shall have a recursive equation in the Fourier domain on the Fourier transform of phi. From this we shall be able to completely characterise the Fourier transform of phi in terms of the discrete time Fourier transform of the sequence H.

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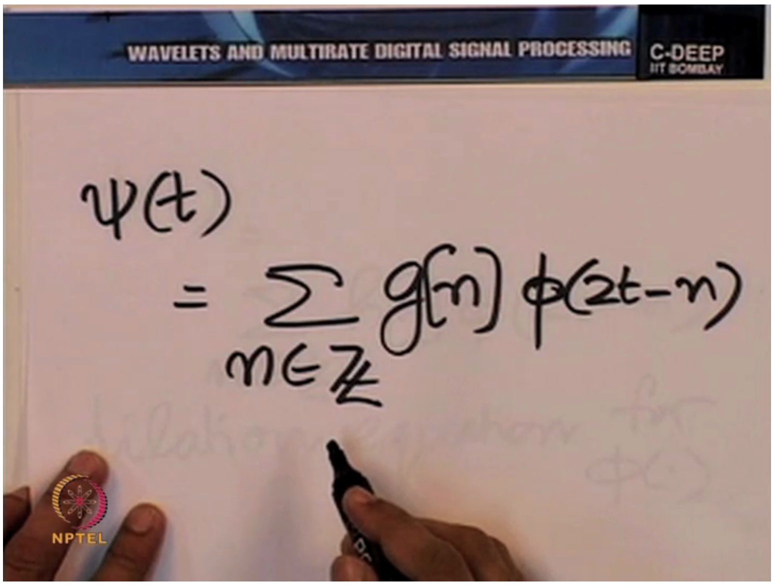

Having done so, we shall then progress to this dilation equation here and we will relate the Fourier transform of the wavelet to the Fourier transform of the scaling function effectively. And then, noting that the discrete time Fourier transform of the sequence Gn can be used to make this relationship, we shall obtain the wavelet from the scaling function. So, it is with these 2 steps that we shall begin the next lecture.

For the time being, let us keep our curiosity alive to see how beautifully we can enmesh the design of the two band filter bank and the design of scaling function and a wavelet for building a whole multiresolution analysis. With that note of curiosity and anticipation, let us conclude this lecture. Thank you.