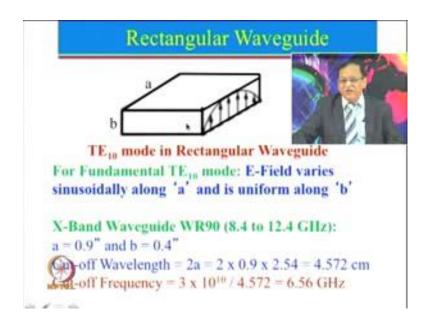
Antennas Prof. Girish Kumar Department of Electrical Engineering Indian Institute of Technology, Bombay

Module – 10 Lecture – 46 Horn Antennas-II

Hello, and welcome to today's lecture on Horn Antenna which is in continuation of the previous lecture. So, in the previous lecture we had started discussing about the E-Plane sectoral horn antenna and before that we actually looked into a rectangular wave guide and we saw what is the field distribution and most of the time we operate these horn antennas in the fundamental mode.

(Refer Slide Time: 00:45)

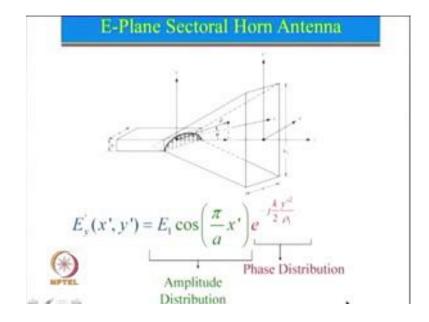


And we have seen that for a rectangular wave guide the field distribution is sinusoidal along the length a and it is uniform along b. And I had also mentioned about them when we talked about E-Plane sectoral when we flair a in this particular direction, we saw that as you keep on increasing the value of b 1 phase errors started increasing and directivity started decreasing. But non lectures also look into the array theory concept of you.

Suppose now we have a field which is uniform over here. So, we can actually think about it is nothing but an array of this particular thing. And along this here field is varying sinusoidally. So, just think about from here to here, let us look into the array theory. In array theory, what we do? We have number of elements here say let say 1, 2, 3, 4, 5 and here there is a sinusoidal variation. Now in the array theory when we are talking about I did mention to you that if the spacing between the element is reduced significantly; array factor for a array was given by sin n psi by 2 divided by n sin psi by 2.

But if number of elements increase and the spacing between the element reduces then that array factor reduces to the sink function which is sin n psi by 2 divided by n psi by 2. So, now, where is these aperture here, what we can actually do we can apply the array theory instead of calling array factor, now we call it space factor. So, we can see that since the field is uniform along this direction. So, for E-Plane radiation pattern side lobe level will be given for the uniform distribution, and side lobe level for uniform distribution we have seen would be of the order of minus 13 to minus 13.5 dB. Whereas, in this plane here since the field is varying sinusoidally and we have seen for cosine distribution side lobe level is less than 20 dB, but of course gain will be also slightly less than this particular plane.

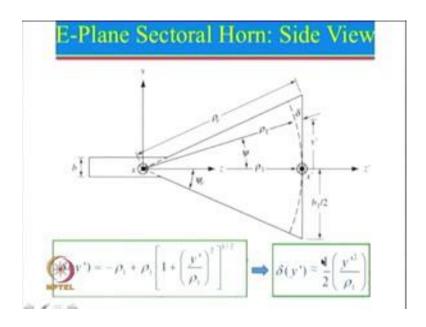
We will remember this particular concept and we talk about E-Field then we will see these things here. So, now, let just very quickly go through what we had discuss in the last lecture.



(Refer Slide Time: 03:14)

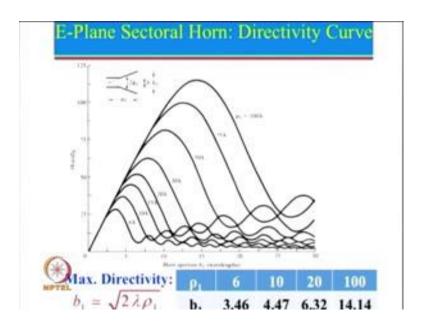
We started with the E-Plane sectoral, we expanded in this particular direction.

(Refer Slide Time: 03:20)



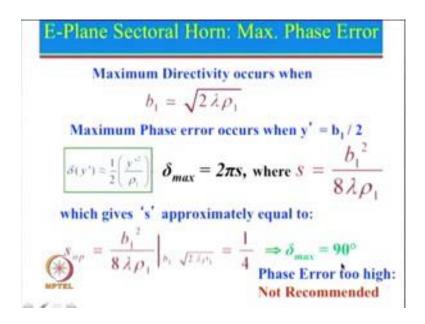
Then we found out what is the phase error.

(Refer Slide Time: 03:23)



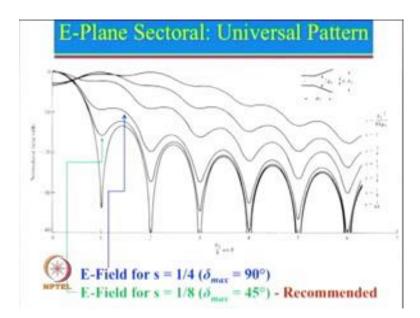
And then we looked at the directivity curve. And we saw that as b 1 increases, directivity increases initially then starts decreasing mainly because phase error is increasing in this particular for these particular values.

(Refer Slide Time: 03:41)



And then we went through the derivation and we saw that delta max 90 degree is the value for which directivity is maximum. However, I also mentioned I do not recommend this much phase error, because this much phase error actually speaking reduces the efficiency of the antenna. So, we generally recommend phase error of about 45 degree.

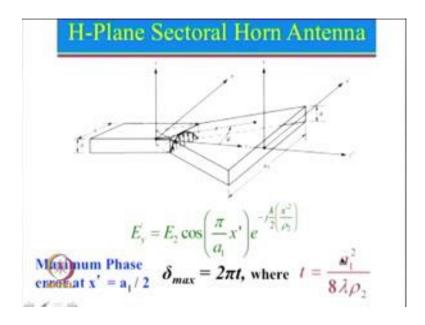
(Refer Slide Time: 04:07)



Then we had looked into the E-Plane pattern in the previous lecture, we will just re look in to a. So, when the phase error; that means, S is equal to very small value which is 1 by 64 here so for that the phase error will be small and this will be the pattern. And you can actually see that this value here if you draw the line here somewhere so that is about minus 13 minus 13.5 dB. So, this is the pattern because, E field is uniform along this direction and that is why we have a side lobe level which is 13.5. But when we look at the phase error of 90 degree, one can actually see over here side lobe level is of the order of minus 10 dB and that is definitely not desirable lot of power gets radiated in this particular direction and hence overall efficiency of the antenna decreases.

If you take delta max equal to 90 degree typical efficiency may be of the order of 50 percent to 60 percent. However, if you take this value here where delta max is 45 degree, we can see for this particular case here you can see that side lobe levels have definitely reduce from here to here, we can see little bit of a dip also; so that means, radiation in this direction is much lesser than radiation compare to this particular value here. So, naturally this will give us a better efficiency. So, I generally recommend this one here and we can get about 70 to 80 percent efficiency if delta max is taken around 45 degree.

(Refer Slide Time: 05:48)

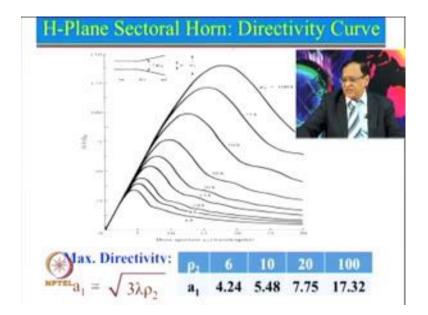


Now, let just look at the next one which is H-Plane sectoral horn antenna. So, what we do over here is that this flooring is done along the H-Plane. We know that this is E-Plane H-Plane is in this direction. So, the flooring is done in this particular direction here. So, in the previous case, b was expanded to b 1, here a is expanded to a 1. Now as far as the field variation is there it will remain same as before, except that earlier it was expanded in this direction here.

So, we can actually see E y that will be the component in this direction. So, this E y, let us amplitude is let say E 2 and the variation is given by this particular term over here. Again when extra which is equal to 0 which is the origin. So, at this particular value, cos 0 will be equal to 1 so we will get a maximum value, and at x dash equal to a 1 by 2 which is at the H is over here this will be a 1 by 2; that means, it will become pi by 2 cos pi by 2 is equal to 0. So, that is the field distribution along this here.

Here again if you look at it, the phase error if we take this as a reference point then the phase error will keep on increasing and the maximum phase error will be obtained when x dash becomes equal to a 1 by 2. In previous case it was y dash equal to b 1 by 2, because clearing was done along this particular direction. So, here also then similarly, we can write delta max to just to differentiate the notation inside of S, earlier we have written here T and rho 1 becomes rho 2 and b 1 becomes a 1 over here otherwise the concept remains the same.

(Refer Slide Time: 07:45)



If you look at the directivity curve we see a very similar trend. So, what we have here? Again now this is the normalized value of the directivity because the dimension b is not changing so it is normalize with respect to that and wave length.

And over here dimension a 1 is changing so that means aperture size is increasing in this particular direction. And what we have here? The length of the horn antenna is increasing. So, from here to here you can see there are different cases, here the length is

100 lambda which would be very very large compared to this which is about 6 lambda. I just think about it suppose you are designing and antenna at say 3 gigahertz. So, at 3 gigahertz wave length will be 10 centimeter. So, if it is 100 lambda that would be this whole length is going to E 10 meter. So, you can imagine, it is a very very long antenna.

At the same time if you think about 3 gigahertz lambda is 10 centimeter, this would imply this is about 60 centimeter which is actually equal to 2 feet long. And now this dimension is increasing. So, we can actually see that as you go over here the directivity increases and then its starts decreasing.

Same thing is followed just as before. So, here again if you look at the point where this is a maximum directivity, so corresponding to again 6 lambda if you not draw the line, this is about 4.24. And if you go over here for 100 lambda if you draw the line somewhere here, it is about 17.32. If you recall earlier, this number was about 14.14 for E-Plane sectoral, whereas for H-Plane sectoral, it is 17.32. And if you draw the line draw from here, if you draw from here, curve which goes through this enter so that will be like an equation which is given by this particular expression.

And now you might wonder why these things are different. Let me just cover the next slide and then I will explain why these numbers are little higher compare to the previous case.

H-Plane Sectoral Horn: Max. Phase Error
Maximum Directivity occurs when

$$a_1 = \sqrt{3\lambda\rho_2}$$

Maximum Phase error occurs when x' = $a_1/2$
 $\delta_{max} = 2\pi t$, where $t = \frac{a_1^2}{8\lambda\rho_2}$
which gives 't' approximately equal to:
 $t_{a_1} = \frac{a_1^2}{8\lambda\rho_2} \Big|_{a_1 - \sqrt{3\lambda\rho_2}} = \frac{3}{8} \Rightarrow \delta_{max} = 4.35^\circ$
Phase Error too high:
Not Recommended

(Refer Slide Time: 10:05)

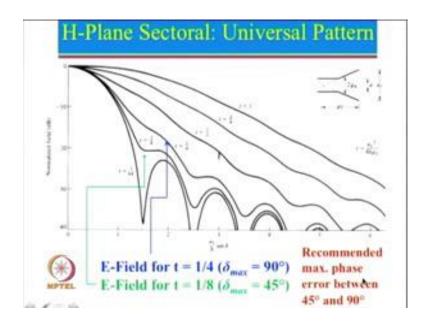
If you use this particular expression, we can find out maximum directivity, we just saw is giving by this expression. We know maximum phase error occurs when x dash is equal to a 1 by 2. So, we know delta max 2 pi t where t is this, so now let us substitute the values. So, t we put over here and here what we are going to do for maximum directivity we know a 1 should be this. So, if you substitute the value here, t optimum comes out to be 3 by 8 and if you put the value of 3 by 8 here and pi, we write in 180 degree. So, 2 into 180 into 3 by 8 gives us 135 degree phase error. And again this is my recommendation that please never ever use this much phase error; phase error is too high not recommended.

Now the question comes; why in the previous case we allowed the phase error of 90 degree for E-Plane and why phase error of 135 degree is allowed for the H-Plane. And the reason for that is that for E-Plane what is happening; field is uniform for the entire region. So, if you look at the center point where the phase is 0 that is the reference and if you look at here this entire amplitude, let say goes to 90 degree and that is not doing contribution. But now if you look at in H-Plane, so in H-Plane, what is happening? Field is varying sinusoidally. So, maximum here and it is 0 over here; and when we expand this whole thing what happens, maxima is also getting expanded, whereas end points are 0. So, at 0 level, there is really no difference, contribution will not be there. Hence we can tolerate little larger phase error because amplitude is going to 0 at the end.

So, even if that 0 if even if that is pointing in the other direction still the middle portion is getting expanded which gives rise to larger directivity. So, theoretically we can say that maximum directivity is obtained for phase error equal to 135 degree. I will tell you my reason also why I do not recommend this value. So, again just look into this here. So, even though at this particular point we say that phase error 135 degrees allowed, but just if you draw the line from here corresponding to this point hard the phase error been 0 instead of getting this value over here for the directivity, it would have gone vertically up and would have gone here. So, it could have been this much directivity which would have been much larger compare to this here if the phase error was relative small or negligible.

Hence I do not recommend phase error of 135 degree at all. And we will see later on what are the implications of these things.

(Refer Slide Time: 13:16)



But let just now look at the radiation pattern which is now as a universal radiation pattern for H-Plane. What we have here? Normalize field in this direction, and this is actually nothing but we are showing the values of a here. And these are the different curves for different value of the phase error. So, this 1 here let us start with here, this corresponds to the very small phase error, you can see that this is a factor is much smaller over here. And now let just see this particular here, if you look at the side lobe level that if I draw the line here you can see it is much below 20 dB.

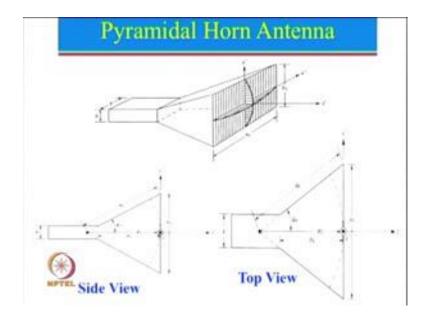
In fact, if you recall now for a uniform distribution we saw the side lobe level was much higher that was close to 13 dB, but if it is a cosine distribution side lobe level are of the order of minus 22 dB or so and these things are very similar to the array theory which you convert to space factor over here. Now corresponding to this here what do we see? So, here phase error is very very small, this is the curve for which you can see that the phase error is 45 degree, and then we see this curve here this is the curve for phase angle error equal to 90 degree.

If you see over here compare to this you can see that the amplitude has increase significantly; that means much more power is being radiated in the undesired direction. And in fact, 135 degree phase error will come somewhere in between this curve and this curve here, because this is 1 by 4 this is 1 by 2 and 135 degree corresponds to 3 by 8

which curve will be in between here. So, you can see that radiation will be taking place in the undesired direction and hence over all directivity of the antenna decreases.

So when you are going to design a horn antenna please remember for E-Plane sectoral horn, horn antenna try to design for a phase or of about 45 degree, for H-Plane sectoral horn antenna I can recommend anything between 45 degree to 90 degree phase error depending upon what are the requirement for the given application, but do not take beyond that.

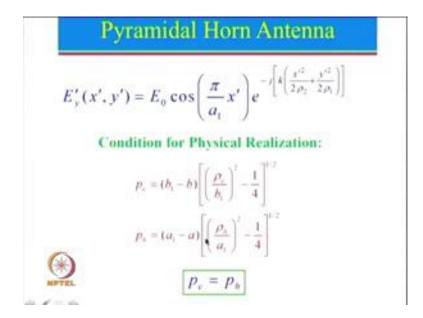
So, now we will see the combination of E-Plane sectoral and H-Plane sectoral and that is known as pyramidal horn antenna.



(Refer Slide Time: 15:57)

Let just look at the pyramidal horn antenna. So, in the pyramidal horn antenna the expansion is done in both the planes. So, this is also expanded this is also expanded. So, this is the side view; side view you will see only this portion which is over here and the top view if you see that will be this part here. So top view we are seeing from a and this is the expansion over here.

(Refer Slide Time: 16:29)



So, now, if you look at the field distribution; field distribution will actually have the similar amplitude distribution because the field variation is only in the one direction which is a cosine distribution. Again we can put it if x dash is equal to 0 cos 0 will be 1 so that will be E 0. And that x dash equal to a 1 by 2, this will be pi by 2 which is equal to 0. But however, if you see the phase error; phase error has increased much more now because what you see now that the phase error now is the sum of the 2 phase errors. So, in one time what we are doing? We are going in the let say in a direction and then we are going in the b direction. So, the phase error will be much more over here for pyramidal horn antenna.

And then comes the next part condition for physical realization, in fcat actually there is a only 1 condition even though you see here 3 equation the actual condition is that P e should be equal to P h where P is given by this and P h is given by this expression here. So, let us what are these things here. So, if you look into here; we actually call it distance from here to here there is actually known as the distance from aperture, aperture is also referred some times in books as mouth of the horn antenna and this one over here is known as meth of the horn antenna.

We actually what we are looking at here is the distance from here to this point so that is you can call it a neck and this is the mouth. So, neck to mouth distance for this case as well as for this case should be same. Now I want to mention here few things here. So, it is this distance from here to here and this distance from here to here that should be constant. Whereas, what we are earlier looking at is the phase center; now phase center for this particular case is different in the phase center in this case here. The reason for that is here the floor angle over here may not be sane as floor angle in this particular direction. The reason for that is we can actually have a larger phase error in the H-Plane and we can have a lesser phase angle error in the E-Plane, because the amplitude is uniform along this. So, that is why since the floor angle will be different phase center will be slightly different in the 2 cases.

However, for physical realization from here to here and from here to here it should be same. So, we can just say it looks like a P, but actually it is rho E and rho H. So, rho E and rho H should be same. Now how do we calculate the expression for rho E.? So, I just want to mention I will give you some hint here. So now, let say this is the angle here, so for this particular angle, if you look this part here, one triangle and you need to imagine this as a another triangle. Tan alpha is given by this distance divided by this distance here. So, this distance is nothing but b 1 by 2 divided by this distance here which has been taken as rho 1.

Also the same thing can be written for this here that this tan alpha of this will be this distance which is b by 2 divided by this distance here that gives you one expression. And now we know that distance from here to here will be nothing but rho 1 minus this particular distance. And this distance we have seen the tan of this angle will be b by 2 divided by them. So, simplify this equation for this term which is rho E; similarly you do the same thing simplify it for the term here rho H and then put this condition over here; and these are the expanded thing. You can see that basically these terms are coming as I said you need to just to little bit of a calculation use that concept of tan angle and then take the 2 ratios simplify it you will get this particular expression. So, this is the only condition which we need to put for physical realization.

Now we need to design let say horn antenna.

(Refer Slide Time: 21:09)

	Alternatively	
Directivity of Pyramidal Horn Antenna can be	$G_0 = \frac{1}{2} \left(\frac{4\pi}{\lambda^2} a \right)$	(b_1, b_2)
obtained using Directivity	$a_1 = \sqrt{3\lambda\rho_2} \approx \sqrt{3\lambda\rho_n}$	$\rho_2 = \rho_0$
curves for E-and b H-Planes	$\rho_1 = \sqrt{2\lambda\rho_1} \approx \sqrt{2\lambda\rho_r}$	$\rho_{\rm f} = \rho_{\rm c}$
	$p_r = (b_t - b) \sqrt{\left(\frac{p_r}{b_1}\right)^2 - \frac{1}{4}}$	

For designing a horn antenna there are 2 important things which we need to know: first is, what is the frequency at which we are going to operate? It is very very important to know at what frequency we are going to operate horn antenna, because we know that let say are rectangular wave guide will be expanded to pyramidal horn. So we must know; what is the cut off frequency of the rectangular wave guide and what is the operating frequency range of the wave guide. Until and unless we know that we cannot just choose arbitrary.

So, for example we had seen that for H band which is from 8.4 to 12.4, any frequency in that particular band 8.4 to 12.4, we can design pyramidal horn antenna in that frequency band and for that we need to take w r 90. Similarly suppose you want to design antenna at say 30 gigahertz or you want to design antenna at 1 gigahertz you must choose proper wave guide which will have that operating frequency range; then comes the next part once frequency is given, so we choose the wave guide dimension.

The next part will be we need to design horn antenna for a given gain or directivity. There are 2 possibilities are there: I have given the directivity curve for E and H-Plane which where the normalize E and H-Plane. So, we can use that to find the overall directivity pyramidal horn. So, if you look into this here so we saw the normalized plot which were lambda d E by one of the term here and then lambda d H by then other term. So, this pat is actually or ready known from those code directivity curves and then we can find out what is the directivity.

Alternatively we can find the directivity of the horn antenna from its aperture. So, what is the aperture? So, aperture area theory if we apply it says 4 pi area multiplied by lambda square. And at the aperture what is the area of the aperture a 1 multiplied by b 1. If you look at the directivity expression over here, the length of the horn antenna does not come into picture. So, one may think that where directivity is independent of the length, but that is not the correct thing.

And we can see here that the gain of the horn antenna is almost half of this particular value. So, if you look in to their, this means that the efficiency of this particular antenna is only 50 percent. And I actually want to mention all those optimum dimensions which we had seen they really give raise to an efficiency of only about 50 percent, but I also want to mention that some books do take instead of 0.5 they take as 0.6 also. So, some books take this 60 percent efficiency some people or some books take 50 percent efficiency, but we show you much later how to get even better efficiency.

But right now I will go with this text book approach, and we will see how things can be simplified and designed. So now, we had seen for a optimum pyramidal horn antenna for first we had looked into the sectoral. So, we have seen for E-Plane sectoral, this was the optimum dimension, for H-Plane sectoral that was the optimum dimension. Now again here an assumption has been made that rho 2 is approximately equal to rho H which is not always true, this will only happen if the aperture size is relatively small compare to the length of the horn antenna.

When we talk about the length of the horn; the length is actually taken from the neck to the mouth. And this we had seen these are the values of the rho E and rho H these values as I mentioned these have been obtained for the physical realization of the horn antenna. Now what we have here equation 1, 2, 3, 4, 5, if you combine these equations and by combining these equations one actually gets equation over here. Now that may look like a fairly complicated equation over here, but just 3-4 steps that will take for you to combine the equation you can actually just see here what needs to be done. We have a 1 b 1 here, so now a 1 b 1 is given by this particular term over here and this term has been approximated by this here. So, these terms over here we can actually take square of this

everything. So, a 1 square b 1 square; square root will get rid of it and then this term here is given by the term over here and this one comes over here. So, you can actually see that all of these things will come in this particular expression.

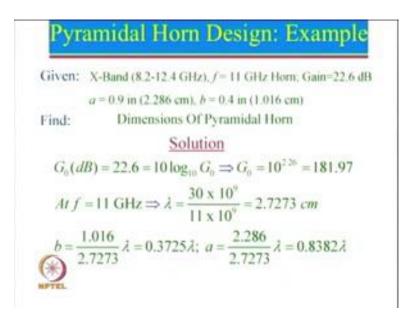


(Refer Slide Time: 25:41)

And then what you can see over here that term psi or you can say x also for simplicity if you see here x is in the numerator and over here x is in the denominator that is how it has been arranged. And where the expression x is it is nothing but rho E divided by lambda you can see that. So now, what we need to do with this we need to solve this equation for I will just use the term x here even though this is known as psi; so we need to solve for x and for that what we need to do we need to start with the trail.

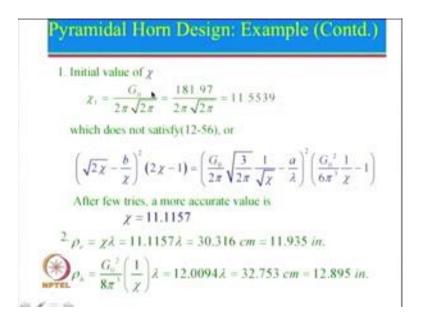
And this is the good trail equation, so if you try with this equation it is possible that this left hand side may not be equal to right hand side so you may have to do some trail. So, I am just going to just show you very quickly, but in the next lecture I will show you more detail. So, if you calculate this from here the trail value then you can calculate rest of the parameter.

(Refer Slide Time: 27:33)



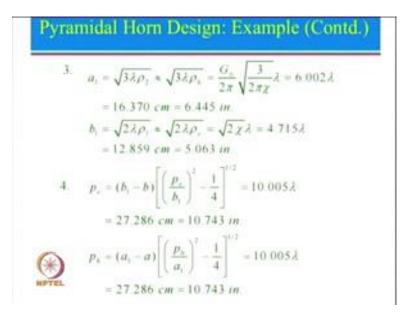
There are some design, equations are given there is a design for 11 gigahertz is given for a given value of the gain over here. So, that is the solution.

(Refer Slide Time: 27:44)

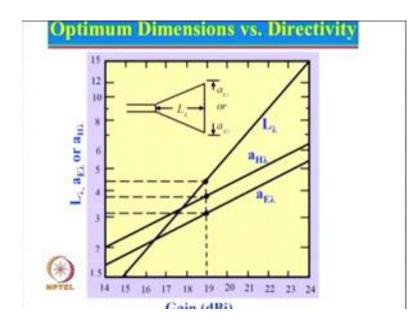


I am just very quickly going to show you the trailer of the next lecture.

(Refer Slide Time: 27:52)



(Refer Slide Time: 27:54)



So, you can choose starting value of x then you do this you do all f these calculation and finally, you can do the design. Alternatively if you actually read the cross book, in the cross book this graph is actually given over here. So, for any value of the game which is over here draw the vertical line and read the corresponding values design is over.

It is a very very simplified way of doing the thing; however, I want that you people look in to the derivation from this part. First I would like that you people do some practice drive this particular expression for this condition and then derive this particular expression from the previous equation and in the next lecture we will continue from here.

So, with that just to quickly summarize. Today we restarted with the E-Plane sectoral horn antenna, we looked at the field distribution and we saw that for very phase small phase error the side lobe level will be of the order of minus 13 dB or so, but as the phase error increases side lobe level increases and efficiency decreases.

Then we looked at the H-Plane sectoral horn antenna, and we notice that because of the cosign distribution side lobe levels were below 20 dB and phase error can be little larger compare to the E-Plane sectoral antenna. And then we talked about the pyramidal horn antenna, and there is a 1 main condition for physical realization you can say that for E-Plane sectoral or H-Plane sectoral the length from the mouth to the neck should be same. And then I showed you the equation and very quickly I showed you how graphically you can solve the problem also.

But in the next lecture we will see these things in more detail, till then bye do some practice because as this say 'practice makes man perfect'. I still do not know why they do not say- 'practice make woman perfect'; in fact I said that some time and one lady responded that 'women are perfect so why mess with the perfection'. But I would like to say 'practice make people perfect'. So, please do some practice, read this examples from the book and we will continue from here in the next lecture, till then bye.