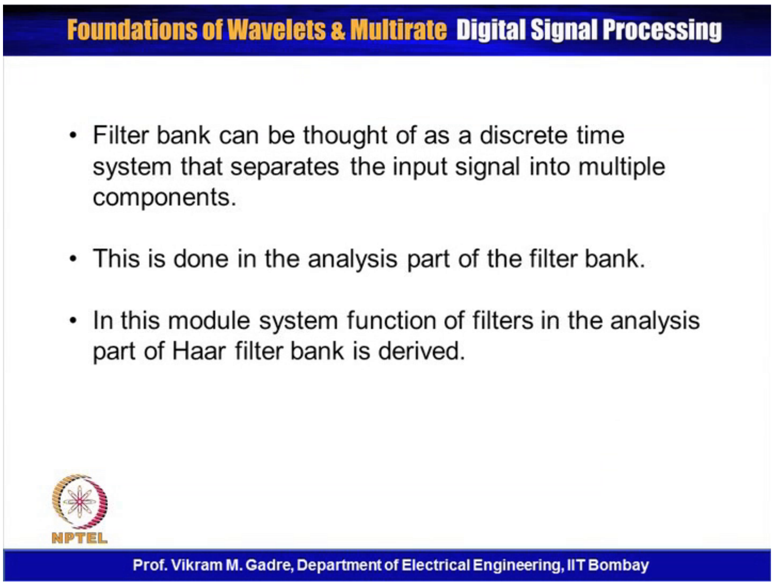
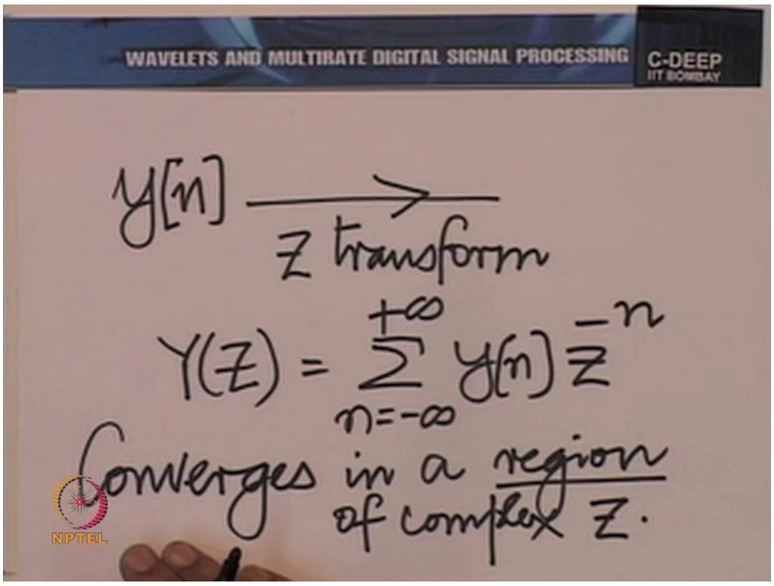
Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
Professor Vikram M. Gadre.
Department Of Electrical Engineering.
Indian Institute of Technology Bombay.
Week-2.
Lecture-6.2.
Haar Analysis Filter Bank in Z-domain.

(Refer Slide Time: 00:18)



Now, I shall just recapitulate a few concepts from Discrete Time Processing to refresh our memories and enable our discussion. Recall that if we have a sequence y of n, its Z transform is described by Y of Z.

(Refer Slide Time: 01:00)

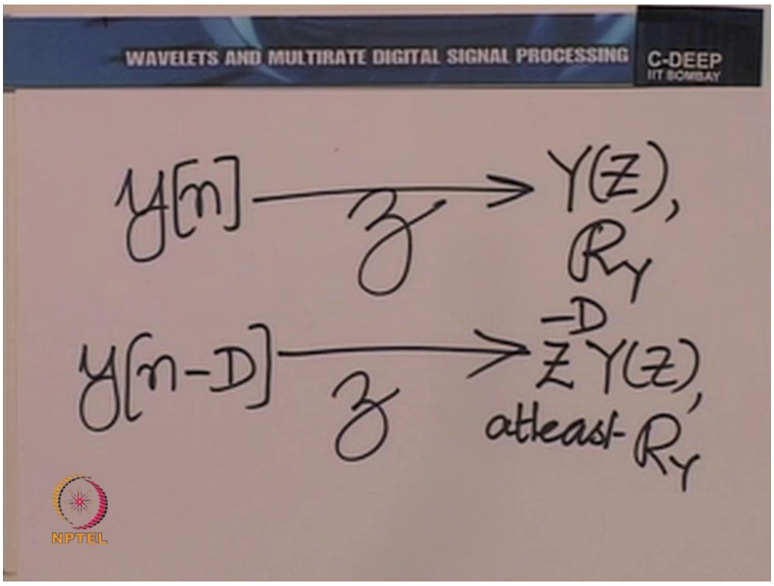


Normally we use the small letter to denote the sequence in time and the corresponding capital letter to denote the sequence in Z domain or also in the frequency domain. And Y of Z is summation and going from - to + infinity y n Z raised to power - n. Now we must remember that this convergence in a region of the Z plane, it doesn't converge all over the complex Z plane and that region in which it converges is called the region of convergence. So a Z transform is always defined by an expression and a region of convergence. Outside the region of convergence the expression has no meaning.

Recall also that both of them are absolutely necessary to complete the Z transform. If we only give the expression and do not specify the region of convergence, there is a possibility that 2 sequences could correspond to that expression, 2 or more in fact. Therefore, it is only after we specify the region of convergence, that the Z transform is completely specified, a few points that we noted in connection with Z transform.

We are also familiar with several properties of the Z transform. For example, when we shift a sequence, the Z transform is multiplied by an appropriate power of Z. So let us recall that property of the Z transform. So if y n has the Z transform and you know having the Z transform we shall denote by script Z like this.

(Refer Slide Time: 03:15)

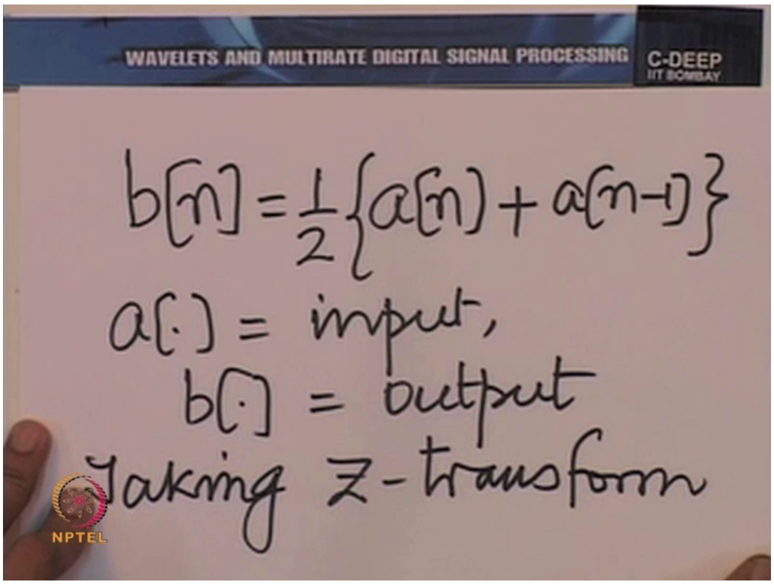


So when we when we write this, what we mean is that y of n has the Z transform given by Y of Z with a region of convergence R. Maybe if you want to be specific, we could say region of convergence of capital Y. So if y of n has Z transform given by Y Z with a region of convergence R y, then y of n - D has a Z transform given by Z raised the power - t Y of Z and the region of convergence is at least R y if not more. Sometimes, the region of convergence might expand a little beyond R y.

The Z transform is a linear operator, so if we take a linear combination of sequences, the same linear combination occurs in the Z domain. As far as the regions of convergence go, the regions of convergence are at least T intersection of the region of the convergence of the individual sequences which are linearly combined if not more.

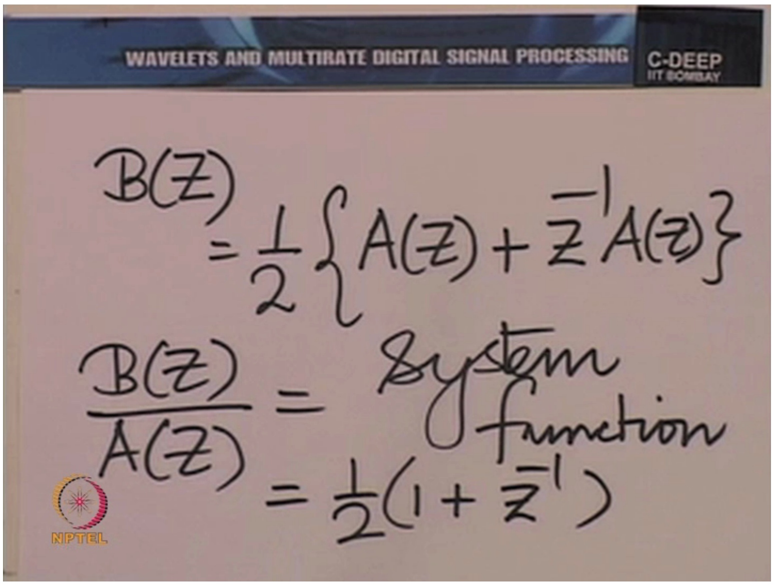
So when we have an operation being done on sequences and the corresponding Z transform is recorded, it is possible the region of convergence might expand beyond the intersection of the region of convergence of the sequences. Anyway, using this let us transform the filters that we had a minute ago into the Z domain. Let me put down the 2 filters explicitly once again.

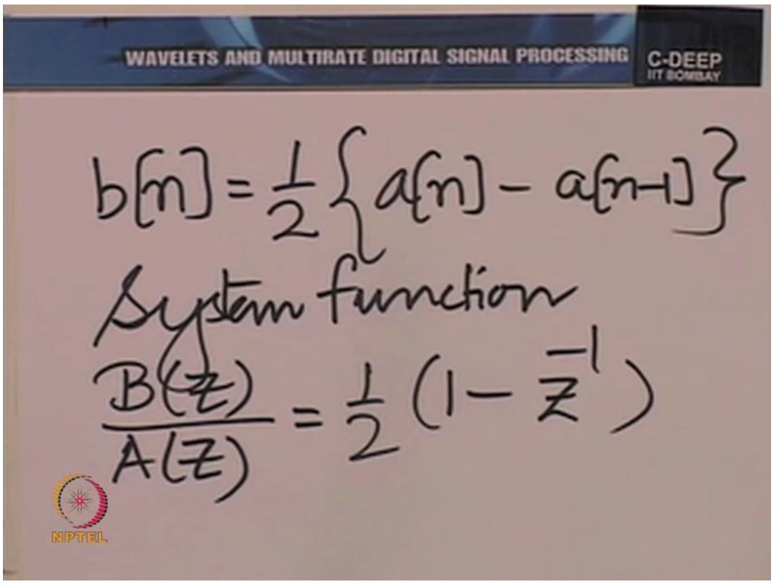
(Refer Slide Time: 5:11)



I have the first filter given by b n is half a n + an - 1, where a is the input and b the output. If we take the Z transform on both sides, we get what is called the system function of this filter. So let us calculate that system function.

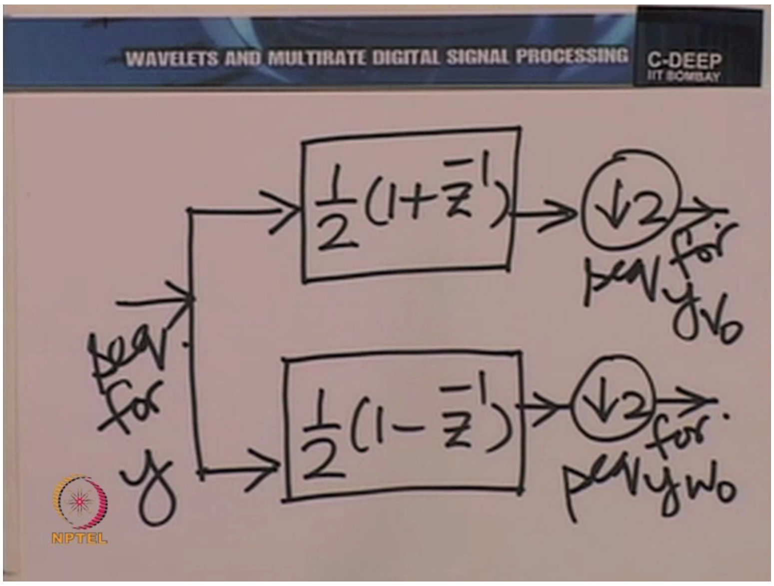
(Refer Slide Time: 5:55)





We get B Z is half A Z + Z inverse A Z whereupon B Z by A Z can be obtained and this is called the system function of the filter. And the system function is easily seem to be half 1 + Z inverse. In a similar manner we can calculate the system function of the second filter. That is described by b of n is half a n - a n - 1.

(Refer Slide Time: 6:45)



And there the system function would be B Z by A Z given by half 1 - Z inverse. So all in all, we have a pair of filters followed by a pair of decimator and the structure looks like this, half into 1 + Z inverse here, half into 1 - Z inverse there and a decimator.

So here we have the sequence for y, here we have the sequence for y V 0 and here we have the sequence for y W 0. Coming out here, here and here respectively, now this is what decomposes or analyses the function y t into its components. And therefore, this pair of filters followed by the decimation operations is referred towards the analysis filter bank corresponding to the Haar multiresolution analysis.

Now we should also bring out the synthesis filter bank for the same multiresolution analysis. In other words, if we have the projection on V 0 and the projection on W 0, how do we reconstruct the y t function? So in other words, if I have the sequences corresponding to Y V 0, and the sequences corresponding to Y, the sequences corresponding to Y V 0 and Y W 0, the projections on V 0 and W 0. And if we wish to reconstruct the sequence corresponding to Y, what are the things that we need to do?