Fundamentals of Wavelets, Filter Banks and Time Frequency Analysis.
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Week-2.
Lecture-6.1.
Introduction to Filter Banks.

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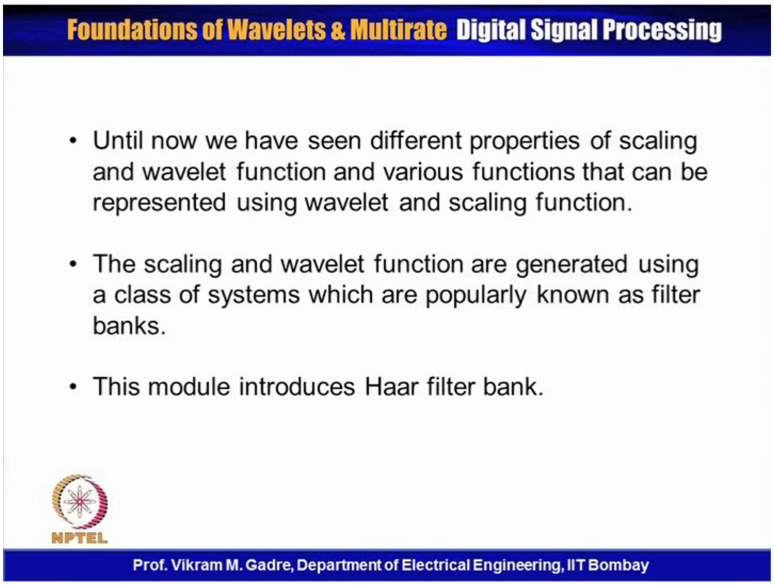

A very warm welcome to this lecture on the subject of Wavelets and Multirate Digital Signal Processing. In this lecture, we continue to build on the idea of connecting multiresolution analysis and a set of filters and therefore we shall call this lecture a lecture based on the Haar filter bank.

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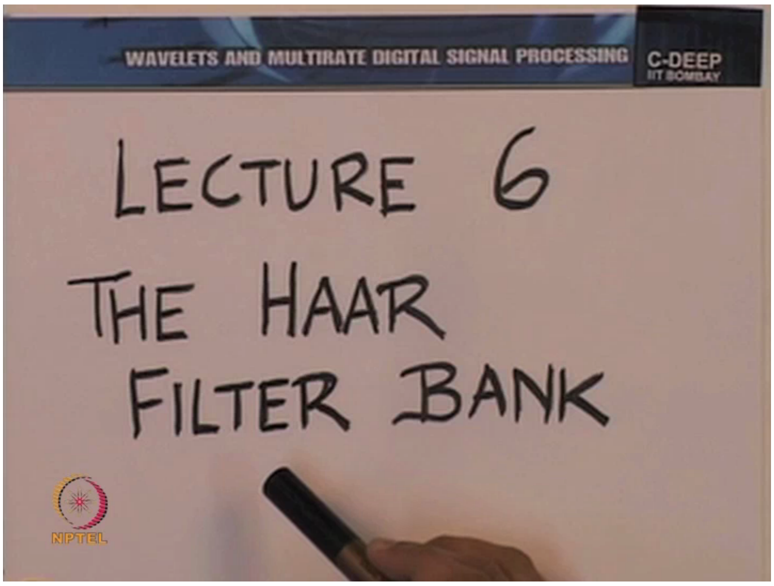

Haar if you recall is the multiresolution analysis that we are discussing and we have talked about filter banks earlier, a collection of filters with certain mutual and individual characteristics, either with a common input or a common point of output summation. So we are going to build up the connection between the Haar multiresolution analysis and the filter banks as we understand them in the discrete domain. Now towards that objective, let us go back to the example that we bought up last time. Now I shall highlight before you the example once again, let me reiterate the example.

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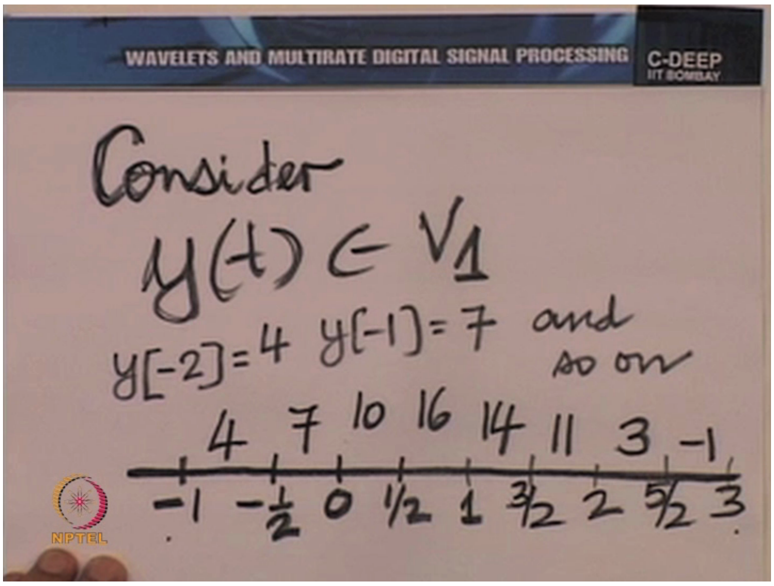

So we consider a function y t belonging to V 1 as understood in the Haar multiresolution analysis. And remember the function V 1 that we were talking about yesterday, we said on the real axis, we would of course in principle define it over every half interval.

But then we can be content with looking at a segment of this real access between - 1 and 3 and we will do exactly that. So we have this segment between - 1 and 3 and the values in the successive half intervals here, starting from the half interval immediately next to - 1, the piecewise constant values are 4, 7, 10, 16, 14, 11, 3 and - 1. We said it is quite adequate for us to write down the piecewise constant values in each half interval.

And that would define the function completely, in fact as far as a function belonging to V 1 is concerned; it is these that constitute the coefficients of expansion. So in fact, the sequence as we understand it is here. For example, y at the sequence corresponding to this would have y at - 2 equal to 4, y at - 1 equal to 7 and so on. y at 0 is equal to 10; y at 1 is equal to 16 and so on so forth.

And of course this sequence could be used in constructing the function from the basis. So we have y of t is summation n running from - to + infinity y of n Phi 2 t - n.

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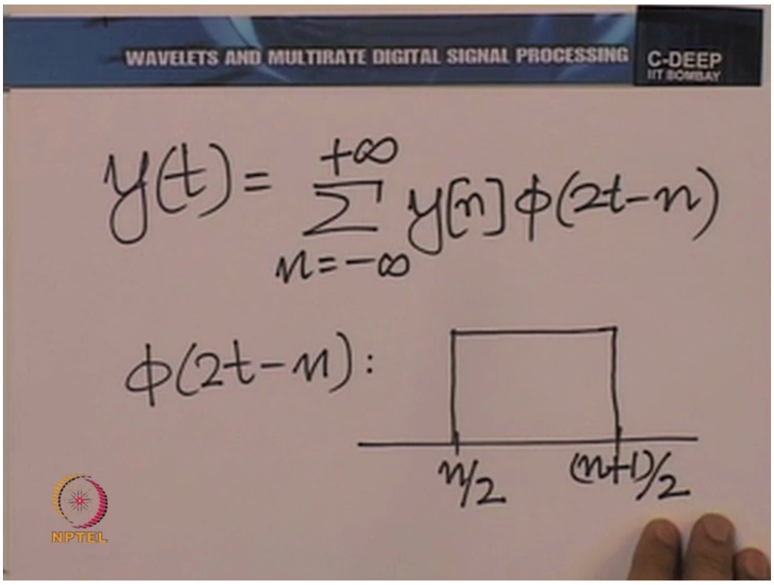

Recall that Phi of 2 t - n look like this. It was 1 over a half interval, a half interval defined by n by 2 to n + 1 by 2. Now after reflecting on this, we must now do exactly what we did; we put down a scheme yesterday for going from the function in V 1 to its components in V 0 and W 0. So we said we could make an orthogonal decomposition of V 1. We said we could write

V 1 as V 0 + that is orthogonal sum W 0 and both V0 and W 0 could be defined on the standard unit intervals.

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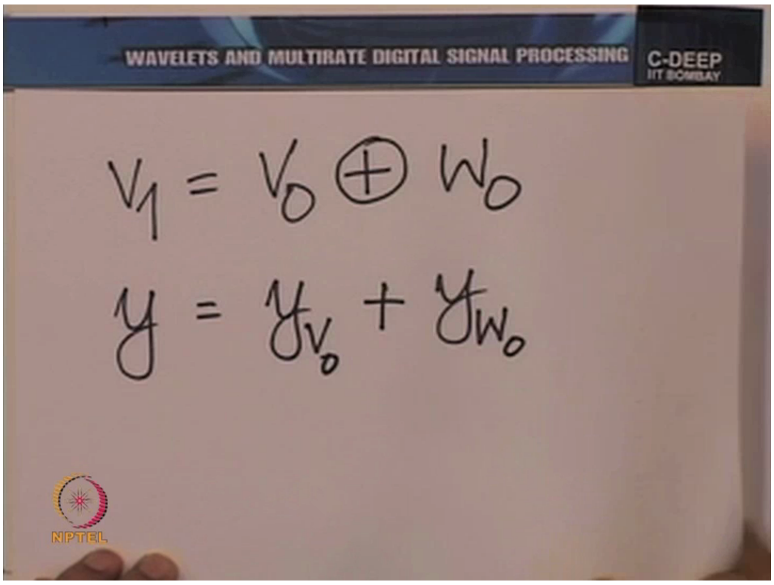

So for example, for this particular function that we have here, let us put down explicitly the projection on V 0 and W 0. You see, we now use the word projection, we have made an orthogonal decomposition of V 1 the space V 1 into the spaces V 0 and W 0 and we have explained the meaning of V 0 and W 0 yesterday. We have also talked about their basis. We have also shown how to make this decomposition in each standard unit intervals.

So for example, if we go back to this function which we were discussing a couple of minutes before, what we would now need to do is to take each standard unit interval. So for example, the interval - 1 to 0 and then the interval 0 to 1, 1 to 2, 2 to 3 and each of them we need to put down how the projection on V 0 would look and how the projection on W 0 would look.

And recall for example, if you consider the interval from - 1 to 0, the projection on V 0 would be piecewise constant on that interval and would take the value given by the average of 4 and 7, namely 4 + 7 by 2. On the other hand, the projection on W 0 would be given by a multiple of the Haar wavelet located between - 1 and 0 and with a coefficient 4 - 7 by 2 associated with it.

So let us proceed to put down these 2 projections. So we have the projections y on V 0 which we shall call y V 0 and y on W 0 which we shall call y W 0. Let us sketch y V 0 first. Let me keep this for reference on top.

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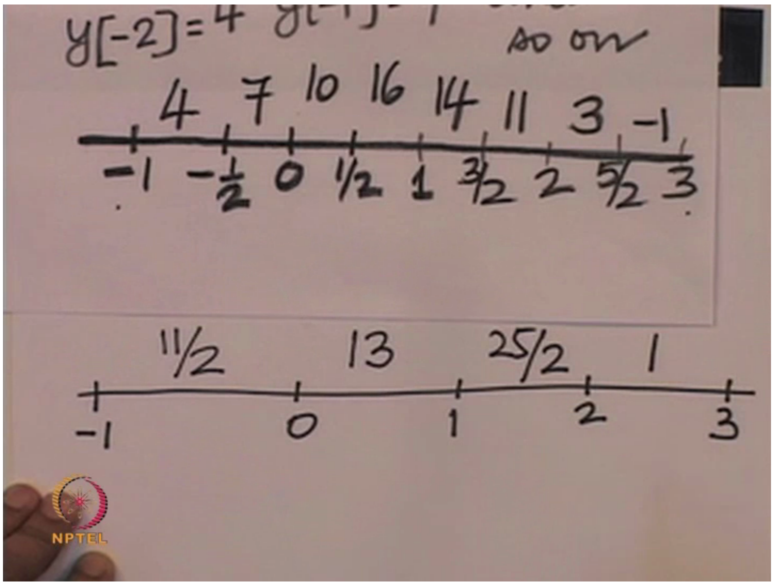

Now between - 1 and 0 as we see, the projection would be 4 + 7 by 2. Between 0 and 1, the projection would be 10 + 16 by 2, so let us put down the values. I keep this for reference on top and put down the values here. So 4 + 7 by 2 that is 11 by 2. Here, 10 + 16 by that is 13. Between 1 and 2, it is going to be 14 + 11 by that is 25 by 2 and between 2 and 3, it is going to be 3 + - 1 by 2 that is 2 by 2 that is 1. This is how the projection on V 0 would look. y projected on V 0 as a function of t. Piecewise constant on the unit intervals and these are the piecewise constant values.

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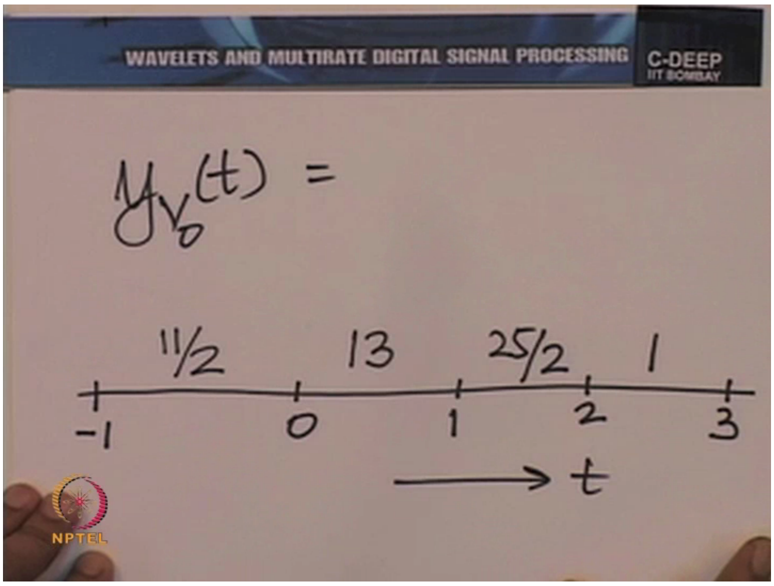

Now of course I am not actually drawing the function here, it is easy to visualise the piecewise constant values. But let me now go to the projection on W 0. So I shall once again use this as a reference here.

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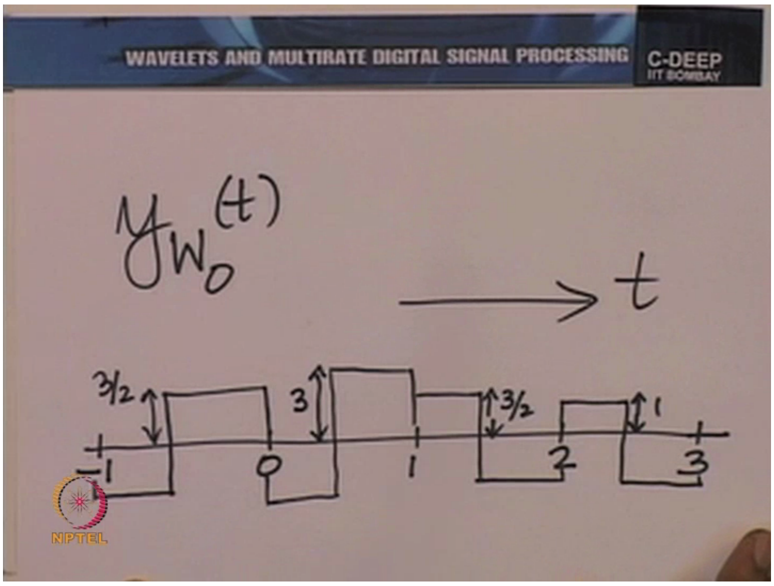

And I put down the values, this time I will explicitly indicate that we are using this basis, so I have - 1, 0, 1, 2, 3 and I am using a basis. So between - 1 and 0, I am going to use a proper translate of psi t with a height given by 4 -7 by 2. This height is going to be 3 by 2, of course with a negative sign. Between 0 and 1, it is again going to have a negative sign and the height is going to be 10 - 16 by 2 that is 6 by 2. So of course I am not drawing this to scale. I am just drawing it to be indicative, this height would be 3. Here, between 1 and 2 you would have 14 - 11 by 2 that is 3 by 2 but with a positive sign. You would have a multiple of psi t placed in this unit interval with a height of 3 by 2.

And finally, in the interval between 2 and 3 you have a height of 1 and a positive multiple of psi t placed 3 by 2, 3, 3 by 2 and 1. So this is the projection of y on the space W 0 as a function of t. Simple, now we can also write down the sequences, so in fact we should do that. If we go back to this projection, so now what we must do is to construct the sequences which describe these projections.

And in fact by constructing these sequences, we will also understand how the 2 discrete filters that we talked about in the previous lecture work. So let us go back to this function, the projection on V 0. How would the sequence here look? The sequence would be this.

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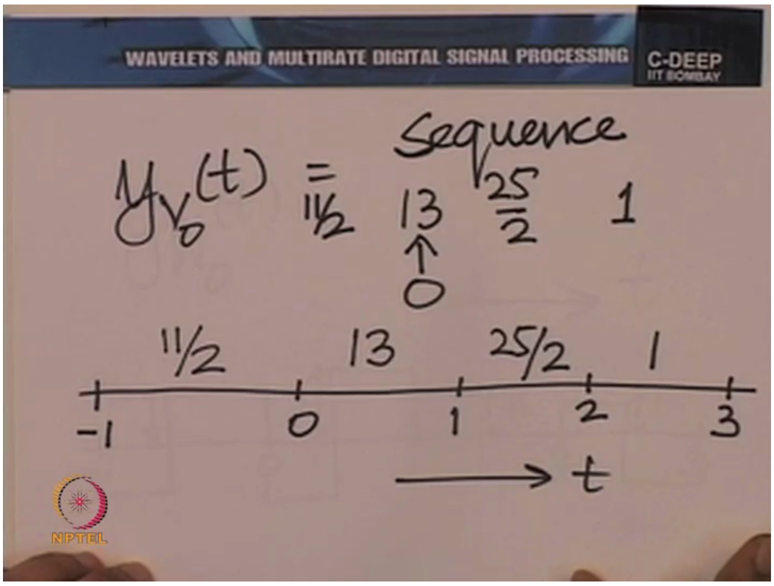

13, 25 by 2, 1, 11 by 2, 13 marked at 0, this is the sequence. And similarly we can put down a sequence corresponding to the projection on W 0. So the sequence here would be.

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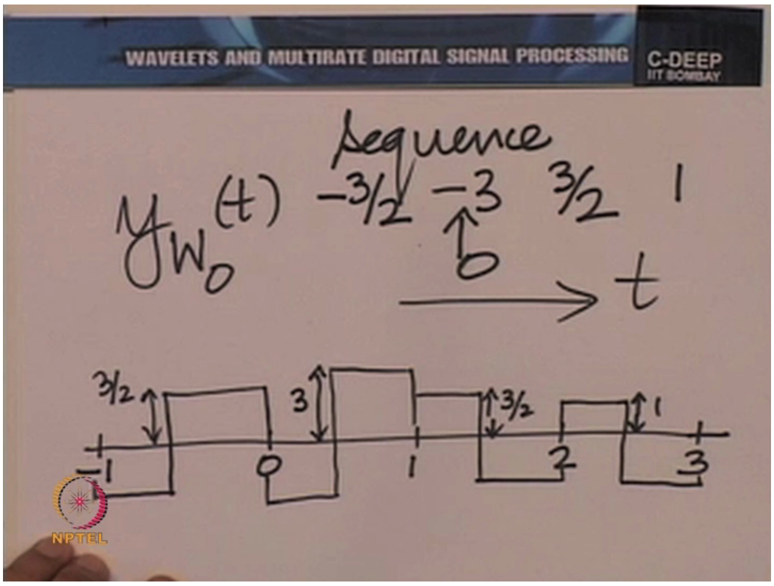

Well remember this is - 3 by 2 at - 1 - 3 at 0 and so on. So - 3, - 3 by 2, + 3 by 2, and + 1 with - 3 put at the point 0, that is the sequence. Now, we have also seen the filters that correspond to this, so yesterday we noted that if we put down the sequence y n, let us now put it down in the language of discrete time processing. If you have a sequence y n, sequence corresponding to y t.

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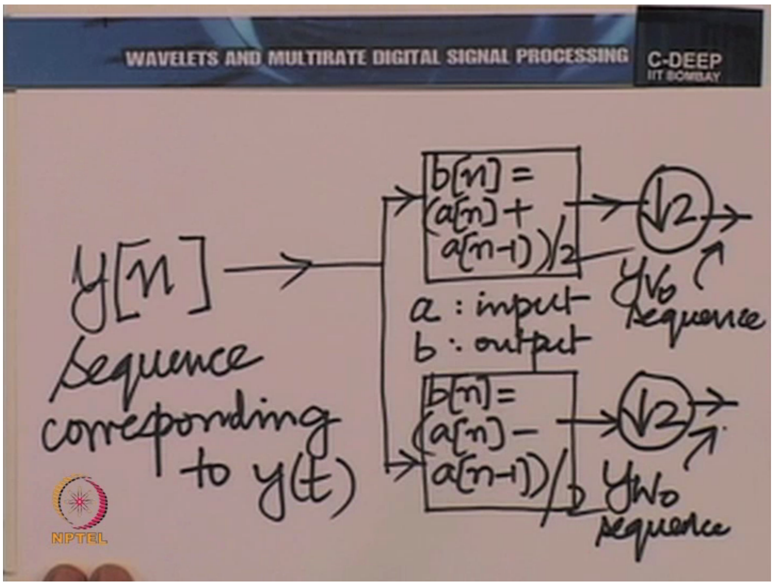

And if you pass it through 2 filters, the upper filter is described by the equation well you know, now I cannot use y n for the output because I am using y n to describe this. So I use different input output notations here. I will say in these 2 discrete time filters A is the input and B is the output. So I have B n is A n + A n - 1 by 2. And here I have B n is A n - A n - 1 by 2.

And then I have a decimation operation as I talked about yesterday, a down arrow followed by 2. So this is the structure that gives me the sequence for V 0 here and the sequence for W 0 here, let us write that down. The y V 0 sequence there and the y W 0 sequence here. In fact, what we should do is to describe these filters in terms of their system functions rather than describe them in the time domain as we are doing here.