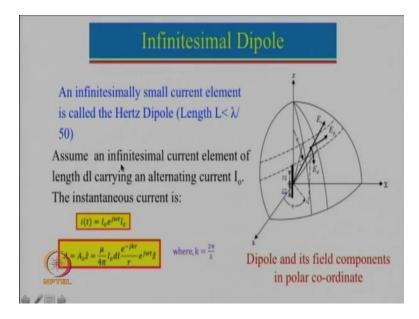
Antennas Prof. Girish Kumar Department of Electrical Engineering Indian Institute of Technology, Bombay

Module – 02 Lecture – 08 Dipole Antennas-I

Hello, and welcome to today's lecture. Now in the last lecture we had discussed about antenna fundamentals, we saw how directivity can be calculated from half power beam width, we also saw the link budget; how we actually have a transmit antenna, receive antenna, and then what is the distance between them and depending upon the transmitted power we can calculate what is the received power. And then we also looked at what are the radiation norms in India and Abroad, and we found out that there are lot of radiation hazards because of the over use of cell phone as well as to the people who are living next to the cell tower, and it is also affecting birds, bees, animals, plants, trees, and environment also.

Now today we will talk about the fundamental antenna which is a Dipole Antenna. So, let us start with the dipole antenna.

(Refer Slide Time: 01:13)



In general we start with the infinitesimal dipole; the definition of infinitesimal dipole is that the length of the dipole should be less than lambda by 50. So, if length is less than that then we consider that as an infinitesimal dipole. And here what we are assuming, we are assuming an infinitesimal current element which is actually means very small current element of length dl and it has a uniform current which is I 0; that is the amplitude but otherwise it is a sinusoidal current depending upon whatever is the frequency. So, then we can actually define the instantaneous current as I 0 e to the power j omega t.

Basically, this current element is along the z direction. So, what we have access here the dipole is placed at the origin, so this is an x axis this is a y axis and this is a z axis. And just to mention again angle phi is measured from this axis. So, anything component along this will be resolved on this direction here and the angle theta is measured from z direction. So now, since the current element is placed in the z direction we write this as a current in the z direction. And e to the power j omega t can be written as cos omega t plus j sine omega t.

So, for this particular current, but I want to mention here that I disagree with a lot of books which they claim this to be an infinitesimal dipole and they actually say that for infinitesimal dipole we can assume that the current is uniform, but in a reality that is not a correct assumption because current at the open end will always be equal to 0. Suppose if you feed here than the current will be 0 here and then it will have a maximum value. But nevertheless for the derivation purpose we will assume that this element is carrying a constant current.

So, please remember it is only for the derivation, but not the real thing. So, corresponding to this current element we can actually find out what is the vector magnetic potential.

(Refer Slide Time: 03:33)

Uniform Current – Magnetic Vector Potential

$$\underline{A} = \hat{a}_z A_z = \hat{a}_z \frac{\mu I_o \ell}{4\pi r} e^{-jkr}$$

$$A_r = A_z \cos\theta = \frac{\mu I_o \ell}{4\pi r} \cos\theta \ e^{-jkr}$$

$$A_\theta = -A_z \sin\theta = -\frac{\mu I_o \ell}{4\pi r} \sin\theta \ e^{-jkr}$$

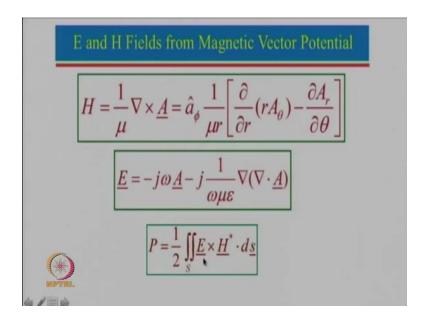
$$A_\theta = 0$$

So, this is how we can find the vector magnetic potential and since the current is in the z direction we have only component which is A z and that is given by this 1 over here. And since the length l which we have assumed is dl very small and then that is integrated it will become l, because current is uniform so I 0 will come as it is. Now this is a z direction from here we can find out the spherical coordinate A r A theta phi. So, from z we can actually say A z will be cos theta, so we can actually look into this here. So, let us say any element which is in the z direction, so then and this is the direction which is E r it is shown here e field, but think about this as A r then A r can be found out simply by z component multiplied by cos theta. So, that is the component which we can find out.

Then regarding the E theta component; so E theta component or in this case A theta component can be resolved now that will be sine theta and since it is going down a minus sign will come here. Now here it is electric field, but for A phi it will be perpendicular and A phi will be in this direction which is actually nothing but theta equal to 90 degree and cos 90 is equal to 0. So, hence the A phi component will be equal to 0.

So, once we know the A component vector magnetic potential then we can use the Maxwell's equation.

(Refer Slide Time: 05:13)



And along with the waves equation we can find out what is the H field what is the E field; and if we know E field as well as H field then we can find out what is power. So, power is nothing but you can say double integration E across H conjugate and that is the surface integral.

(Refer Slide Time: 05:34)

$$Uniform Current - E and H Fields$$

$$E_{r} = \eta \frac{I_{o}\ell}{2\pi r^{2}} \cos\theta \left[1 + \frac{1}{jkr}\right] e^{-jkr}$$

$$E_{\theta} = j\eta \frac{kI_{o}\ell}{4\pi r} \sin\theta \left[1 + \frac{1}{jkr} - \frac{1}{(kr)^{2}}\right] e^{-jkr}$$

$$E_{\phi} = H_{r} = H_{\theta} = 0$$

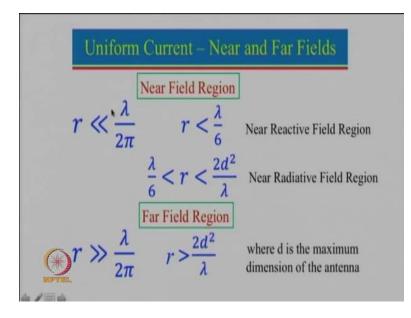
$$H_{\phi} = j \frac{kI_{o}\ell}{4\pi r} \left[1 + \frac{1}{jkr}\right] \sin\theta e^{-jkr}$$

So, from here we can by using that equation or we can find out E r E theta and these are the components which are 0 and these are here and H phi.

Now I am just giving you this expression, you can see the details derivation of these expressions in several books. I can just mention let us say the book of balance or cross and other thing. So, I will get into the more of the concept part and you can see these derivations in these books.

So now, I just want to bring to the next point here. If we see here this has a component in the denominator which is r square and here it is r. For E theta the component in the denominator is r here and then there is another are here, so r will multiplied by r will become r square and there is a r square this component will become r cube. Now when we are designing antenna most of the time we are concerned about far field radiation pattern. So, at far field we can assume that r is very large and if r is very large then r square will be extremely large, so this entire thing can be equated to 0 for far field. And for this particular case here this term and this term they can be ignored again at power of a distance where r is very large.

Similarly for H phi, so this has a 1 by r component this will be 1 by r square component, so we can ignore this particular term here. So, if we do that we can actually see far field pattern will be much simpler, but from here we can also define various areas and these various areas are defined in terms of the distance.



(Refer Slide Time: 07:19)

So, if r is much smaller than lambda by 2 pi. And this is coming basically because of the 2 pi by lambda which is nothing but k r. So, actually speaking you should think about if

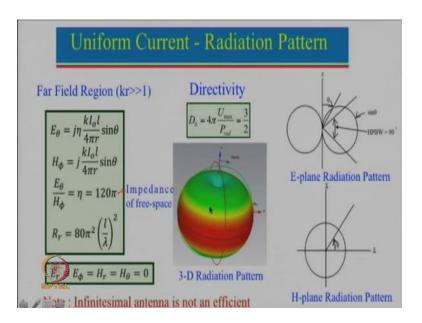
k r is much less than 1 and k is nothing but 2 pi by lambda so it goes over here. So, if this is the condition applied and over here also this region is defined into two different parts here; one is r less than lambda by 6 in 2 into pi is approximately 6. So, that is the near reactively region, and from lambda by 6 2 up to this distance here that is known as near radiative region. And any distance which is more than this which is r greater than 2 d square by lambda will be a far field region. And what is d? D is the maximum dimension of the antenna. Now, one should also remember that far field region also implies r should be much much greater than lambda by 2 pi

So, let just take some example here; suppose if we take an example of just let us say a lambda by 10 antenna which are dipole antenna. So, if I put here lambda by 10 so that will be lambda square by 100 the quantity will be reduced to lambda by 50. Now we cannot say that r greater than lambda by 50 will satisfy far field region criteria, because we also have another criteria which is lambda by 2 pi.

So, do not always go with this equation this equation along with this equation define far field region. But however, especially for antenna which are very large suppose d is equal to 10 lambda, if d is 10 lambda then this will be 200 lambda. So, you can see that that will be very far away distance. So, dimension plays very very important role. So that is why a many a times for larger antenna it is very difficult to do the radiation pattern measurement within let us say an anechoic chamber which is inside a room. So, for larger antenna invariably people do the measurement in the either in the far field or at the open area.

Or many a times if we want to calculate far field region and if the distance is very large then near field measurements are done and from the near field measurement far field can be obtained by using the concept of Fourier transform; and that way we can get the far field radiation pattern.

(Refer Slide Time: 10:04)



So now let us just see; what are the different things here. So, I have used the term uniform current, I am not mentioning here infinitesimal dipole because I do not agree that current will go to 0 at the edges. But for a uniform current let us say; what are the far field region. So, please see here this is k r much much greater than 1. So, we can now say there will be only two components; one will be E theta component another one is H phi component. And if you really look into over here what are the different terms; so eta is nothing but free space impedance which is equal to 120 pi or equal to 377 (Refer Time: 10:46) k is equal to 2 pi by lambda. So, you can actually see that lambda is coming into picture so that 2 pi can be there and lambda will be down below. So, it will be I by lambda.

So, please remember for all the antennas in general it is always the normalized length which is more important. And now you can see something interesting if you take the ratio of E theta by H phi you can see that most of the terms are similar except for eta here. So, E theta by H phi is equal to eta which is 120 pi and this is impedance of free space; so that means a wave which is propagating in the free space E theta and H phi will be perpendicular to each other; that means E plane and H plane will be perpendicular to each other; that means E plane and H plane will be perpendicular to each other at the impedance ratio will be equal to 120 pie; let us first just look at the radiation pattern for a dipole antenna. This is for infinitesimal dipole, so this is the 3-D plot and these are the plots in the H plane pattern and E plane pattern or we also call it

azimuthal. So now, just imagine that dipole antenna is over here. So, at this particular point here we can actually think about a dipole antenna as a pen in my hand.

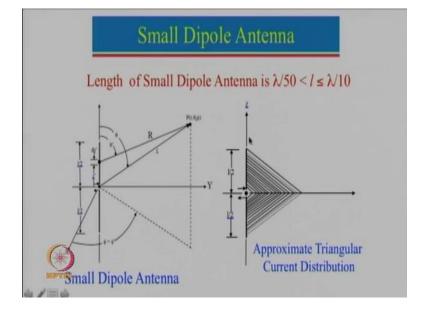
So, you can imagine that this dipole antenna is like a pen in my hand and change the color. Now, if you look at this pen from let say this direction what you see full length of the pen. If I see from my said I see the full length, from your side if you see you see the full length of the pen. So, if you actually look all around you will actually see the full length of the pen. Now please apply this concept only to dipole antenna, do not try to apply for all and every other antenna.

Now for this dipole antenna let say we are standing here we see the full length now as we move along and if we see from the top all we really see is the tip of the pen; that means we will see very little and as we move along we will start increasing little more. In fact, dipole radiation pattern is very very similar to the way we look at this particular pen. So, the dipole pattern will be uniform along this here which will be maximum and then it will go from maxima will go to 0 then again it will go to 0 go to maxima.

So, one can actually see now the pattern. So, here is the dipole antenna, so you can see that or you can imagine a pen over here. So, we see the maximum intensity or over here maximum radiation in this, and then as we move along we hardly see anything. So, this is the 0 radiation in this side and then this is basically is repeating in this side. So, this is also known as a figure of eight. However, this is just a one plane, in the one plane we are showing. And for H plane pattern you can also think about again. If there is a current carrying conductor which is placed in the z direction. So, where will be the magnetic field? Magnetic field will be given by this finger here. So, this is the (Refer Time: 14:10) you can say electric dipole antenna current and the magnetic field will be like this. So, that is why H field is uniform along this particular plane here.

So now, let us just look at the 3-D pattern. So, here a red color implies maximum radiation and then as you can move along this one here it almost becomes slightly blue which is the least radiation zone. So, from red it is turning to orange, yellow, green, and blue. So, basically intensity is reducing. So, this is the real 3-D plot of a infinitesimal dipole antenna, this is the H plane pattern this is the E plane pattern also known as this one is known as azimuthal pattern, this is known as elevation pattern.

I also want to mention infinitesimal antenna is not at all an efficient antenna. So, please do not try to use infinitely small dipole antenna ever.



(Refer Slide Time: 15:14)

Now let us just see a case of a small dipole antenna. Generally a small dipole antenna is defined between lambda by 50 to lambda by 10. So, that is the length of the dipole antenna. Here again as before x axis y axis dipole is placed along the z axis here and the current is again a not assumed uniform. Now, the current will be 0 over here and it will be maximum. So, if the dipole antenna length is small this will be the triangular distribution. As the length increases reaching to this, we can still think that it is a triangular distribution.

I just want to tell you if the length is increased further; for example, if the length becomes lambda by 2. Now for lambda by 2 the variation will not be triangular, but it will be actually a perfect sinusoidal waveform like this here. Why? If the length this is lambda by 2; that means, the current will satisfy the boundary condition of 0, sinusoidal value here which is maximum and then it is going to 0 and that will be if we know that a sinusoidal waveform let say goes from 0 will go to a maxima then go to 0 and then it will repeat the cycle. So, this distance from here to here will be then lambda by to and from here to here will be if it is sinusoidal it will be lambda by 4.

So, when we discuss about a lambda by 2 dipole antenna will not be assuming a triangular distribution, but we will be assuming a sinusoidal distribution, but any

sinusoidal distribution for a shorter distance can be approximated as a triangular distribution. So now, once it is a triangular distribution, now we can write the current.

Small Dipole – Radiation Resistance	
Small Dipole Current Distribution $ \begin{aligned} l_e(x',y',z') &= \hat{a}_x l_0 \left(1 - \frac{2}{l} z' \right), & 0 \le z' \le l/2 \\ & \hat{a}_x l_0 \left(1 + \frac{2}{l} z' \right), & -l/2 \le z' \le 0 \end{aligned} $	Far Field Region (kr>>1) $ \begin{bmatrix} E_{\theta} \simeq j\eta \frac{kl_{\theta}le^{-jkr}}{8\pi r} \sin\theta \\ E_{r} \simeq E_{\phi} = H_{r} = H_{\theta} = 0 \\ H_{\phi} \simeq j \frac{kl_{\theta}le^{-jkr}}{8\pi r} \sin\theta \\ R_{r} = \frac{2P_{rad}}{ l_{\theta} ^{2}} = 20\pi^{2} \left(\frac{l}{\lambda}\right)^{2} $
Small Dipole Vector Potential $A(x, y, z) = \frac{\mu}{4\pi} \left[\hat{a}_x \int_{-l/2}^{0} l_0 \left(1 + \frac{2}{l} z' \right) \frac{e^{-jkR}}{R} dz' \right]$ $+ \hat{a}_x \int_{0}^{l/2} l_0 \left(1 - \frac{2}{l} z' \right) \frac{e^{-jkR}}{R} dz'$	For $l = \lambda / 10$, $R_r = 2\Omega r^2 \left(\frac{l}{\lambda}\right)^2$ $l = \lambda / 4$, $R_r = 12.3 \Omega$ Dipoles also have reactive impedance

(Refer Slide Time: 17:11)

So, we can write the current now in the triangular distribution form. So, I 0 is not constant anymore it is varying along the z direction. So, this is for 0 to 1 by 2 and this is for minus 1 by 2 to 0; that means, this is above a region and this is below region or the central point there. And so from here again we can find out the value of vector magnetic potential by integrating over the length. However, just to make life very simple if we actually look at we are taking the average over the length. So, we can actually assume very similar to what we had done for the previous case except that now I 0 will become I 0 by 2 which is the average value of a triangular distribution.

So, that is why E theta, E phi all the components remain the same, so the difference is only that now I 0 is nothing but I 0 by 2, so earlier it was 4 pi r now it is 8 pi r. So, otherwise E theta E r remain exactly the same as before. And then we can actually see also similarly H phi component can be there in between other components are 0, and from here if we take the ratio of the two we will still get the equal value of eta which is 120 pi.

Now from here we can actually find out what is the radiation resistance, but before that we can find out the power radiated. I will just go back to slides and the power radiated just to tell. So, what is power radiated is nothing but I 0 square r divided by 2; I will

define what is r also, first just let us look at what is power radiated. So, power radiated can be found out by using this particular expression here, and since there is only one component now so E cross H conjugate will actually be replaced by magnitude of E square divided by eta, because H is nothing but E divided by eta which is 125. So, by using this we can find out what is power because we know what is E and H field; and then power is now given by half I 0 square times R r.

Now, what is this R r? I just want to mention here that R r is actually nothing but a radiation resistance it is not a physical quantity at all, it is something like from circuit point of view. When we say power radiated, power radiated from antenna point of view is a fantastic thing it is radiating in the free space; that means antenna is radiating. But from circuit point of view what we think the radiated power is actually a power lost. So, just a circuit representation is that power loss can be represented as I square r by 2.

So, that is the R r is nothing but it is known as a radiation resistance, but this quantity is not a physical quantity it is just a representation of radiated power which is considered as a loss power from circuit point of view. So, just by integrating that this term comes out to be 20 pi square l by lambda square.

Now I just still want to go back a little bit over from here to the back side. Please remember this expression has a I 0 square. Now here is the radiation resistance which is equal to 80 pi square 1 by lambda, this is for uniform current. And now you might wonder why I did not talk about this here, because in reality in no situation for a dipole antenna for very small a dipole antenna current will be 0 at the end and current will be maximum or triangular distribution. So, in general you should not be using this formula at all, but you can actually say that this value is about 4 times more than the expression which I showed you for a triangular distribution, because I is here increased by 2 times because there it is triangular here it is uniform; so 2 times current increase means 4 times radiation resistance will increase.

But please do not use this particular expression even for infinitesimal dipole, I always recommend people to use this particular expression over here. So, this is the expression to calculate; what is the radiation resistance. So, I have taken a few example here: let us say if the length is equal to lambda by 10. So, if length is lambda by 10 we can substitute

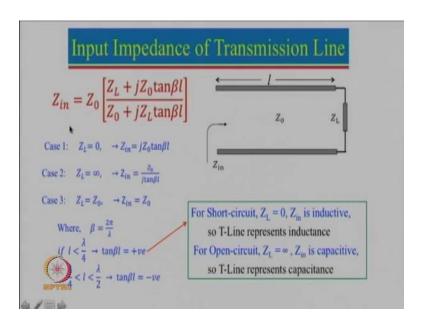
over here lambda by 10, so this term will become actually approximately 1 by 100. Pi square is approximately 10, so 20 into 10 200 by 100 will give rise to 2 ohm resistance.

Now that is a very small resistance, and if we try to feed this antenna with let us say a 50 ohm line well you know that power radiated will be very less, most of the power will get reflected back. So, just for the example I have taken 1 equal to lambda by 4. Please remember lambda by 4 does not fall under the category of small dipole, I have just taken this as an example to show you what will happen if we use this expression here. So, if you use this expression for lambda by 4 simplify that comes out to be about 12.3 ohm. And just for as a discussion again if you think length is equal to lambda by 2; that means if the length is increase by 2 times this should increase by roughly 5 times, so that will become approximately 50 ohm.

But now this is the calculation for radiation resistance, but dipoles also have a reactive impedance. In fact, all the antennas have real part as well as reactive part. So, how do we calculate the reactive part? So, for that what we will do will actually think about again a dipole antenna and just to look at this here. Now there are two arms are there; there is a one arm going up one arm is going down here, and this is actually has to be fed with the current from here. So, what we really need is a, a current going in here which will be let say in this direction and then the current will be going like this. And it has a reverse current here which is going in this, so current will be flowing in this direction.

So, you can actually see that current flows over here and goes like this, and over here current flows from here and goes up here. So, this one here really needs a balanced current here. So, we need a source which has to have a balancing. Now for analysis point of view we can actually think that if it is a symmetrical thing, so this is a plus here there is a minus. So, we will try to analyze this portion over here. So, this portion will be nothing but equal to the length will be now equal to 1 by 2, and there is an open circuit here so we can think about a transmission line which is an open circuit over here. So, by using this concept we can find out what is the reactive part.

(Refer Slide Time: 24:55)



So, for that we will actually take this as a transmission line. The transmission line has a characteristic impedance of Z 0, length is 1 and it is terminated in a load Z L. For this transmission line we know how to calculate input impedance. So, input impedance is given by this particular expression here; I am sure most of you might have studied in electromagnetic waves course or electromagnetic field for a transmission line one can find the input impedance. So, I will just take a few cases here: Z L equal to 0, what Z L equal to 0 implies here? This will be short circuit. So, if it is a short circuit we put Z L equal to 0 here, Z L equal to 0 here. So, this expression gets modified to j Z 0 tan beta 1.

Now this is the case which we are interested at this moment which is Z L equal to infinity. Infinity means this is open circuit. So, right now in our case for a dipole antenna this tip is open circuit. For that we can calculate z input as given by Z 0 divided by j tan beta. You can actually put Z L equal to infinity here Z L equal to infinity here. So, in terms of infinity this will be negligible and again in terms of infinity this term will be negligible. So, these two will be negligible, what we will left with is infinity divided by j infinity tan beta 1 and that is the term coming over here. Now if Z L is equal to 0 that is what we would like from antenna point of view the load impedance should be equal to characteristic impedance in that particular case we can say that if you put Z L equal to Z 0, which is not dependent on the length of the antenna.

So, what is beta here? It is equal to 2 pi by lambda. I just want to highlight here that many books use beta or some books use k also so k is equal to beta is equal to 2 pi lambda. So, let us just take a case when length is less than lambda by 4, because we know that we are talking about its small dipoles. So, length will be definitely less than that. So, for this lambda by 4 if you put here beta is 2 pi by lambda this will be lambda by 4; that means, tan beta 1 will be always positive. And if it is always positive; that means for short circuit if this term is positive this can be written as z input equal to j omega l; that means it will be impacted. That means, any small transmission line which is shorted at the end will represent inductance.

And for open circuit Z L is infinity. So, that we can see from here if it is infinity and this is positive, this will be capacitive. That means, a open circuit line here which is small will have a capacitive thing. So, a small dipole antenna will actually in a reality will have or the small radiation resistance along with a capacitance which is associated with this transmission line.

So, we will continue from here in the next lecture, where we will see that how lambda by 2 dipole antenna is designed, how we can actually even use higher order modes of dipole antenna, how to calculate the half power beam width of the dipole antenna, and then folded dipole antenna.

Thank you very much; we will see you next time.