Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-2.
Lecture-7.2.
Frequency Response of Haar Analysis Lowpass Filter Bank.

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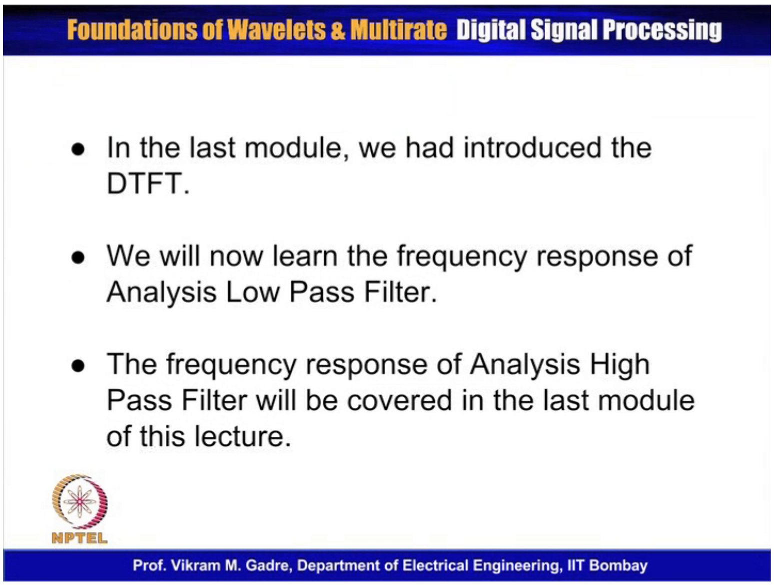

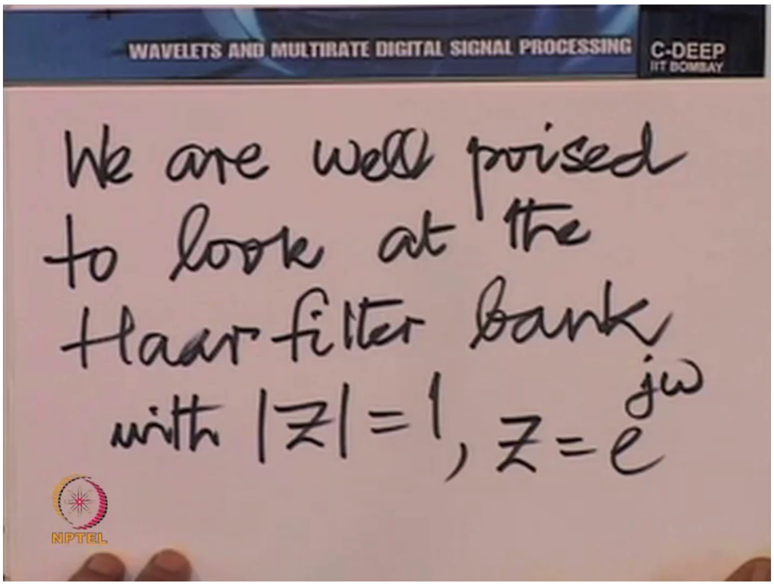

So, we are now entitled or we are now well poised to look at the Haar filter bank in the frequency domain. And what do we mean by the frequency domain, with mod Z equal to1 or Z equal to e raised to the power J Omega. Indeed, consider the analysis side, the analysis filters.

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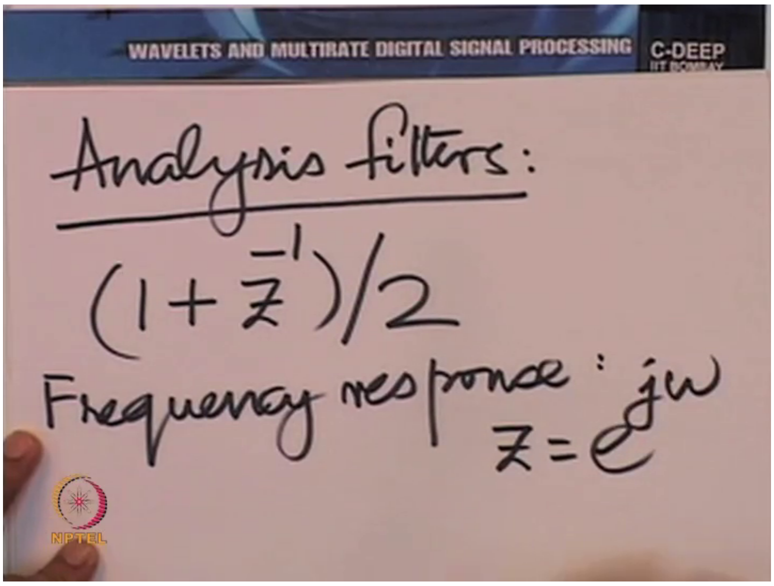

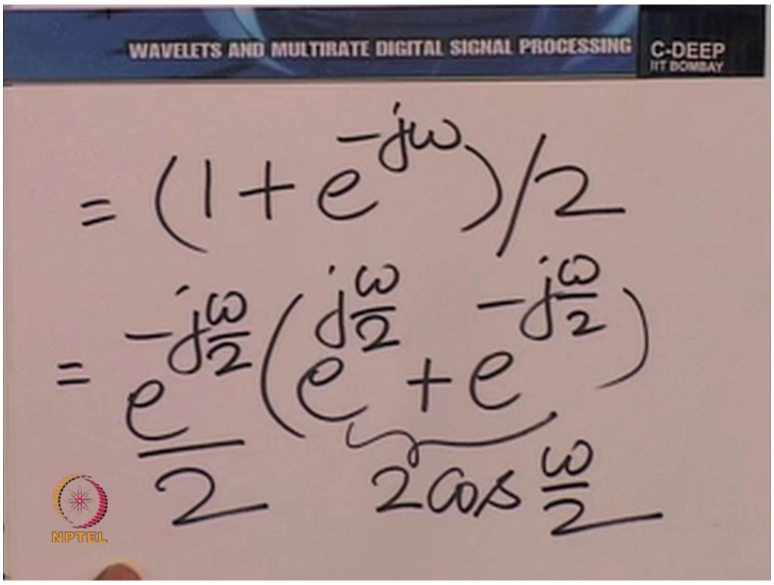

Both of them have a frequency response, obviously. Let us take the filter 1 + Z inverse by 2 and let us look at its frequency response. Of course we would obtain the frequency response by substituting Z equal to e raised to the power J Omega, that would give us 1 + e raised to the power J omega by 2, - J omega by 2 if you like.

Which we can simplify, we can take e raised to the power - J Omega by 2 common here and then put e raised to the power J Omega by 2 + e raised to the power - J Omega by 2 inside the bracket, leave the half factor as it is. So, this is what we have here. Now, recognise that this is essentially 2 times cos omega by2.

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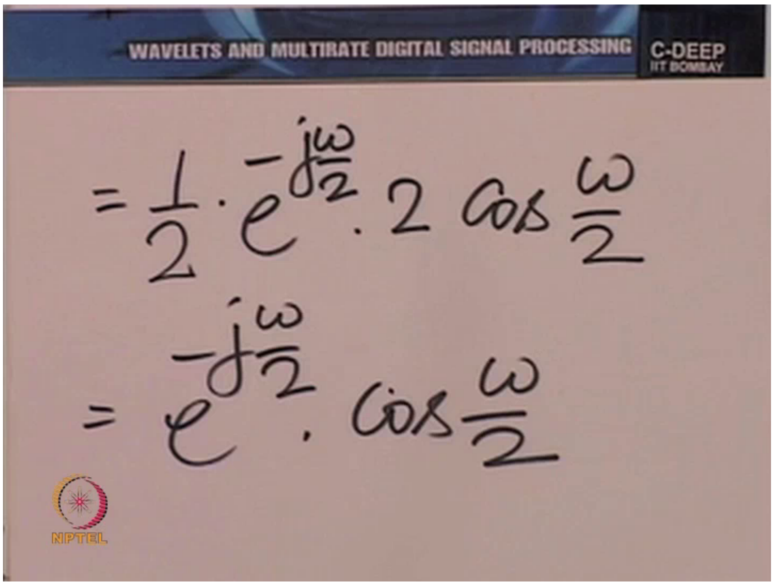

And therefore we have half e raised to the power - J Omega by2, 2 times cos omega by2 which gives us e raised to the power - J Omega by 2 times cos omega by2, this is the frequency response.

Let us sketch the magnitude and the phase of this frequency response and recall that for discrete systems, it is adequate to sketch the magnitude and phase or for that matter to determine the magnitude and phase response in the region omega going from - pie to + pie. We do not need to go outside that range, because after all, the frequency response is periodic with the period of 2 pie, so whatever occurs between - pie and pie is going to be repeated around every multiple of 2 pie.

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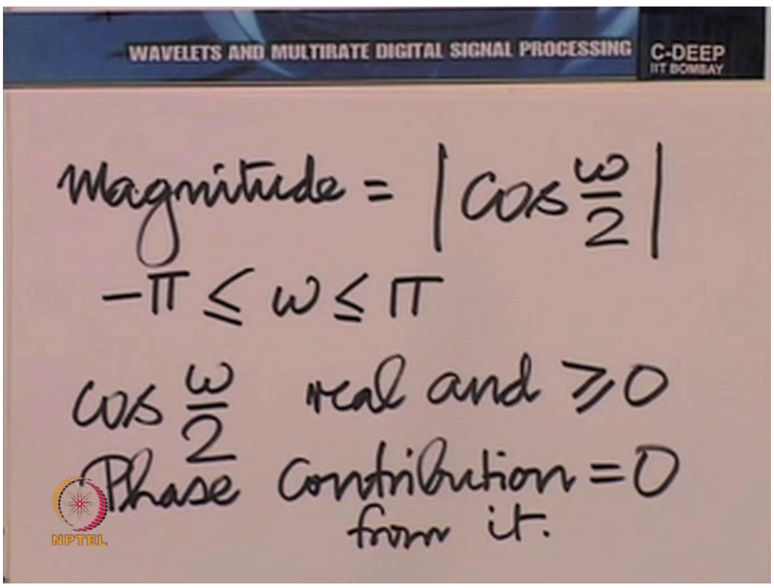

So, the magnitude of this response would be mod cos omega by2. And let us confine as I said omega between pie and - pie. Whereupon cos omega by2 turns out to be nonnegative, in fact essentially cos omega by2 is real and greater than equal to 0 in this region. That is phase contribution from this term is 0.

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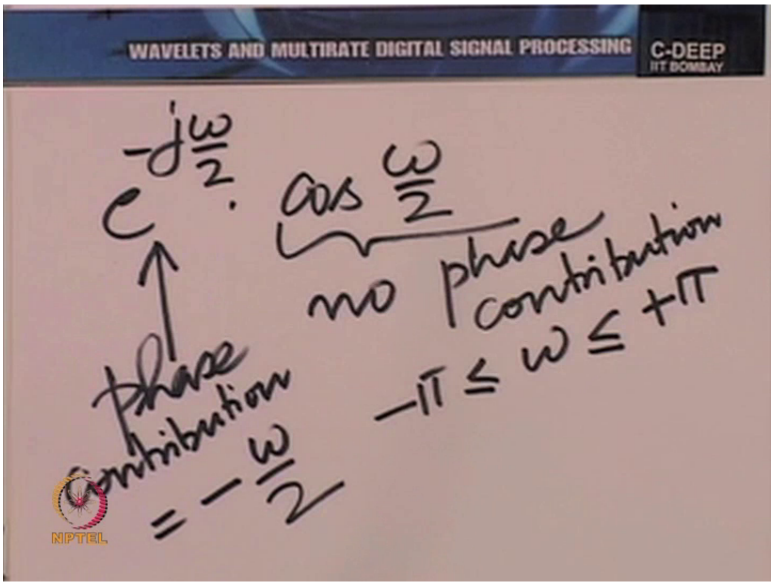

So, if you look at the overall frequency response here namely e raised to the power - J Omega by2 times cos omega by2, this has no phase contribution for - pie less than equal to omega less than equal to + pie and the phase contribution comes only from here. The phase contribution from this term is going to be equal to - omega by2. So, now we can sketch the phase and magnitude contributions very clearly.

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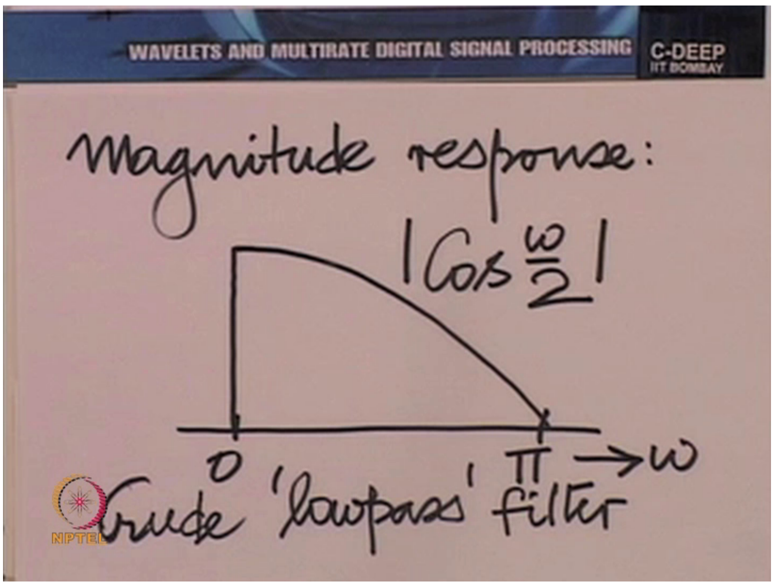

Let us sketch the magnitude response. The magnitude again, you know, this is a real filter, this is a filter with a real impulse response. Now, when an impulse response is real, the frequency response is conjugate symmetric. So, whatever is at omega is also at - omega in magnitude, whatever is at omega in phase is the negative of what is there at - omega in phase.

So, we need only sketch the magnitude response and the phase response between 0 and pie, we can ignore - pie to 0 because there is going to be conjugate symmetry. So, let us sketch it only between 0 and pie. You can see that this is the kind of pattern that is going to show, essentially of the form cos omega by2. And for the moment we can already see that this is something like a crude lowpass filter. I say it is a lowpass filter because it emphasises the lower frequencies and de-emphasises the higher frequencies, I say its crude, it is far from ideal.

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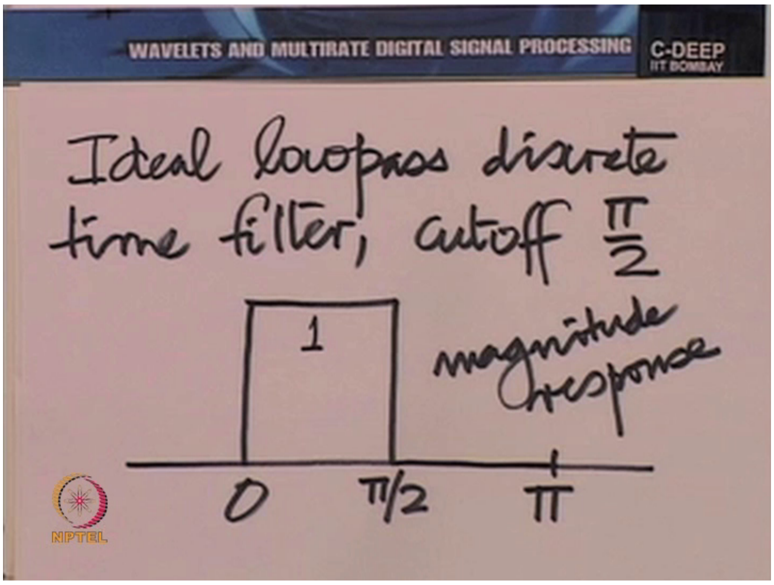

In fact if you have an ideal lowpass filter, then of course you need to specify the cut-off too, so an ideal lowpass discrete time filter with a cut-off of pie by 2 will look something like this, I mean in the magnitude sense. So, it would have a magnitude response that looks like this. And ideally the phase response would be 0, so it would be one, the response itself would be 1 between 0 and pie by 2 and 0 between pie by 2 and pie and of course mirrored as it is between - pie and 0. So, ideally we would have this magnitude and phase equal to 0.

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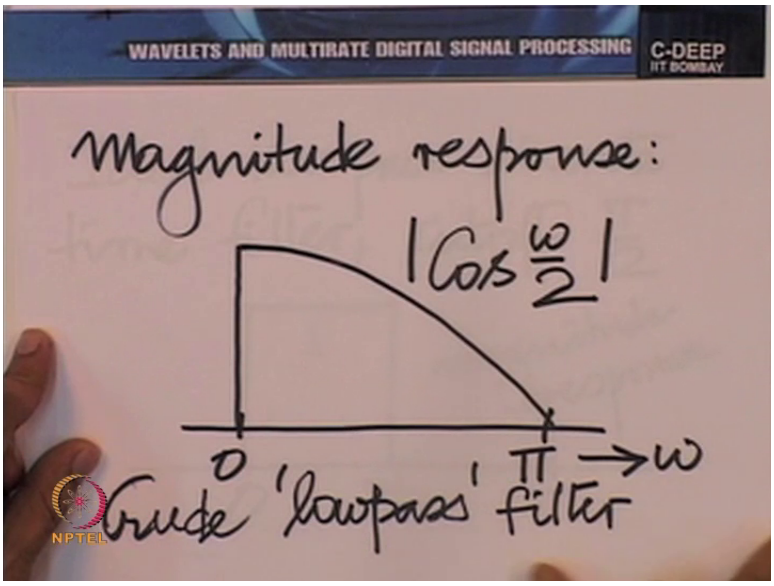

Whereas, we in reality, we are trying to replace this ideal filter here with this crude approximation to the lowpass filter here. So, please remember this is the ideal towards which we are striving and this is where we have reached in some sense with this very simple Haar multiresolution analysis or Haar filter bank. Now we shall understand this even better when we look at the magnitude response of the high pass or the other filter in the filter bank. But for the moment, let us complete our discussion by also drawing the phase response.

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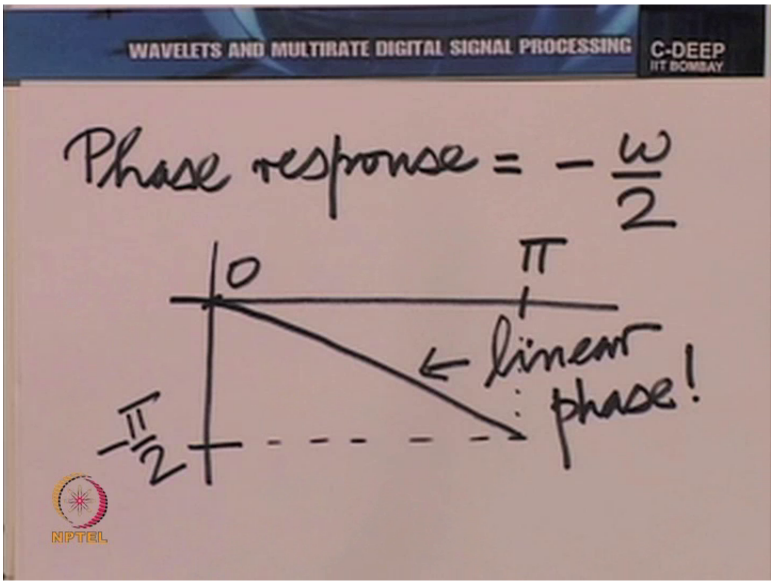

So, the phase response of this filter is as follows. The phase response as we noted was - omega by2, so in the region from 0 to pie, at pie, it will of course take on the value of - pie by 2. And this is a straight line.

So, in fact you get what is called linear phase. Now, linear phase is something very attractive in discrete time signal processing. In fact, one of the reasons why people go to discrete time signal processing from analog signal processing is because you can get linear phase. Why is linear phase important? Let us spend a minute in reflecting on this. You see, what does the phase response denote? Or for that matter, what does the frequency response denote? The frequency response tells us what happens to a sine wave when it passes through the system.

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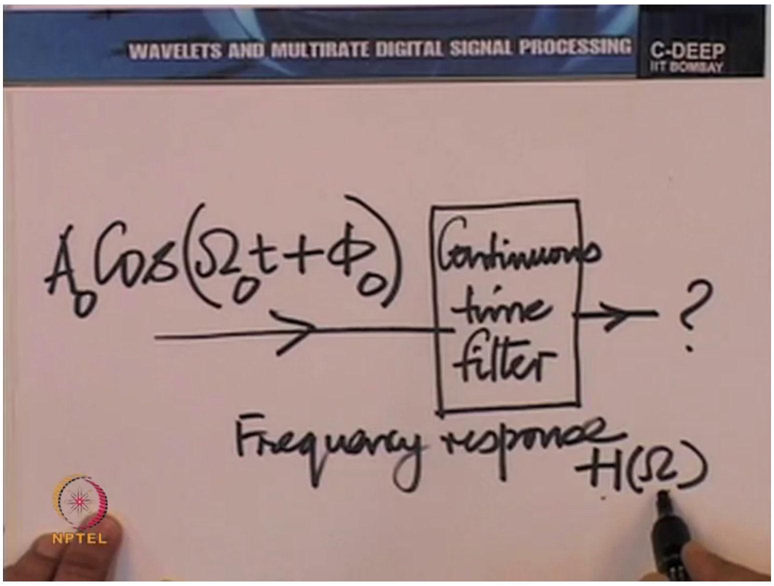

So, in fact to understand this better, let us go to continuous time 1st. Suppose we fed a sine wave, let us say cos omega 0 t + phi 0 times A 0. So, amplitude A 0 frequency capital Omega 0 phase phi 0 to a continuous system, to a continuous time filter with frequency response, let us say H as a function of omega, what would come out? Very simple, you would evaluate the frequency, this is of course a complex number as a function of capital Omega. You would evaluate this at omega equal to omega 0, the magnitude would multiply the magnitude, the angle would add to the angle here.

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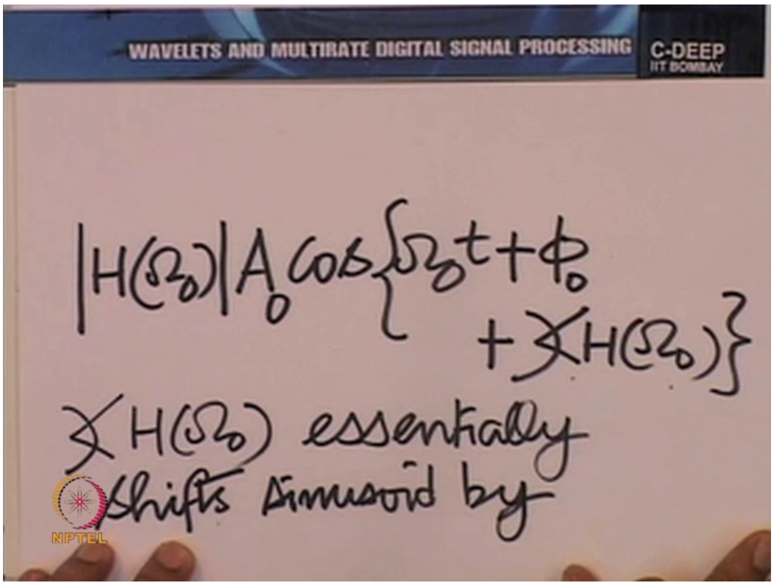

So in other words, what would come out is mod H evaluated at omega 0 A 0 cos omega 0 t + phi 0 + angle of H evaluated at omega 0. What is the effect of this angle? As you can see, the effect of this angle is to change the phase and a change of phase is equivalent to a change in time, so it is like shifting the time, shifting the sine wave on the time axis.

So, angle H omega 0 essentially shifts the sine wave, shifts the sinusoid by well you know t is to be replaced by t + something and that + something is given by this divided by omega. So, I mean, let us write it out explicitly that way. Let us reason out what it should be explicitly.

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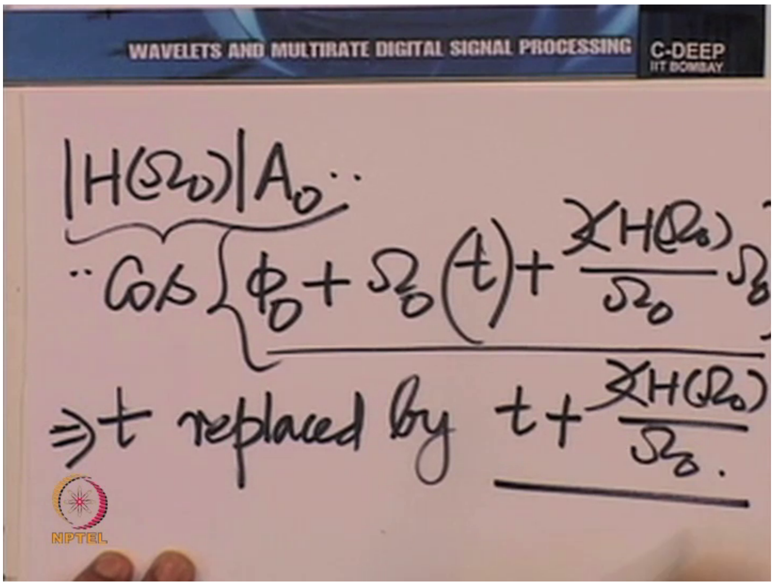

So, we rewrite that expression, we write as mod H omega 0 times A 0 cos, we keep phi 0 as it is, we will take omega 0, and then we will write t + the angle of H omega 0 divided by omega 0, I am sorry, ya. Well I will write omega 0 t + this into omega 0. So, again you have angle here, so this into omega 0, I will write it this way. So, I will rewrite soforgetting about this part, this part is essentially a change of magnitude, it is only this part which I wish to focus on. t has been replaced by t + angle H omega 0 by omega 0. So, essentially this quantity, angle H omega 0 divided by omega 0 is like a timeshift, it is an important observation we make.

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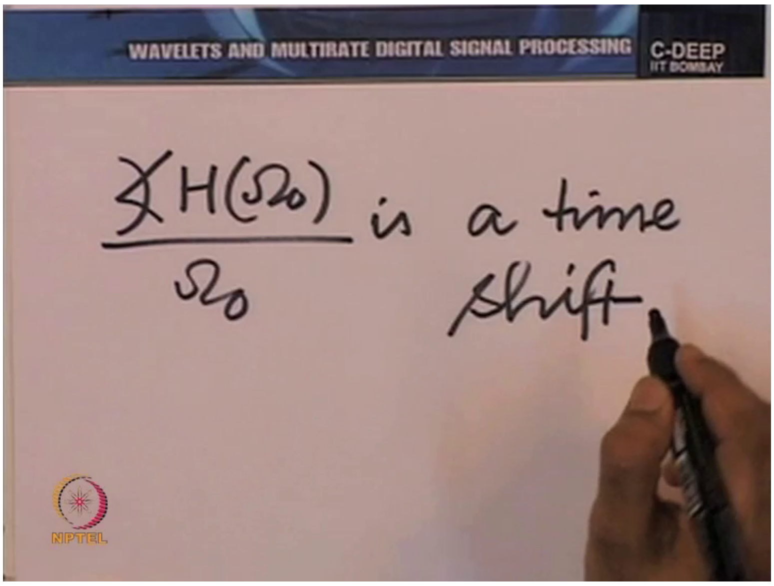

Now, just in case, it is independent of omega 0, we have a good situation. So, you know, this timeshift actually is what is called the necessary evil in the frequency response. You know most of the time when we give specifications for designing a filter, whether it is in the analog domain or in the discrete time domain, we do not really want a phase response. The phase response comes as a necessary evil and we have to work around it. Now, what does a phase response do?

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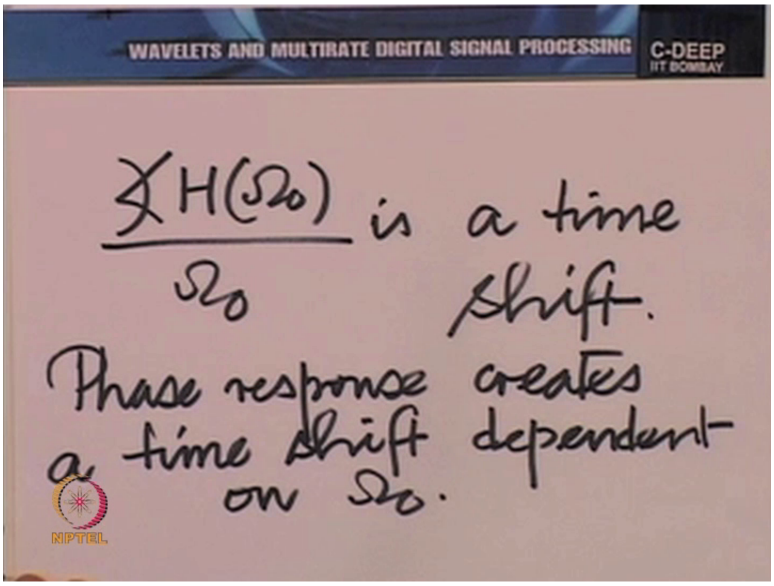

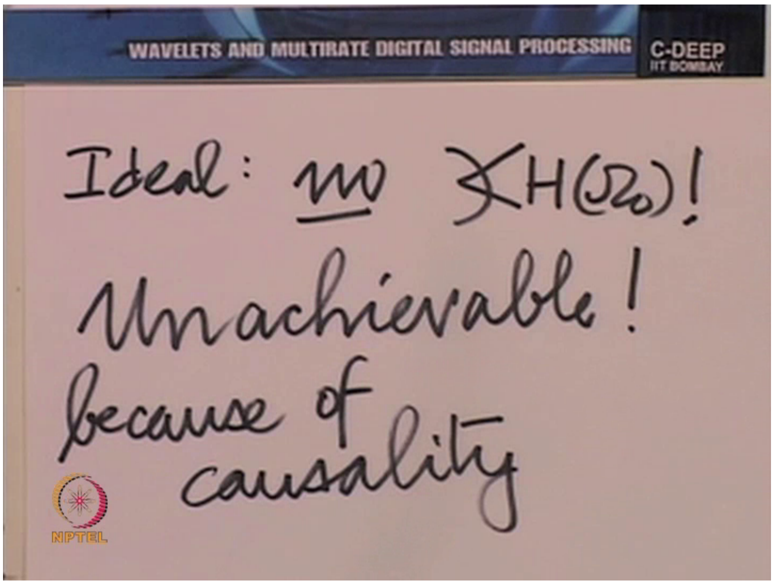

At each frequency is creates a timeshift, the phase response creates a timeshift dependent on frequency. What is a good situation to have? An ideal situation is no phase response, this is unachievable. In fact it is unachievable because of causality. You know if you want the filters to be causal, then you cannot ask for 0 phase. Incidentally, that is why you can use 0 phase filters in dealing with images, in dealing with two-dimensional data, because in two-dimensional data, causality is not an important requirement.

But when you are dealing with one-dimensional data in time, causality is essential, you cannot avoid it and therefore you cannot avoid the phase response either. Now, if you must live with the phase response, what kind of a phase response would you like to live with? You would like to make sure that phase response does not treat different frequencies differently.

So, if it has to shift a sine wave in time, so be it, but then shift all sine waves by the same time and that is exactly what we are saying when we talk about linear phase.

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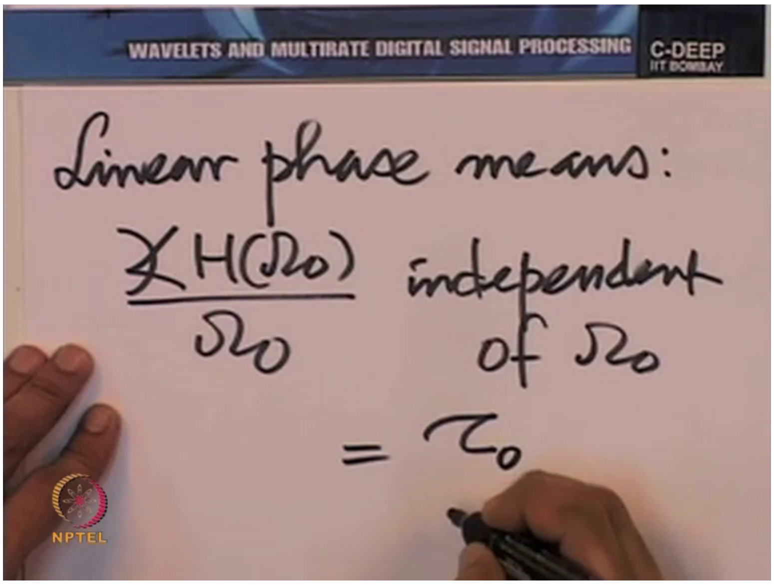

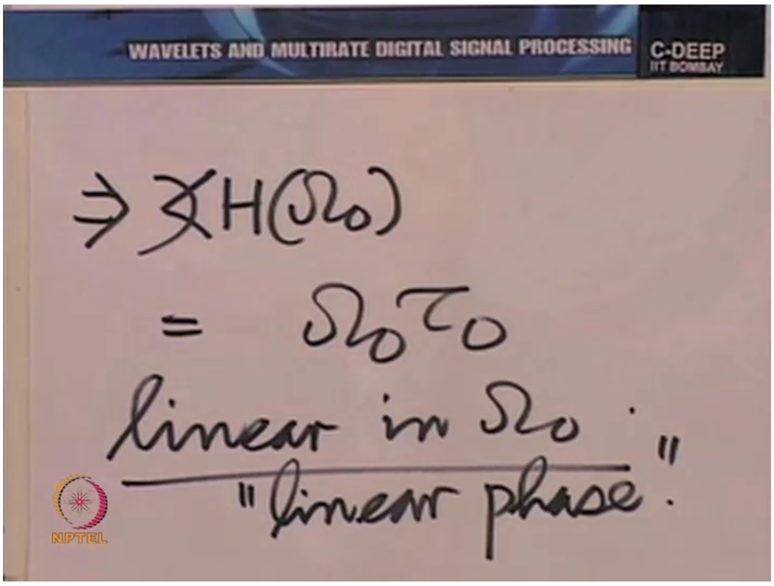

Linear phase means angle H omega 0 by omega 0 is independent of omega 0. That means it is a constant, let us say it is some tau or tau 0 if you like. Which means that the angle of H omega 0 is of the form some omega 0 times tau 0. It is linear in tau 0, in omega 0. That is why it is called linear phase. Now, this is what I am trying to emphasise at this point. This very simple filter bank, the haar filter bank has this beautiful property of linear phase. By the way, this is something unusual in filter banks.

It is not easy to design filter banks of larger order with linear phase, in fact only one class of filter banks has this property. Linear phase in some sense has to be compromised with something else. So in the Haar, you can have your cake and eat it too, so you can have linear phase, you can have orthogonality but later we will see that if you want something else out of your filter bank, if your filter bank must be better in some sense, you have to sacrifice linear phase.