

Antennas
Prof. Girish Kumar
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Module – 12
Lecture – 58
Reflector Antennas – III

Hello, and welcome to today's lecture on Reflector antennas. In fact, today's lecture is in continuation with the reflector antennas which we were discussing in the last couple of lectures. So, we have started with the plane planar reflector antenna. Basically which was a flat reflector, and in the flat reflectors we talked about two different categories: one was planar reflector and then other one was corner reflector.

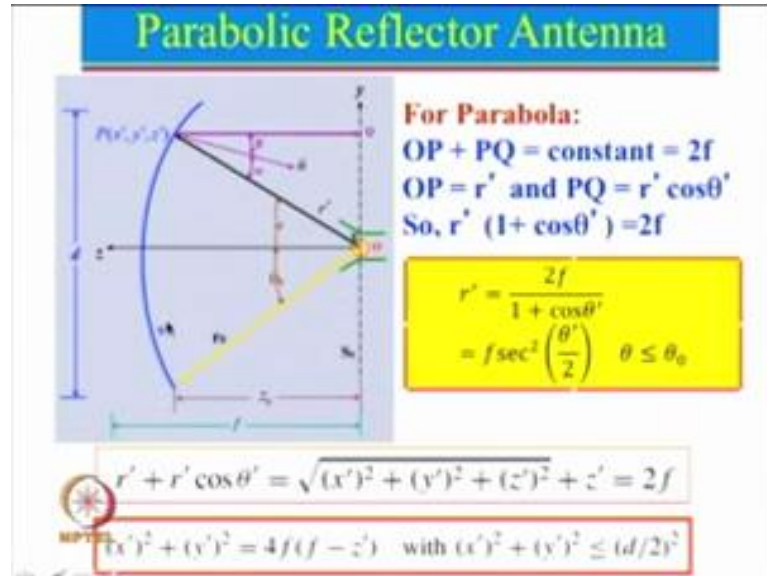
So, for planar reflector we actually looked into if the dipole antenna is kept in parallel with the ground plane or in perpendicular to the ground plane. And we saw what is the effect on the gain and the radiation pattern. After that we looked at different corner reflector antennas and we saw that 30 degree, 45 degree, 60 degree, 90 degree reflector antennas will form different number of images. And number of images is nothing but equal to 360 divided by α minus 1 ; where α is the corner reflector angle.

So, we have seen that for 90 degree corner reflector there were 3 images and for 30 degree corner reflector there were 11 images. And basically if numbers of images are more than that would really imply that the gain of the antenna is large. A typically a corner reflector antenna may give a gain of the order of 10 to 12 dB. But today we will talk about parabolic reflector antenna which can give gain of 20 dB, 30 dB, 40 dB, 50 dB, 60 dB and even 70 dB. And just to look into the number so 50 dB corresponds to 100000, and 60 dB corresponds to 1 million. So, these are the very large values of the gain and of course, it requires very large reflector antenna.

So, let us start with the reflector antenna. And before we start with the reflector antenna please remember one thing reflector antenna majority of the time uses a parabolic dish antenna, and parabola has a property that if we have a parabola and something which is coming from the infinity then the waves after reflecting from the surface will focus at a focal point. Or if we put a feed at the focal point and if it is transmitting then after reflection from the surface it will go and in parallel and it can cover a very large distance

depending upon the fresh transmission equation which we discussed in the earlier few lecture.

(Refer Slide Time: 02:54)



So, today now let us see the parabolic reflector antenna. So this is; what is a parabolic reflector antenna which is shown over here, and we can actually see that there is a feed which is put at the focal point. And the property of the parabola reflector is that if the feed point if you looked at the focal point then the distance from here let us say; to this one up to the surface and reflect back here. So, this distance remains constant for any angle of theta. For example, if we say this is O, this is P and this is Q. So, OP plus PQ is constant, and that would be OP. If you take this point here let us say this is P dash and if this will be Q dash then OP dash plus P dash Q dash will be same as OP plus PQ. And then we can generalise that from here to here that distance is f which is the focal point. So, this plus this will equal to OP plus PQ.

So, for a parabola we can actually write a condition that OP plus PQ is constant and that is equal to 2f. So now, we can actually write this in a slightly different form. So, OP is nothing but let us say we define this distance as r dash and that will be OP is r dash. And what is PQ? PQ is over here. So, if this angle is theta then if you draw the line here this angle will be also theta. So, if this is r dash then r dash multiplied by cos theta will be equal to PQ. So, that is PQ is equal to r dash cos theta dash.

So, we can combine this particular equation as OP is r and PQ is $r \cos \theta$, so we can take r outside and the right hand side will be $2f$. And this can be simplified as r is equal to $2f$ this whole thing comes over here. And from here the simplification comes in the form of; so we know that let us say $\cos^2 \theta$ is given by $2 \cos^2 \theta - 1$. So, this is what it is; so this is $\cos \theta$. So, that will be $2 \cos^2 \theta - 1$ and 1 will cancel. And now $2 \cos^2 \theta$ will be in the denominator so that will become $\sec^2 \theta$ by 2 in the numerator and the two term which came here will get cancelled.

Now, this is the equation of a parabola in the polar coordinate. In fact, many a times people are familiar with more like $y^2 = 4ax$ form that is a Cartesian coordinate, but this is a polar coordinate. So, here a just few other things I want to highlight. So, the focal length here is f here, and this angle which is maximum angle will be $\theta = 0$ that is the angle made by the outer h of the parabola.

Now that if you look at the parabola even though it looks like a line, but if you look from the complete 3D point of view this whole thing will look like a circle. So, we can see that this is nothing but a circle and whose diameter is given by d . So, we can actually say that from here to here distance will be $d/2$.

So if you look at the aperture; the aperture of the parabolic reflector will look like a circular aperture. And for that circular aperture we can also find out what will be the gain. So, what is the gain for a circular aperture, we know that the gain is defined as $4\pi A / \lambda^2$ multiplied by efficiency. So, where A is area; so in this case area will be A is nothing but πr^2 or that will be equal to $\pi d^2 / 4$. So, from that we can find out what is the gain value.

But right now let us just look in to this equation one more time. So, we can expand this equation in the Cartesian coordinate. So, how do we define r plus $r \cos \theta$ that is the left side; so that is defined that r is nothing but square root of $x^2 + y^2 + z^2$. And $r \cos \theta$ is nothing but in this particular thing which is z that comes over here and $2f$ is as before in the right hand side.

Few more things I just want to highlight here. So, here z is measured in this direction, y is measured in this direction which is a y axis, and perpendicular to this will be x axis. So

x is here, y is here and this is z axis. So, now if you take this z dash term on this side and square both the side. So, if you square then square root will go away we will be left with x square and y square, but now z square gets cancelled because this term which is going in this side so this will be now like a minus b whole square which is given by a square minus 2 ab plus b square. So, b here is z dash, so that will be z square on this side; that z square gets cancelled over here. And this term over here will be one term will be 4 f square and then minus of 4 f z dash which is simplified over here.

So now, one can see that this equation is given by x square plus y square which is equal to 4 f times f minus z i. And what is the limiting condition of this? Limiting condition will be when x dash comes at the end, so x dash square plus y dash square will always be less than or equal to d by 2 square. So, this point here you can say if it is somewhere here then; that means we have a at this particular points x dash will be close to you can say 0 and y dash will be also close to 0 at this particular point. But when we look at the extreme and lower here we can say that this will be the equation of the circle which is given by x square plus y square equal to d by 2 square.

Now, this equation can be further simplified and will see what happens in the next slide.

(Refer Slide Time: 09:44)

Parabolic Reflector Antenna Equations

$$\theta_0 = \tan^{-1} \left(\frac{d/2}{z_0} \right)$$

$$\theta_0 = \tan^{-1} \left| \frac{d}{2} \right| \left| \frac{1}{f - \frac{d^2}{16f}} \right| = \tan^{-1} \left| \frac{\frac{1}{2} \left(\frac{f}{d} \right)}{\left(\frac{f}{d} \right)^2 - \frac{1}{16}} \right|$$

$$f = \left(\frac{d}{4} \right) \cot \left(\frac{\theta_0}{2} \right)$$

f/d	0.4	0.5	0.6	0.7	0.8	1.0
θ_0	64.0	53.1	45.2	39.3	34.7	28.1

So, here now we are few quantities to be defined, this is a tan inverse if we take this side tan theta 0 is given by d by 2 divided by z 0. So, let us go back what it is? So tan theta 0 theta 0 is this one here. So, tan theta 0 will be this distance divided by this distance here.

Now this distance we know is nothing but half of the diameter. So, that will be $d/2$, and distance from here to here we have defined that as z_0 over here. So, that will be this distance z_0 .

And now we can further simplify this whole thing. So, $\tan \theta$ you can see that $d/2$ and then z_0 these terms you will need to solve this particular thing, use the equations given in this particular side we know that what is z' ; z' can be simplified in terms of r and $\cos \theta$ and then simplify you can use this particular equation to find out the limiting case. You can see that over here we can put the limiting case. So, if we put the limiting case here this will be let us say at the $h^2 + y^2$ which is nothing but equal to $(d/2)^2$. So, that will be this term over here. And this will be then $4f^2 - 4fz'$. And we need to (Refer Time: 11:13) and z' will be equal to z_0 when we are looking at this particular point.

So, by using this equation finding the value of z_0 from here substituting the values over here and doing little bit simplification we get this particular equation over here. And that is the next step we are basically what has done is simply this has been divided by f/d^2 . If you divide by everything f/d^2 will be left with $-1/16$ here and if you divide here f/d^2 then x will become square, and if you divide this f/d^2 you can see that d one term will go away this is what we will get.

So, what we can see here there is a relation between θ_0 and f/d . So, if f/d is given for a parabolic reflector we can find the value of θ_0 . And this equation can be further simplified and we can write this whole thing as $\tan \theta_0$ over here and that gets simplify to this term. And this term what we can see here that if θ_0 is known then we can find out what is the value of f for a given d , or we can bring this d this side. So, that will be f/d will be equal to $1/4 \cot^2 \theta_0$.

In fact this is the very very important thing, and one should really look at these values because this will help us in designing a reflector antenna. It is very very important to choose a proper value of f/d , as we will see later on. So, let us just see I have just done some calculation here.

So, f/d ; so d comes on this side, so if f/d is 0.4 you can either use this equation and we have given the different values of f/d 0.4, 0.5. And then correspondingly the value of θ_0 has been calculated so that comes out to be 64.43 and so on. Now, the question

is what value of f by d we should choose? Or alternatively what value of θ_0 we should choose? So I will just give a general recommendation. Generally f by d equal to 0.5 is chosen for majority of the practical parabolic reflector which are prime focus reflector. Later on we will also talk about Cassegrain reflector.

And we will see that for majority of the time for Cassegrain reflector f by d is tending closer to value of 1. So let us just see here, let us go back to the last slide and we will see that; what is f by d 0.5 and 50.1 really implying. So, what we are looking at is this angle of roughly 53 degree and that will give the value of f by d . That means, now we should know what should be the diameter and then correspondingly we should choose f by d .

So, this actually the whole thing starts as a design concept. And if you recall in the very beginning when we are talking about the efficiency and we were talking about the fresh transition equation we had also talked about directivity related with the aperture. So, let us just look very quickly in to a design of the reflector antenna.

Now, majority of the time actually speaking the problem starts with the design specification. So, the design specification could be; let us say design of parabolic reflector antenna for a gain of say 40 dB at frequency of say 4 gigahertz. So, that would be the only thing which would be given to a designer, nobody will give you what is the f by d ratio or what you should do where you should feed point; all it is specified majority if the time is well gain is given and you design the whole reflector antenna.

So, this is where the starting point is. So, let us say now we know g is equal to $4\pi a$ by λ^2 , and a which is area and the area what we have to take we have to only take the area of the surface here, not this depth does not come into picture at all as far as the gain equation is concerned. But it does come indirectly in the form of the efficiency. So, let us just re look into it. So, the gain is nothing but $4\pi a$ by λ^2 multiplied by efficiency, area is πd^2 by 4, so gain is now equal to πd by λ whole square multiplied by efficiency.

Now we know the gain we know the frequency, and now we can calculate the value of d . Now of course, we need to know the efficiency, and as we will see a in the next few slides typical efficiency of a reflector antenna can be about 0.7 to 0.8. However, they are some papers which theoretically do claim that you can get an efficiency of close to 100 percent, but those are really very very TDS thing. Right now we will focus on the prime

focus reflector where we can get typically efficiency of 70 to 80 percent and not more than that.

Now this equation which we just said g is equal to πd by λ square multiplied by efficiency. So, that gives us for a given value of gain and frequency we can find out what is the value of the d . It does not talk about f by d . So, how do we calculate and how do we know what should be the value of f by d ? So, this is where I gave the general guideline that f by d typically can be taken as 0.54 prime focus. In fact, the configuration which I am showing you is a prime focus.

So, now what really happens? So, suppose if f by d is small, so what will happen? f by d small that means focal point will be closer. So, the focal point is closer than the reflector will be more of this shape which will have a larger bend, but if f by d is increasing; that means this focal point is changing in that particular case this reflector will keep opening up. So for a larger f by d the reflector will look more like a flatter thing, more like a flat plate; and if you bend like this it looks more like a flat bowl. So, you can think about soup bowl, so soup bowl will have a f by d is close to even 0.5 . And if you look I can soccer plate or plate in which you eat the food and if that is made as a parabolic shape then that flatter plate will have a larger f by d ratio.

As I said majority of the time you are starting can be 0.5 as f by d . However, it actually depends upon what kind of a feed we have used. So, we will now see one by one what is the effect of the feed and what is the effect of the radiation pattern of the feed; which is very very important. So, choosing all these parameters are not independent they really depend on each other, the only thing you can say that diameter finding is relatively easy, if you know the gain if you know the frequency and if you take efficiency roughly say 0.7 or practically as we will see 0.6 or 0.5 people also got it.

In fact, recently we bought one commercial antenna a reflector antenna at 2.45 gigahertz we did not see the specs in detail. And finally, once they think arrived at our place we were doing the testing and we realize the efficiency of that was only 0.3 ; that means we are talking about 30 percent efficient antenna, so which was really a very poor choice to by that antenna. But nevertheless we could do some initial testing because of course it did give us gain and it did give us a directive beam. So, except for that gain reduction we could do other testing with that particular antenna

But that really means that you have to design properly the antenna. So, what are the different factors which govern the efficiency? Let us see one by one these things.

(Refer Slide Time: 20:01)

Gain and Aperture Efficiency of Parabolic Reflector Antenna

$$G = \epsilon_{ap} D_u = \epsilon_{ap} \frac{4\pi}{\lambda^2} A_p \quad \epsilon_{ap} = \epsilon_s \epsilon_t \epsilon_p \epsilon_x \epsilon_b \epsilon_r$$

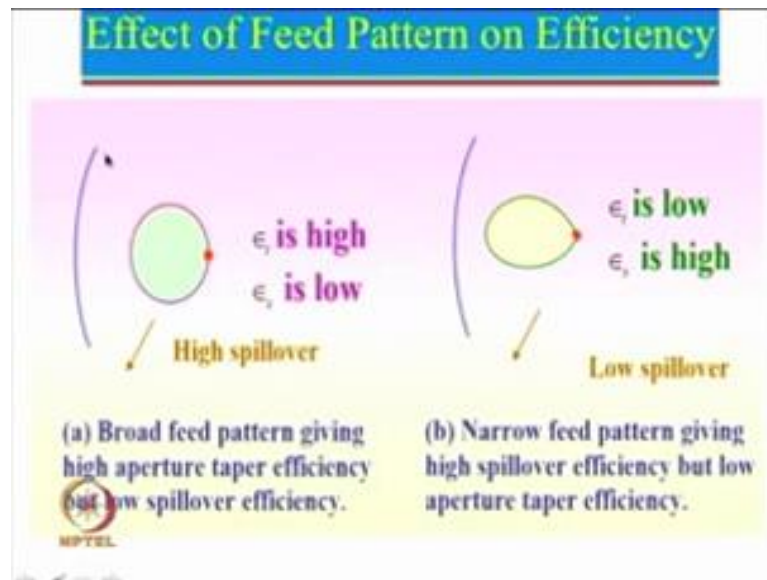
- **Spillover efficiency (ϵ_s)**: fraction of the total power that is radiated by the feed, intercepted, and collimated by the reflecting surface
- **Taper efficiency (ϵ_t)**: uniformity of the amplitude distribution of the feed pattern over the surface of the reflector.
- **Phase efficiency (ϵ_p)**: phase uniformity of the field over the aperture plane
- **Polarisation efficiency (ϵ_x)**: polarization uniformity of the field over the aperture plane
- **Blockage efficiency (ϵ_b)**
- **Random Error Efficiency (ϵ_r)**

So, now gain and aperture efficiency of parabolic reflector antenna: as I mentioned earlier. So, gain is nothing but efficiency multiplied by directivity, and directivity is given by $4\pi A/\lambda^2$. And for a parabolic reflector A will be nothing but $\pi d^2/4$. So, you can see that this d^2 by 4 will get cancelled and this whole thing will become $\pi d/\lambda^2$ multiply this.

Now there are so many things which are associated with the efficiency, this is known as aperture efficiency. And aperture efficiency depends upon so many of this efficiency there are total 6 different efficiency are there. So, let us start one by one; this is a efficiency with suffix s that is known as spillover efficiency. Then epsilon t or efficiency t that is t stands for taper efficiency P is phase efficiency and then x is polarisation efficiency; b is blockage efficiency and then r is random error efficiency.

So, in the beginning we will look at these two in more detail and then we will talk about these efficiencies later on. So, let us first look at spillover efficiency and taper efficiency.

(Refer Slide Time: 21:33)



Now, spill over and taper efficiency depends very strongly on the radiation pattern of the feed element. Here are two examples. So, this you can see parabola reflector here; this is also same parabolic reflector. So, these two reflectors are exactly same. We are going to operate at the same frequency. Now the only difference between these two cases is you can see here, this is the pattern of the feed you can see that this beam is relatively wider compare to this beam here. You can see that this beam is relatively narrower.

So, one can just see that reflectors are exactly same, efficiency frequency is exactly same, but now the feed has different pattern. So, let us see what really happen. So, let us start with this particular thing here. Suppose if you look at this particular feed pattern. So, one can see that the maximum radiation in this direction which will go the reflector and reflect back. Then from here this radiation will go here and reflect back, then it can go over here reflect back, and this part here which is going that will be lost which is also known as spillover.

So, anything which you can see at this particular angle that is nothing but it is spillover. But if you look at this particular side here now, in this particular case you can actually see that s maximum radiation is. In this direction which will reflect back this part is same as this side and, but if you look at this one here at this angle the radiation at this angle is much more compare to the radiation at this angle. That means there will be a larger spillover here that means, there will be high spill over here and high spillover means that

spillover efficiency is low. That means, more power is going in this particular direction compare to this particular case here. So, it has poor spillover efficiency. The other one is a taper efficiency. So, let us see what is happening over here.

Now, in this particular case we have written here it is high and low, but let us see what is that really mean. So, in this particular case one can see that radiation is maximum here, but even if you see at this particular point, if you just imagine a line going from here to this particular point you can see that the amplitude reduction is relatively small compare to this over here. At this particular angle you can see that amplitude has reduced drastically.

So now, what is really happening? So this way which is transmitted it goes here reflects back, it goes over here reflects back. Now you imagine a plane at this particular line here, so you look at this particular plain here same thing we look at the plain over here. Now, we know that the property of the parabola is that the point which goes from here comes back or the thing from here to this particular point, all of them are same distance; so that means the reflected wave at this particular plane will be in the same phase.

However, this particular thing you can see the reflected wave here, it will have a maximum amplitude and this amplitude will keep on reducing. But in this case reduction in the amplitude will be relatively less, whereas in this case we can see reduction in the amplitude will be much larger because amplitude going over here and reflected back is relatively less.

Now recall the array theory; and in the array theory I had also mention that array theory becomes the space theory in a sense that we had seen that array factor actually becomes a space factor for continuous sources. So, I had mentioned that if the number of elements are increased drastically and the spacing between the element is reduced drastically then that would almost become like a continues source.

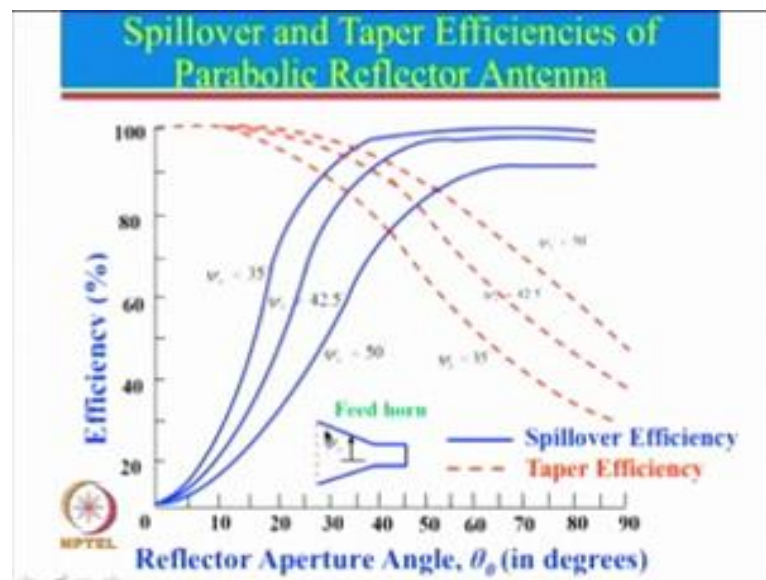
Now, if you recall that what we had discussed about the non uniform taper distribution. We had seen that if there is a uniform distribution that leads to maximum gain, and if there is a non uniform radiation; that means suppose the radiation is a relatively less; that means from maxima to this value here it is relatively less or if it is total cosine function which is 0 to maxima is back to 0. So, we had seen that for a cosine distribution gain was less, for uniform distribution gain was more.

Or we also discuss about cosine distribution over a pedestal where it at in the end it does not go to 0, but it goes to relatively lower value. So, you can see between the two cases: one case is almost becoming closer to cosine distribution and the other case is becoming more closer to the cosine on a pedestal. So, cosine on a pedestal will have a relatively higher gain compare to the cosine distribution.

So, that is what the taper efficiency really implies that in this particular case since the field is relatively uniform or you can say reduction is relatively less, so the reflected wave will have maxima here and amplitude will decrease slightly. But in this case amplitude will maximum here, decrees will be much more over here. That means, if this case taper efficiency will be low, but the spillover efficiency is high.

So, you can see that for the same reflector antenna depending upon the beam pattern in one case this is high this is low, in other case this is low this is high; That means, if you look at the product of these two now which sometimes some of the books defined as aperture efficiency and I just show the plot here quickly.

(Refer Slide Time: 27:59)

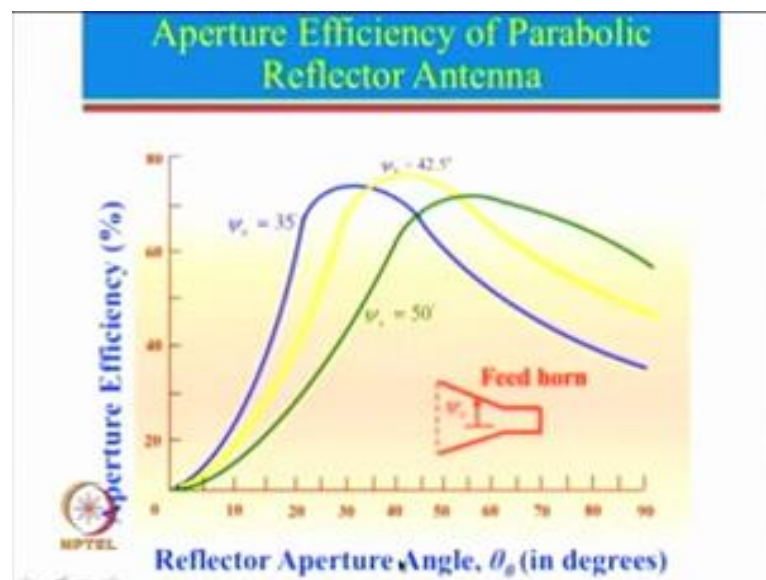


So, here is the point this is the spillover and taper efficiency. So, we have shown efficiency on this side and we were shown reflector aperture angle on this particular side. You can see that this is the angle sign here which is actually denoting the half power beam width of the feed horn, and here we have different plots here. These are the plots for half power beam width of the antenna. So, in one case it will 35 degree then 42.5 and

50 degree. Along this angle this shows angle theta 0. Now theta 0 is varying from 0 to 90 degree and these are the plots over here. One can see this is the spill over efficiency plot and this is the taper efficiency plot over here. And the product of these two in some places they actually say that aperture efficiency epsilon AP is nothing but equal to epsilon s multiplied by epsilon t, and sometimes that 10d to ignore the other four efficiencies but that cannot be ignored, but right now let us just look at the product of the two.

So, if you look at the product of two: this is one this is close to 0 you can see that the efficiency will be very very small. And over here you can see that the efficiency will be relatively small somewhere in this area efficiency will be maximum.

(Refer Slide Time: 29:30)



So, I just show you the efficiency curve here. So, even though it says aperture efficiency please remember, this is only two efficiencies we are talking about epsilon s multiplied by epsilon t. So, one can see that this aperture efficiency is close to 0 here, if the reflector angle is just close to 0 and efficiency relatively you can see is somewhere here this kind of flat which is actually nice for the antenna designer engineer. And you can see that this number is somewhere if I draw the line around 70 percent, you can see that the efficiency is of the order of 70 percent to about 75 percent.

So, we will continue from here in the next lecture. Just to summarise: today we started discussing about the parabolic reflector and so far we have talked about prime focus

reflector, they have many other types of reflector antenna which we will discuss in the next lecture. And then we actually looked into a very simple concept that this is nothing but $OP + PQ$ should be equal to constant, and from there we derive the relation between f by d and θ_0 . And θ_0 is very very important and related to f by d that depends upon the radiation pattern of the feed. And we saw that the gain is primarily determined by the diameter of the parabolic reflector which is but multiplied by efficiency. And efficiency intern depends upon θ_0 and θ_0 intern depends upon f by d .

So, we will see all of these things in the next lecture. And in the next lecture we will try to conclude the reflector antenna and also that will be the conclusion of this particular course also. With that thank you very much and we will see you next time, bye.