

Antennas
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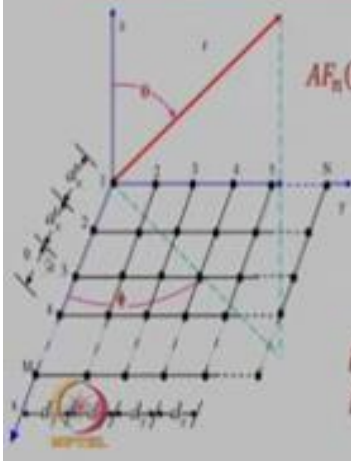
Module – 04
Lecture – 18
Planar Arrays

Hello everyone and welcome to today's lecture. Today I am going to talk about planar antenna arrays. In the last few lectures we have been talking about linear antenna array and in those different cases we had discussed about for example, we started with uniform amplitude array then we had looked at non uniform amplitude array, then we also looked into what happens if the phase between the different elements change. So, the beam changes from broad side towards the end fire direction. And also we looked into how the gain gets affected. Because as you change the amplitude of the different elements. So, yes side lobe levels reduce, if we use for example, cosine distribution or triangular distribution, but gain also reduces. So, those are the different things which we talked about.

Now, today we will discuss about how these linear antenna arrays can be made into planar configurations. So, let us start with this planar antenna array.

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Rectangular Planar Array



$$AF_n(\theta, \phi) = \left(\frac{1 - \sin\left(\frac{M}{2}\psi_x\right)}{M \sin\left(\frac{\psi_x}{2}\right)} \right) \left(\frac{1 - \sin\left(\frac{N}{2}\psi_y\right)}{N \sin\left(\frac{\psi_y}{2}\right)} \right)$$

where, $\psi_x = kd_x \sin\theta \cos\phi + \beta_x$
 $\psi_y = kd_y \sin\theta \sin\phi + \beta_y$

$\beta_x = -kd_x \sin\theta_0 \cos\phi_0$ for $\psi_x = 0$
 $\beta_y = -kd_y \sin\theta_0 \sin\phi_0$ for $\psi_y = 0$

So, in the planar antenna array, we had discussed little bit about in the last lecture. We will continue from there. So, let us assume that we have an array which has m number of elements along x axis and we have n number of elements along y axis. So, the total number of elements in this plane will be m multiplied by n . Now the analysis of this can be thought in a very simple way that. What we do at is we take these linear arrays along the x axis the elements are from 1 to m . So, it is a linear axis with uniform spacing of d x all along now all of these elements 1 to n can be combined with a single expression and that is given by it is array factor $1 + m \sin m \text{ by } 2 \psi_x$ divided by $\sin \psi_x \text{ by } 2$. So, now, we can apply the pattern multiplication to find out the array factor of the combined array.

So, what we do the first step is that consider 1 to m elements as now a single element placed over here whose array factor is given by this term. And now all of these m elements come over here, this one comes over here, this one comes over here and now what we do we apply the array factor for n element which is now along y axis. So, if we put the array factor of this that is the array factor for the elements along the y axis. So, the total array factor for this planar array will be product of the 2.

Now, the difference from an previous array and this here is actually in the term ψ_x and ψ_y earlier we had only ψ and we had only one access here we have 2 access. So, for example, let us say we want to find out the far field act at point r . So, this particular distance is r and we take a projection along the x y plane. Now since this angle is θ when we take projection of this one x y plane, that projection will result into θ is here. So, that will be $\sin \theta$. So, you can see that the term $\sin \theta$ has come into picture. Now when this projection has taken over here, if you want to consider the element along x axis, this angle is ϕ we multiplied this by $\cos \phi$. And that is why we have term $\cos \phi$ for y axis element we take the projection along with this here. So, we multiply by \cos this is $\cos \phi$ this will be now $\sin \phi$. So, that is the term coming over here.

Now, β_x and β_y are same as δ which is a phase difference. So, here β_x is the phase difference along x axis and β_y is the phase difference along y axis. So, now, we can find out if we want to know in which beam direction we want the maxima to be. So, normally what we do we put ψ_x equal to 0 ψ_y equal to 0. So, we put that 0 over here and if we put ψ_x equal to 0, we can write β_x as minus of this term over

here and β_y is given by this term. So, now, let us see we want the beam direction in particular angle. So, to start with let us say if we want to be maxima in the broad side direction. So, in the broad side direction θ_0 will be equal to 0, and $\sin 0$ is equal to 0. So, β_x will be 0 and β_y will be 0 so; that means, all the element should be fed with same phase then the beam maxima will happen in the broad side.

Now, suppose we want the beam maxima at let us say an angle 10 degree from the broad side. So, θ_0 will be 10. So, we put that value over here that will be $\sin 10$ and let us say we want 5 to be let us say 20 degrees; that means, the beam maxima will be now in this ϕ direction along with θ . So, then these are the ϕ_0 values. So, from here we can find out what should be the desired value of β_x and β_y for the given values of θ_0 and ϕ_0 . So, that is the simple way and one another thing. So, let us say point here let us say we start with the 0 here. So, then this element here element number 2 shown along x axis will be β_x then this will be 2 β_x this will be 3 β_x and so on. Similarly, elements along m axis will be the phase will be here β_y then 2 β_y then 3 β_y and 4 β_y .

Now, let us just look at this point here, what will be the phase at this particular point. So, this will be β_x coming from here and β_y coming from here. So, that will be β_x plus β_y . If you move along this axis here this point, this has travelled β_x and this is 2 β_y . So, that will be the phase difference of this particular element. So, that is what the principle of the phased array is that we by changing the phase of β_x and β_y we can change the beam direction to any desired direction of θ_0 or ϕ_0 .

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Rectangular Planar Array

$$\tan\phi_0 = \frac{\beta_y d_x}{\beta_x d_y}$$
$$\text{and } \sin^2\theta_0 = \left(\frac{\beta_x}{kd_x}\right)^2 + \left(\frac{\beta_y}{kd_y}\right)^2 \quad \text{where } k = 2\pi/\lambda$$

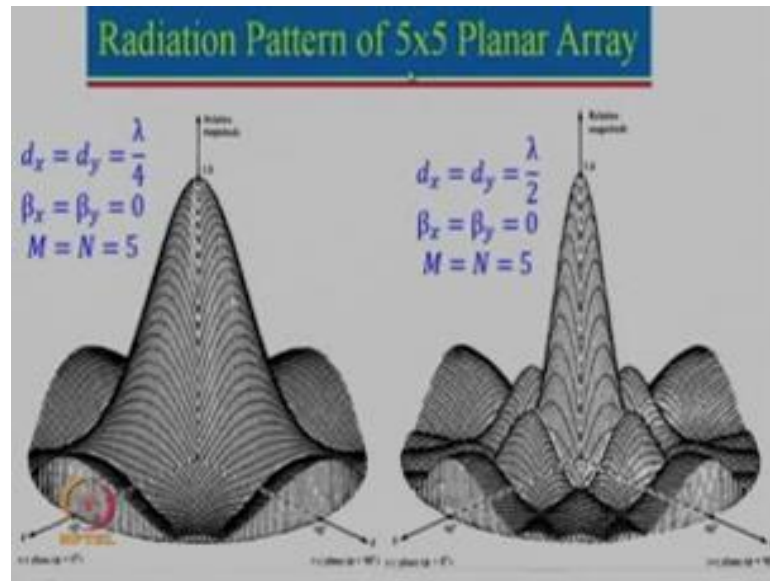
The principal maximum ($m = n = 0$) and grating lobes can be located by:

$$kd_x(\sin\theta\cos\phi - \sin\theta_0\cos\phi_0) = \pm 2m\pi \quad m = 0, 1, 2, \dots$$
$$kd_y(\sin\theta\sin\phi - \sin\theta_0\sin\phi_0) = \pm 2n\pi \quad n = 0, 1, 2, \dots$$

Now, suppose if beta x beta y is given in certain case. So, in that case you can calculate and phi 0 and theta 0 from the previous equation, if we take the ratio of the 2, if we take the ratio this term will get canceled with this here and we can actually see that sin divided by cos will be tan phi 0, that is the expression given over here. So, beta x and beta y are known we can calculate what will be the principal maximum. Now we can find the value of theta 0 from here. So, if we know that sin square phi 0 plus cos square phi 0 is equal to 1. So, we square both sides and then add those terms that give rise to the term sin square theta 0, which is given in the form beta x beta y and other terms are k which is 2 pi by lambda d x is distance along x direction, d y is distance along y direction.

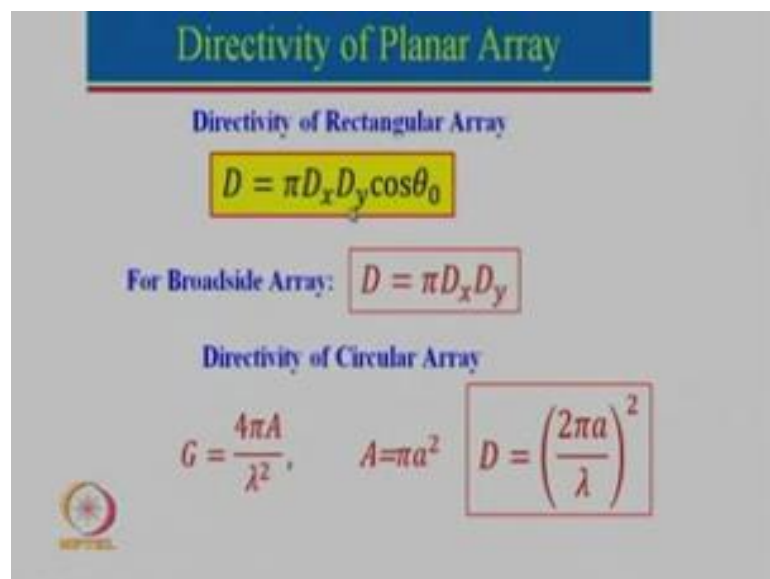
Now, the principal maximum can be obtained when we put m equal to n equal to 0 and we can put over here, but the grating lobes can be obtained when m and n are equal to 1. And we should try to avoid grating lobes as much as possible. So, we should try that this condition should never ever happen.

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So, let us just take an example of 5 by 5 planar array. So, this is a m equal to n equal to 5 because array size is 5 by 5 and beta x beta y is given they are equal to 0 so; that means, the beam maxima will be in the broad side direction. The 2 cases which are shown over here, distance between each element is lambda by 4 here distance between each element is lambda by 2. So, if we look into this here the total aperture will be more and if the total aperture is more beam width will reduce. Here total aperture is less. So, beam width is wide. So, this will give us less gain this will give us more gain.

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So, directivity of a planar array can be calculated. If know use the concept of the earlier. So, directivity along x axis which is a one linear array directivity of the elements along the y axis, so that will be d_y , the total directivity can be formed by $d_x d_y$ where $\cos \theta_0$ is the angle which is measured from broad side. So, for broad $\cos \theta_0$ will be equal to 1. So, that will be the directivity term. So, that is broad side director.

In the same we can find out the directivity of circular array. So, we know that for circular array the concept is aperture area theory says d or g if we assume d is equal to g for 100 percent efficiency it is given by $4 \pi a^2 / \lambda^2$. And for a circular array area will be πa^2 . So, we can just say directivity is given by this particular expression.

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Hexagonal Array – 7 Elements

Example: Calculate the array factor of a 7-elements hexagonal array (2 elements in first and third rows, 3 elements in the second row).

$$AF_c(\theta, \phi) = \left(\frac{1 \sin\left(\frac{M}{2} \psi_x\right)}{M \sin\left(\frac{\psi_x}{2}\right)} \right) \left(\frac{1 \sin\left(\frac{N}{2} \psi_y\right)}{N \sin\left(\frac{\psi_y}{2}\right)} \right)$$

$$\psi_x = \frac{2\pi}{\lambda} d_x \sin\theta \cos\phi$$

$$\psi_y = \frac{2\pi}{\lambda} d_y \sin\theta \sin\phi$$

Total Array Factor = Array Factor of (Group 1 + Group 2)

So, now let us just take some special cases. So, here we have taken an hexagonal array which has total 7 elements. And these 7 elements are placed like there are 3 elements placed over here, and the 2 elements are upside 2 are in the downside. And the term here hexagonal array is used that this distance between this element and this element that is exactly equal. So, this is equal to $\lambda/2$ this is $\lambda/2$. So, all the dimensions are $\lambda/2$ or in a general case the dimensions can be deal. This can also be thought in another way equilateral triangle. So, this all these 3 dimension are equal. So, this is also known as equilateral triangular array or hexagonal array.

So, now we want to find out what is the radiation pattern or array factor of this one. Again we can think this whole thing as a planar array and if we apply the similar concept which we just studied for the rectangular array, we can apply the same concept. So, these 7 elements can be broken into 2 groups. So, group 1 consists of 3 elements here and group 2 consists of 2 by 2 array over here. So, if we can find out individually array factor for group 1 and group 2 then the total array factor will be sum of these. So, let just see now what will be the pattern.

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AF of Hexagonal Array - 7 Elements

Array factor of Group 1: $M=3, N=1$

$$AF_1(\theta, \phi) = \left\{ \frac{1 \sin\left(\frac{3}{2}\psi_x\right)}{3 \sin\left(\frac{\psi_x}{2}\right)} \right\} \quad d_x = \frac{\lambda}{2}$$

Array factor of Group 2: $M=2, N=2$

$$AF_2(\theta, \phi) = \left\{ \frac{1 \sin\left(\frac{2}{2}\psi_x\right)}{2 \sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1 \sin\left(\frac{2}{2}\psi_y\right)}{2 \sin\left(\frac{\psi_y}{2}\right)} \right\} \quad d_x = \frac{\lambda}{2} \quad d_y = \sqrt{3} \frac{\lambda}{2}$$

Total Array Factor = $AF_1 + AF_2$

So, array factor for group one m is 3 n is equal to 1. So, we simply use the previous expression 1 by m sin this function here. So, we substitute m equal to 3 we got that here and what is d x is in this case, d x is the distance along an x is shown here as in this side and y is x s this side here. So, along x the distance is lambda by 2. So, d x is equal to lambda by 2.

So, now array factor of group 2 which is nothing, but 2 by 2 array. So, both m and n are equal to 2, we substitute the value and what is d x, d x is equal to lambda by 2 which we can see over here. And what is the distance along y axis since this is an equilateral triangle the height of the equilateral triangle will be square root 3 by 2 times d or the length here. So, the total will be nothing, but square root 3 times lambda by 2 and that is what we put here the total array factor is nothing, but sum of these 2 array factor. So, this is the one of the easiest way to find out the array factor.

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Hexagonal Array – 19 Elements

Example: Calculate the array factor of a 19-element hexagonal array (3 elements in first and fifth rows, 4 elements in the second and fourth rows and 5 in the third row)

$$AF_n(\theta, \phi) = \left(\frac{1 \sin\left(\frac{M}{2}\psi_x\right)}{M \sin\left(\frac{\psi_x}{2}\right)} \right) \left(\frac{1 \sin\left(\frac{N}{2}\psi_y\right)}{N \sin\left(\frac{\psi_y}{2}\right)} \right)$$

$$\psi_x = \frac{2\pi}{\lambda} d_x \sin\theta \cos\phi$$

$$\psi_y = \frac{2\pi}{\lambda} d_y \sin\theta \sin\phi$$

Total Array Factor = Array Factor of (Group 1 + Group 2 + Group 3)

Now, we just took 7 elements. Let us say if we take larger array now this is a 19 elements array here. So, where what we have we have 3 elements then 4 elements then 5 elements then 4 then 3 now all of these elements are actually forming an equidistance. So, what we have here all these are distances are lambda by 2, and then these distances are also lambda by 2. In that I just want to tell you. So, I had designed this 19 element array using a circular micro strip antenna way back in 1985. So, more than 30 5 years back I had designed this array not just for isotropic element, but for circular micro strip antenna.

So, let just see now how we can do the analysis, first we need to do the grouping. So, here what we do we make a one group here and that group is of 5 elements, 5 elements into one array. Then we have a group 2 which consist of 4 elements along x axis and 2 elements along y axis. So, that is the 4 by 2 array. Then the group 3 group 3 consists of these 3 elements now do not include the second time here this is already included in the 5 by 1 array. So, here we have these 3 element and these 3 element. So, that is the 3 by 2 array. So, now, all we need to do it is find out the individual array factor for each group 1 2 3. So, let see what we have.


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AF of Hexagonal Array – 19 Elements

Array Factor of Group 1: $M=5, N=1$ $d_x = \frac{\lambda}{2}$

$$AF_1(\theta, \phi) = \left\{ \frac{1 \sin\left(\frac{5}{2} \cdot \frac{2\pi}{\lambda} \cdot d_x \sin\theta \cos\phi\right)}{5 \sin\left(\frac{\psi_x}{2}\right)} \right\}$$

Array Factor of Group 2: $M=4, N=2$ $d_x = \frac{\lambda}{2}$ $d_y = \sqrt{3} \frac{\lambda}{2}$

$$AF_2(\theta, \phi) = \left\{ \frac{1 \sin\left(\frac{4}{2} \cdot \frac{2\pi}{\lambda} \cdot d_x \sin\theta \cos\phi\right)}{4 \sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1 \sin\left(\frac{2}{2} \cdot \frac{2\pi}{\lambda} \cdot d_y \sin\theta \sin\phi\right)}{2 \sin\left(\frac{\psi_y}{2}\right)} \right\}$$


So, array factor of group one we have 5 elements and one element. So, all we need to do it is put m equal to 5 and what is d x that is equal to lambda by 2. Then we have a second group where we had a 4 element let just look back here. So, 1 2 3 4 along x axis and we have a 2 elements along y. So, m is equal to 4 n is equal to 2 we put 4 over here and we put 2 over there and then we need to know what is d x. So, d x is equal to lambda by 2 in this side and what is d y that is this distance here, which is square root 3 lambda by 2 that is all we put at there that is for group 2.


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AF of Hexagonal Array – 19 Elements (Contd.)

Array Factor of Group 3: $M=3, N=2$ $d_x = \frac{\lambda}{2}$ $d_y = \sqrt{3} \lambda$

$$AF_3(\theta, \phi) = \left\{ \frac{1 \sin\left(\frac{3}{2} \cdot \frac{2\pi}{\lambda} \cdot d_x \sin\theta \cos\phi\right)}{3 \sin\left(\frac{\psi_x}{2}\right)} \right\} \left\{ \frac{1 \sin\left(\frac{2}{2} \cdot \frac{2\pi}{\lambda} \cdot d_y \sin\theta \sin\phi\right)}{2 \sin\left(\frac{\psi_y}{2}\right)} \right\}$$

Total Array Factor:

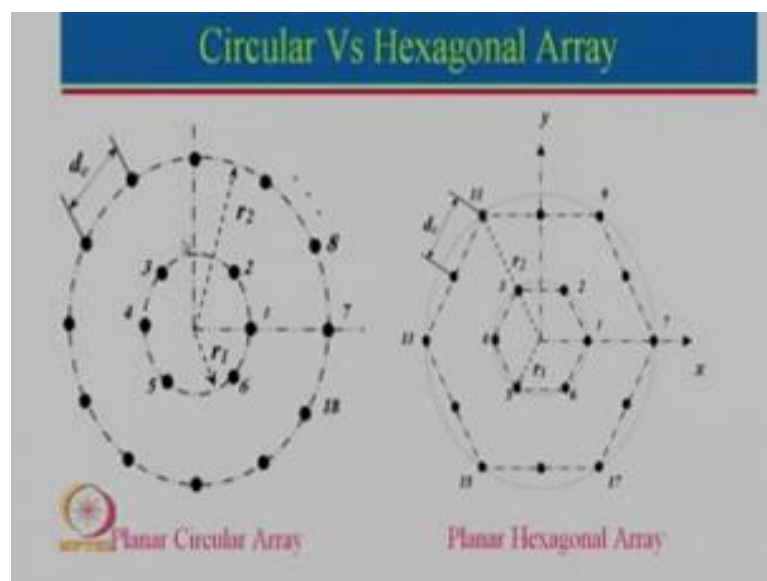
$$AF_{Total} = AF_1 + AF_2 + AF_3$$


Now, we need to find out for group 3. So, for group 3 now the dimension is 3 by 2. So, we put m equal to 3, we put n equal to 2 again d_x remains same which is λ by 2, but d_y now is square root 3 times λ , why you can see that the distance is from here to here. So, this one here each one of them you can think about double of this. So, which will square root 3 λ .

So, all we need to do put these terms over there, and that will give the array factor for this here. So, we got array factor. So, you can think other way round also there are total 19 elements. So, what we have here 6 here 6 plus 8 14 14 plus 5 19. So, that takes care of all the elements.

So, same thing now we have got all the array factor which takes care of 19 elements. Please do not multiply because that will not give a correct answer we have to add array factor of individual terms.

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So, now we actually can look at 2 different arrays one is hexagonal array which we just covered and then another is a planar circular array. If we just look at these 6 elements of the elements along the circular axis then this one if you look at hexagonal they are exactly same so; that means, the radiation pattern or array factor of hexagonal array and this circular array will be exactly same, there will be no difference.

Now, sometimes the center element is also placed here. So, correspondingly we can put a center element here, but now for the next one that is the outer arcs above see just to tell. So, there are 6 elements here each element is at 60-degree angle. Now there are 12 elements. So, each will be at a 30-degree angle. So, if you now try to compare this with an hexagonal array which is the outermost here. So, we can see that these elements are exactly same as along the circle. It has these elements here in between they are not really the same as the circular array.

So, now there are 2 ways to do it, one is we can need to find out let us say the array factor for this here. So, one way is that circular array it is defined in terms of Bessel function. So, you can write this particular function in the form of a Bessel function those expressions are available in the text books which I had mentioned Balanis and the cross up, but; however, the other elements which are missing over here. So, what we can do? Either we can use the Bessel function concept or we can consider these elements and for this element and this element we can consider them separately and now we can add them together.

So, one concept if you have learnt properly which is this concept of the array factor here you can apply that to any general array and you can actually consider these 2 as another element array for the other one you can consider which are not part of the hexagon. So, you can use any of these technique to find out the overall radiation pattern or array factor of course, now for all these cases what we have assumed that all of these are isotropic elements, but in reality there will be practical element array. And I also want to just say that let us just look at a little bit of a design also.

How do we design these particular axes? So, let just go back here little bit and look at this array over here. So, for example, now there are 2 ways the problem can be defined one is known as a analysis problem. So, in the analysis problem the problem will be given as there are let say 4 elements on this side and then 8 elements on this particular side. So, you find out what is the overall directivity. So, for 4 element you can find out what is the directivity using the formula for linear array. Then for this 8 element you can find out the directivity for this and then you can use the combined directivity formula which is given over here and that way you can find the directivity. Now this is a analysis problem then there is an another thing which is known as a design problem. So, in the design problem, generally what it will be said that design and array which actually along

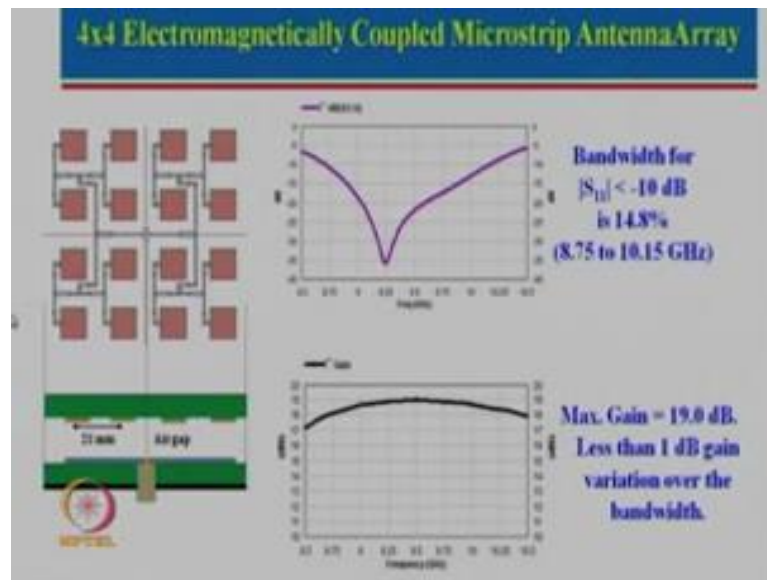
x axis it should give me let us say beam width of say 30 degrees. And it may say that along this axis it should give me a beam width of 10 degrees.

So, now we need to design a rectangular array which should give me a beam width of 10 degrees in this side and let us say beam width of 20 degrees on this side. So, now, you can apply the concept of the linear array which I have given you the expression. So, you know that for a linear array a beam width relationship is there with the total length. So, from that you can find out the total length and then you can see what are the number of elements required you can use as a starting point you can take d as $\lambda/2$, but you can always adjust that. So, instead of 0.5λ you can take 0.6λ or 0.7λ or may be 0.45λ , but not more than 0.7 or 0.8 .

So, now you might wonder if I take more than 0.7 , it will still have a no problem of grating lobe, but actually speaking that is true for x axis and y axis. Let see what happens along the diagonal axis here. So, if I take this as let say 0.7λ and this I take as 0.7 and the diagonal length will be now square root to times this here which is almost coming to λ . So, we may satisfy the condition along x axis or y axis, but along 45 degree I will have a problem of grating lobe. So, generally speaking we do not try to take more than 0.7 for this spacing here. And that is the way hexagonal array does help if we look at an hexagonal array which we just looked at. So, over here now this distance is $\lambda/2$ this distance is $\lambda/2$ or if I take this as a d then this is d . So, I can still take this 0.7 and this takes still take 0.7 I do not have that problem of 45 degrees here.

So, many a times hexagonal arrays are preferred and as I mentioned that I had designed this particular array way back in 85 and I did use a micro strip antenna. So, let me just give you a little simpler array, so that you can see how these things can be practically used here.

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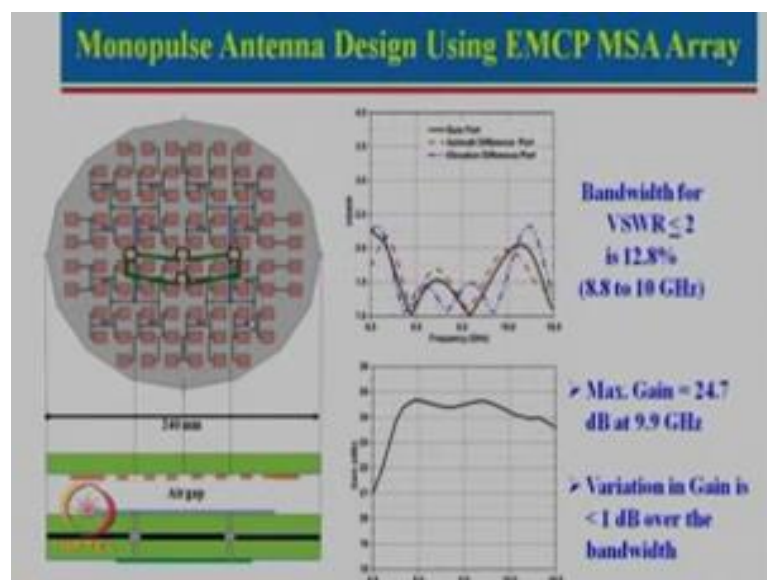
So, I have actually included an example of a 4 by 4 electro magnetically coupled micro strip antenna. I know there are a lot of new terms have come, but let just see what are each thing over here. So, what we have here this is actually a rectangular micro strip antenna and just underneath this also there is another rectangular micro strip antenna. So, these 2 antennas are electromagnetically coupled. So, what we have here there is 4 elements are this side 4 elements are this side. Now we need to excite each of these elements. So, what we have done here is we have used a speed network here. So, let us say these 2 elements. So, there is a feed network over here and when there is a feed network here that excites these 2 element and 2 elements. Then we combine this feed network and then we bring over here.

Now, the same that 2 by 2 set is repeated here as well as repeated here as well as over here. The feed network is symmetrical for this side also and this side. And now what we are doing? We are connecting these feed network like this here for this also we have connected here like this and then we are connecting the feed point and then feeding at this particular point. So, you can actually think that this is my input the power is getting divided into 2 ports here, then this one here is further divided into 2 ways then this is divided into 2 ways then this is further divided into 2 ways. So, that is how 2 by 2 array is built up one by one, because it is electromagnetically coupled. So, these 2 patches are exciting each other it results into a very broad band antenna.

So, one can actually see over here that this antenna is designed in the x band. And what we have got here S11 less than minus 10 dB which is approximately equal corresponds to VSWR equal to 2. So, what we are getting a percentage bandwidth of 14.8 percent and if you look at here that is 1.4 gigahertz bandwidth that is huge bandwidth by using this particular configuration. And also the next important thing is that this 4 by 4 array gives us a gain of about 19 dB maximum. And notice that the gain is fairly flat over the desired bandwidth. In fact, there is a less than 1 dB gain variation over the desired bandwidth of 8.75 to 10.15 gigahertz.

In fact, many a times there are more restrictive sense are there for example, people instead of designing a reflection coefficient for 10 dB they demand even a reflection coefficient for less than 15 dB. So, of course, for that case what will happen the bandwidth will reduce slightly compared to this here, but let us say if the required bandwidth is only say a few hundred megahertz, we can even get a reflection coefficient less than even 20 dB and we get a very flat gain over the bandwidth. So, it is a very nice concept. So, then you might wonder what is this micro strip antenna array and all that. So, after this planar antenna array we will talk about in detail micro strip antenna.

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So, I will have included a one more example here which is a very typical mono pulse radar antenna application and this particular thing we have just recently done. So, what it actually consists of just from the concept point of view.

When we discuss micro strip antenna arrays then will talk about this thing in more detail, but just to tell you what are the applications here. So, here what we have this is a circular dimension which was given to us and that is a 240 mm and in that diameter we have to fit whatever maximum we can get. So, we had actually done that 4 by 4 array. Now that 4 by 4 array completely could not have fitted over here. So, you can see that this actually consists of 4 quadrants. So, is a one quadrant second one third and fourth one here.

Now this is also a concept of mono pulse comparator, where what we do we actually feed one quadrant here and then second third fourth and we take the signal from these 4 quadrant and we generate some pattern and we generate difference pattern. So, by creating this sum and difference pattern we can actually identify the location of the target. So, what we have tried to optimize here that we optimize the area which is available to us. So, if you actually look at the central point here what we have done. So, there were 1 2 3 4 we had a space. So, we added another element here which takes power from this side. And we did not have a much space here. So, we have a 3 element here.

Now, as we mentioned earlier that when we are talking about linear array. That if you use uniform amplitude side lobe levels are relatively high minus 13 dB. But here what we have done. So, we have taken power from the outer one which gives here. So, this actually provides a amplitude taper and the center is maximum it goes down. And that this particular array also gives us the reduced side lobe level and let just look at the VSWR plot for all these thing here. So, what we have got here 8.8 to 10 gigahertz that is nearly 13 percent bandwidth for VSWR less than 2. And for this element we have got a maximum gain of about 24.7 dB. And again in this particular case here if you see the gain variation is less than 1 dB.

So, in the next lecture, we will start talking about various micro strip antenna and just to summarize what we have done today. So, we started with the planar rectangular antenna array now of course, rectangular array will become square if m is equal to n . Then from that planar array rectangular planar array then we discuss about hexagonal where the all the elements are located along the triangular grade, and then we talked about a circular array and we saw some similarities in circular array and hexagonal array and then we took some practical examples of micro strip antenna array. So, we looked at a 4 by 4 array and then we looked at the real application, where these are actually used in a mono

pulse radar system. So, thank you very much will see you next time and will talk about micro strip antenna in detail.

Thank you, bye.