Antennas
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Module – 04
Lecture – 17
Linear Arrays-III

Hello, in the last lecture we looked in to the linear array. And for the linear array we saw how the maximum radiation can be calculated in the main beam direction as well as how the radiation can be calculated in the minor lobes which actually are also known as side lobe level. We also looked at the expression how to find out the null direction and then how to calculate the first null beam width. Then we looked into the different thing and now the let us just look at the again one more time the radiation pattern of the array.

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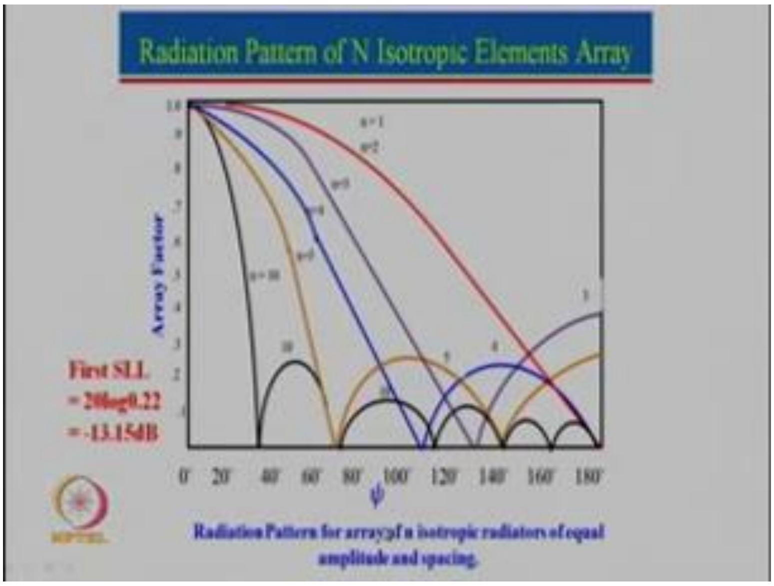

So, we can see here that the array is reducing down to a certain value, but there is a possibility that if we increase the psi value to a greater value.

For example, if psi increases beyond 180 degrees it goes from 0 to 180 and then it becomes let us say 360 degrees. At psi equal to 360 again the radiation will be maximum. So, if that value happens somewhere in this domain for some other value of psi that can give raise to grating lobe. So, we should always try that psi should not become 2 pi. If psi becomes 2 pi, then again will have a maximum radiation. So, that maximum radiation is known as grating lobes.

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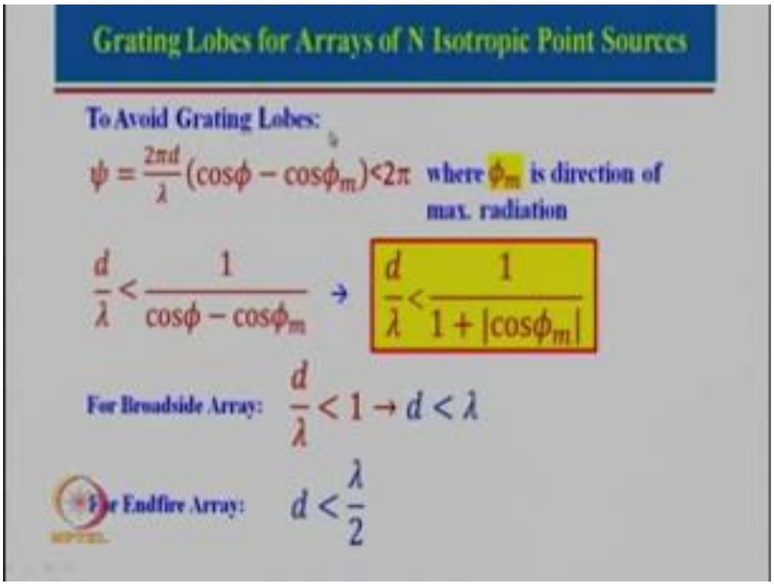

So, we need to avoid the grating lobes in all the situation. So, we have seen that psi is given by this particular expression. And this should be less than 2 pi and where phi m is the direction of maximum radiation, just to recall if phi m is equal to 0 degree that will be an end fire array and when phi m is equal to 90 degrees that will be broad side array. So, over here the condition is that psi must always be less than 2 pi. So, if we simplify this, we can see that d by lambda should be always less than this term over here.

Now, the maximum value of cos phi, as phi varies from 0 to 360 degree can be minus 1. And over here the maximum value of this again can vary from 0 to 1. So, we can say that if d by lambda is always less than, this which is the maximum of the numerate denominator. So, that will be 1 plus the amplitude of this here. So, if this condition is satisfied psi will never ever become 2 pi for any given value of phi m. So, that leads to a condition. For broad side what is phi m? Phi m will be 90 degrees, cos 90 is 0. So, d by lambda should be less than 1. So, if you are designing a broad side array d can should be less than lambda, but if we are designing a end fire array in that case phi m is 0, cos 0 will be 1. So, this should d should be less than lambda by 2.

And this is an important thing that for end fire array d must be less than lambda by 2 and for broad side d should be less than lambda. And if d is equal to lambda by 2, it will give raise to the grating lobs. In fact, we can actually look at the very first example of the 2 element which we had seen that in this particular case.

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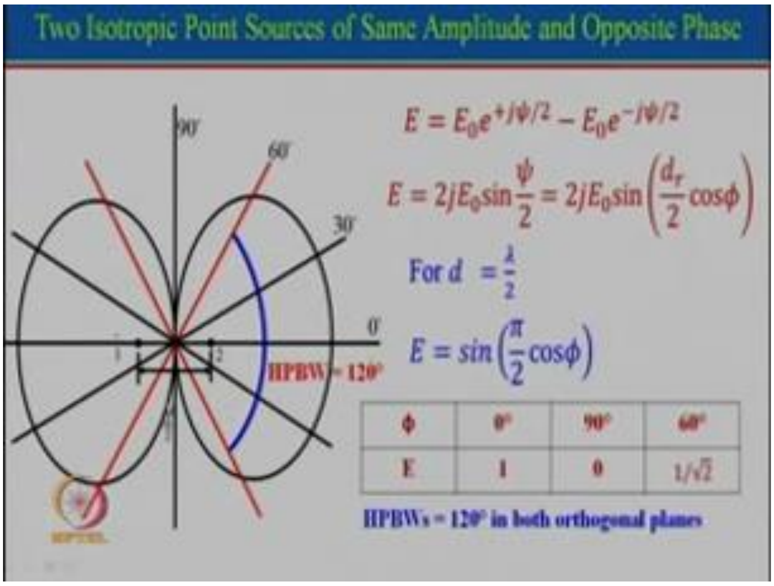

If you see that this is the end fire array. And the distance taken was d equal to lambda by 2, and that is why there is a grating lobe on this direction. Had we taken d less than lambda by 2 then this grating lobe would not have occurred. So, you have to remember that that d must be less than lambda by 2 for end fire array and if you are scanning the beam. So, let us the beam is being scanned to say 60 degrees. So, cos 60 will be 1 by 2 and that will give rise to that this is 1 by 2. So, that will be 2 by 3. So, d by lambda should be less than 2 by 3 for a beam maximum to be at 60 degrees. So, that is why majority of the time, when we design a broad side array invariably we take d as may be 0.5 lambda or 0.6 lambda or 0.7 lambda, but never close to lambda.

Similarly, when we design a end fire array, majority of the time we take d less than 0.25 lambda or may be 0.3 lambda, but never close to 0.5 lambda.

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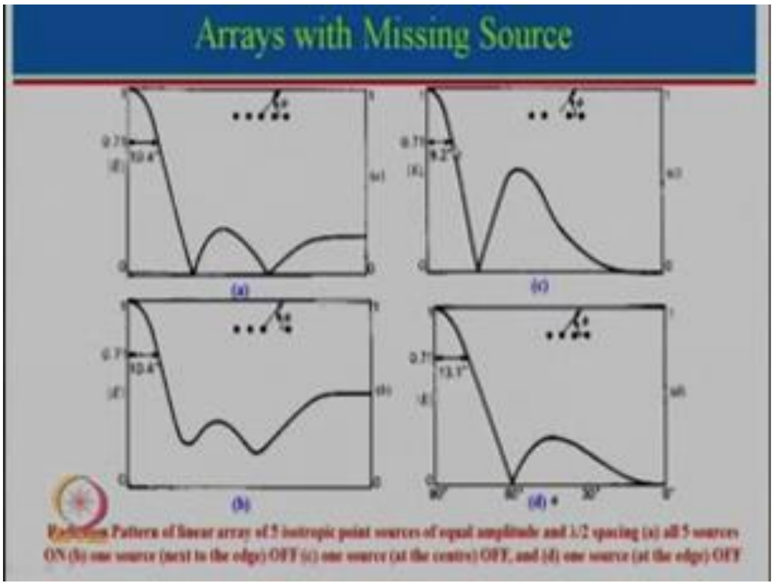

So, now let us see what happens when one power to one of the element is missing. So, here is the case when all the 5 elements are spread with equal amplitude and equal phase, and in this particular case just to tell. So, there are 5 sources each source is space at lambda by 2, the 5 elements. So, from here we can just look at. So, this array factor is very similar to the array factor which we have derived earlier, and for this case you can see that the beam here is going to at 0.71, at after a 10.4 degree here this angle is varying from 0 to 90 degree. So, half power beam width will be double of this here.

Now, instead of all the 5 elements, let us say that if one of the element here is not getting the power. So, array length still remains the same. So, one can see interesting that, this particular beam width remains almost the same, but what is the difference side of this might the nulls are almost disappeared, and we have a side radiation which is actually fairly high. So, if you try to calculate directivity always using the expression d is equal to 4 pi divided by theta e theta h, it is not a good idea you should also look at what is happening to this. So, compared to this pattern here directivity of this is actually higher compared to this because, unnecessarily radiation is going in the undesired direction. So, over here, now there is a different situation the central element is not fed, but all other elements are fed.

Now, if central elements is not fed let see what is happening. This one has reduced from 10.4 to 9.2, but the side lobe level has increased drastically. So, again if we blindly apply the concept of that half power beam with reduce, directivity will increase. Then this will give us a wrong answer. In fact, the directivity here is reduced compared to this here why? Because lot of power is getting wasted in this particular minor lobe which has increased significantly, over let just pay little attention here. So, what we are doing we have putting 1 1 1 0 1 1. So, the central element is 0 what we are noticing side lobe level increases.

So, now think other way round. So, if the central element is fed more power then what will happen? Side lobe level will decrease. And over here half power beam width actually reduces. So, but if we use more power here than half power beam width will increase, we will see in the next slide, what happens if we actually use an non uniform amplitude, but let just take another one last case here, that the last element is not fed. And when the last element is not fed, this whole thing is equivalent to 4 element array. And we know that for 4 element array the half power beam width will be more, and there will be just one complete side lobe. And that you can see from the array plots. So, let us just now see what happens if we use uniform amplitude or we use non uniform amplitude.

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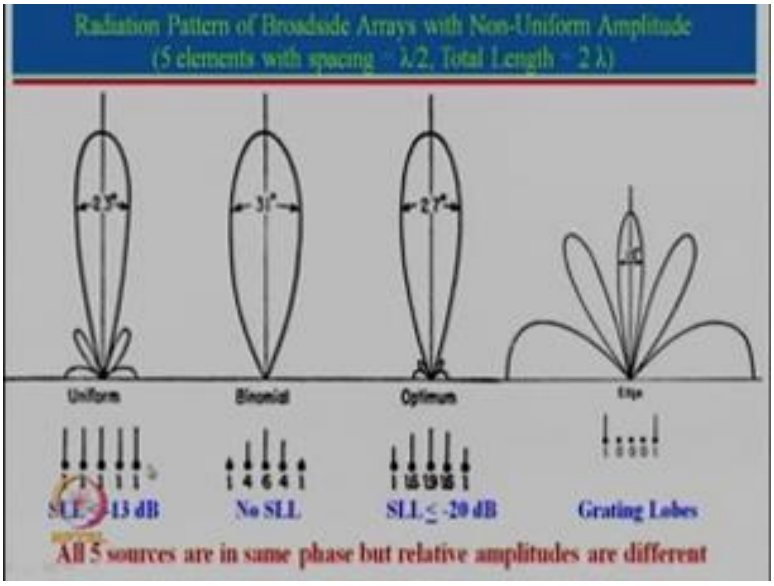

So, again let us take a case here there are 5 elements are there. So, for these 5 elements a equal amplitude is fed and the spacing between the element is lambda by 2. And if the spacing is lambda by 2 total lengths will be 2 lambdas from 1 to 5 elements. So, for this case we can see that this is the radiation pattern. And the side lobe level here would be less than minus 13 dB. If we use binomial distribution, what is binomial distribution we will see in the slide, but over here let us just look at this distribution here where we are feeding at 1 4 6 4 1; that means, the amplitude fed over here is 6 times more than the end element. And in this case if you see there is absolutely no side lobe level. So, that is a great thing.

But there is a difference is that the half power beam width increased from 23 degrees to 31 degrees. So, even though there is a no side lobe, but just because half power beam width increased. So, drastically the gain of this antenna is relatively less. And I want to tell you here except for 3 element binomial practically binomial distribution is not used. So, generally a practical case is somewhere over here, even the terms is optimum in this case here the levels are 1, 1.6, 1.9, 1.6, 1 so; that means, the central element is approximately twice of the power fed compared to the end element. And in this case you can see side lobe levels have reduced which is less than minus 20 dB, but the half power beam width here is slightly more than this one here, but it is less than binomial distribution.

So, gain of this will be slightly less than this, but slide lobe levels are improved further. This is the just one case you can actually think more like from a grating lobe point of view, if only 2 n elements are fed what really would happen. It is like spacing between the element is now equal to 2 lambdas. And we have seen that for broad side grating lobes happens if the spacing between 2 elements is lambda, but here it is 2 lambdas. So, there are 2 lambdas now, there are 2 grating lobes which have come.

So, generally speaking this is not desired except for an application where you probably want to send maximum beam in multiple directions, which is very few applications require that. So, in general we should never ever have a situation, where spacing between the element for broad side is more than a lambda. Otherwise it will give rise to the multiple lobe. So, half power beam width may look narrow, but the directivity is much less.

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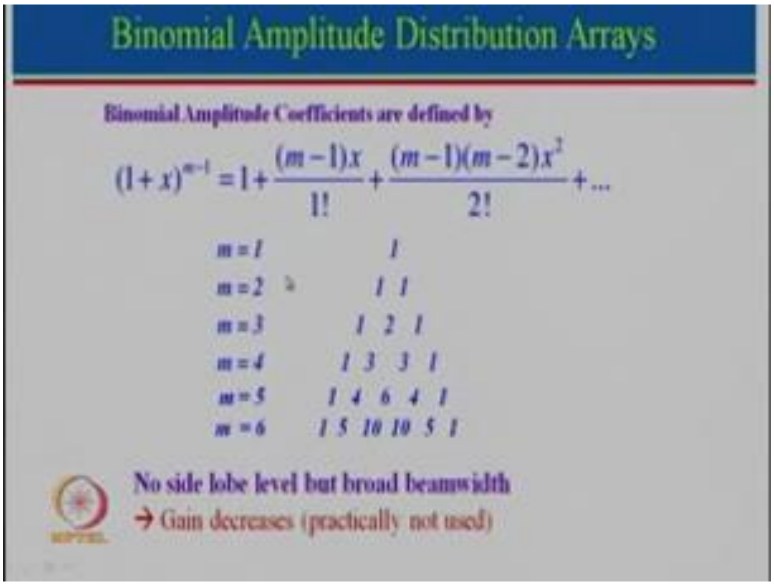

So, how do we calculate binomial amplitude distribution? Well it is something like 1 plus x to the power m minus 1. So, if m is one there is only one element then m 2, 3.

You can actually think about something like a plus b square. What is a plus b square equal to, a square plus 2 a b plus b square? So, a square coefficient is 1 2 a b coefficient is 2 b square coefficient is 1. If you look at a plus b cube, it is a cube plus 3 a square b plus 3 a b square plus b cube, but one can actually also do the derivation in a very simple manner. You can actually make like concept of the tree here. So, what we do here 1 and 1 you add put 2 here and side 1 goes there over here 2 plus 1 is 3 2 plus 1 is 3 put that in between end elements are one next here 3 plus 3 is 6 3 plus 1 is 4 3 plus 1 4 here. Going down further 6 plus 4 is 10 6 plus 4 is 10 4 plus 1 is 5.

So, now think about if I have to go to even a larger element. Suppose if I go to 7. N equal to 7 this it will be then 10 plus 2 20 will be here. 10 plus 5 15 will be there. If I go next it will go to 35. So, the amplitude ratio changes drastically, and that is why power fed to the end element is very small. And if the end elements are not getting power it is almost equivalent to having a lesser number of element. Yes, we do not get any no side lobe levels are there, but beam width becomes very bad gain decreases and hence practically binomial distribution is not used.

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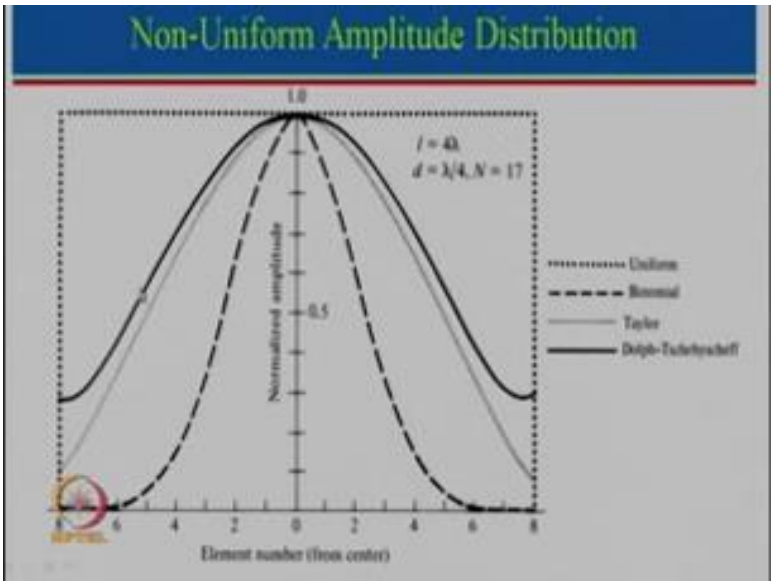

So, what are the different plots here let us just look into it. So, here is an non uniform amplitude distribution for different cases, but first one here is uniform; that means, all elements are fed uniformly. What we have taken here total number of elements 17. So, here are central element 8 elements on the right side, 8 elements on the left side. And the spacing here taken is lambda by 4. So, the total length of the array is 4 lambdas.

So, for uniform all 17 elements are fed with equal amplitude. A binomial distribution which we had just seen in the previous one, if we take that binomial distribution that distribution will look like this. Now you can see that the last 3 elements 8 7 6, this side and 8 7 6 the power fed to these element is almost close to 0. So, it effectively works more like a 13 element array, instead of 17 elements. And that is why the directivity of binomial distribution is very poor. So, now, these are the 2 extreme cases, we have 2 in between cases here. So, we have a Taylor distribution which is shown like this here, just like a Taylor series. And this is the Dolph Tschebyscheff distribution here, just like Tschebyscheff filter, which has maximally flat response. So, these are the different cases here. In fact, if you try to approximate you can also approximate this as a roughly a triangular waveform with some pedestal. This can be almost thought off as a sinusoidal waveform with some pedestal. So, there is a similarity between triangular and cosine function.

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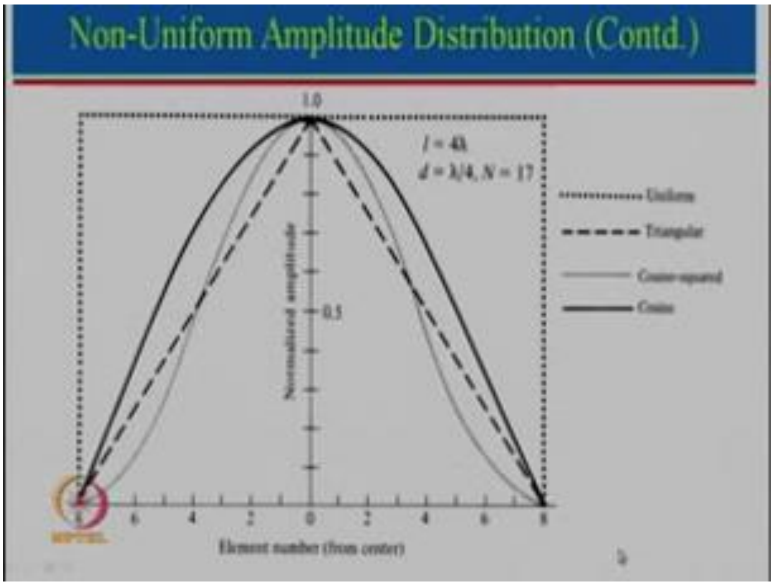

So, let us just see if we actually use the distribution which is given by these here. So, we again as before uniform is over here. If you have a cosine distribution that will be the cosine distribution, at triangular distribution is given by dotted here. That is a triangular distribution and this is a cosine square distribution. So, if you actually see cosine square distribution kind of resembles with the binomial distribution. So, that is why this cosine square gives the lower side lobe level compared to the uniform thing. So, actually speaking instead of using many a times those dot Tschebyscheff tell her it is actually simpler to use triangular distribution or sinusoidal distribution because it is easy to derive these expressions.

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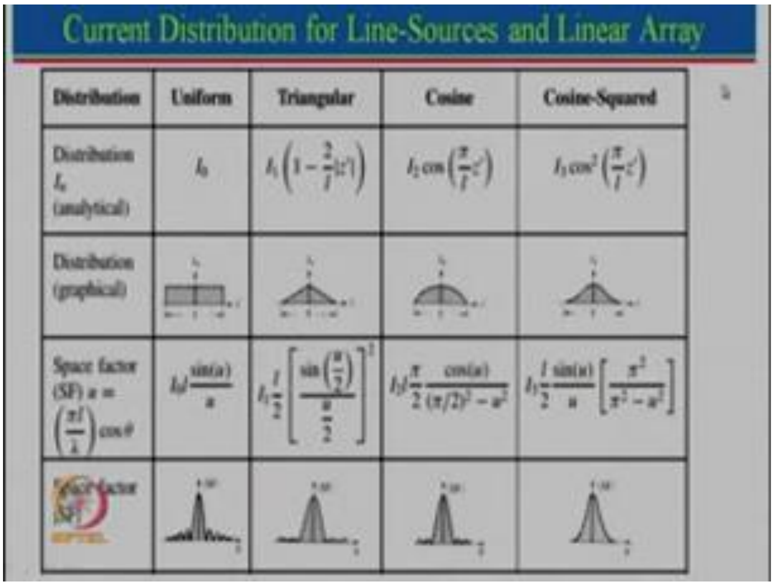

So, here is the plot here. You can see here uniform triangular cosine squared. So, what we have here distribution is given that is psi 0 and triangular will be given by this cosine. I just want to bring to your attention that we started with the linear array which has a number of element, but if number of elements are increased drastically and if we reduce the spacing between the 2 element. Let just take a case here instead of 17 elements if I say that the distance is reduced from lambda by 4 by 10 times let us say lambda by forty. So; that means, we will have very large number of element, but since that distance is reduced, I will use 10 times more element. So, it almost looks like a discrete array is now close to a continuous element array. So, here we are actually showing instead of writing an array factor, we are defining the term space factor. So, array factor limiting condition is space factor. So, array factor in which d is reduced to small value n increases. So, we know for array factor what we had seen the function was sin n psi by 2 divided by n sin psi by 2.

Now, if psi is small, because d is small then n sin psi by 2 will become n psi by 2. So, really speaking it becomes a sync function. So, looking at uniform all elements are fed with uniform or if it is a line source the current is uniform. And for this the array factor response will be a sync function. If it is triangular that is the triangular distribution, this is the cosine distribution, this is the cosine squared distribution. Now these currents can be well defined with the equation and for that then space factor is also very well defined and you can calculate space factor for these cases. So, it is easy to do the next derivations which are as follows.

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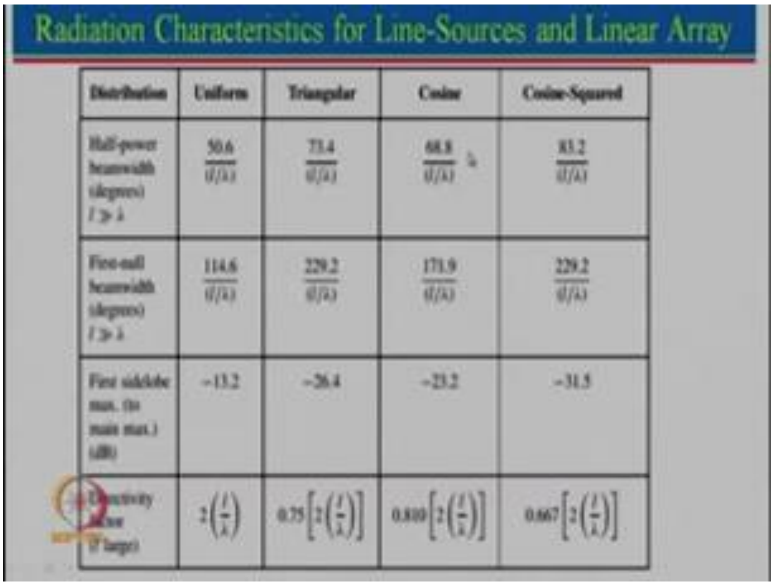

Distribution	Uniform	Triangular	Cosine	Cosine-Squared
Half-power beamwidth (degrees) $l \gg \lambda$	$\frac{50.6}{(l/\lambda)}$	$\frac{73.4}{(l/\lambda)}$	$\frac{68.8}{(l/\lambda)}$	$\frac{83.2}{(l/\lambda)}$
First-null beamwidth (degrees) $l \gg \lambda$	$\frac{114.6}{(l/\lambda)}$	$\frac{229.2}{(l/\lambda)}$	$\frac{171.9}{(l/\lambda)}$	$\frac{229.2}{(l/\lambda)}$
First sidelobe max. (to main max.) (dB)	−13.2	−26.4	−23.2	−31.5
Directivity factor (l large)	$2\left(\frac{l}{\lambda}\right)$	$0.75\left[2\left(\frac{l}{\lambda}\right)\right]$	$0.810\left[2\left(\frac{l}{\lambda}\right)\right]$	$0.667\left[2\left(\frac{l}{\lambda}\right)\right]$

Now, we want to calculate the let say half power beam width. We have seen earlier that half width beam power beam width is given by this expression, but for a triangular you can see that the half power beam width has increased cosine it has increased this as. So, that increase in half power beam width leads to the lower gain, but what is important first to look at is the side lobe, because that is the reason we use this here. So, for uniform if we get a side lobe level as say minus 13 point 2, if we use triangular side lobe level drastically reduces to 26.4. Cosine distribution is in between and cosine squared reduces the side lobe level to below 30 dB which is minus 31.5 dB.

So, now looking at these numbers, we can see that if we use this distribution non uniform, then we can reduce the side lobe level, but one can also see half power beam width increase, which results into the decrease in the directivity. So, this is the directivity factor for large l. So, this is 2 l by lambda which we had seen for uniform. So, now, that is reduced by a factor of 0.75 for triangle for a factor of 0.81 for cosine. So, you can see that the directivity has reduced. So, that is why a many a times what is actually done, that instead of using a simple cosine distribution we actually use a cosine with the pedestal.

So, what we can do instead of using amplitude from 0 to 1 which is normalized here we use let us say 0 to a at constant 1, and then from a you make a cosine function. So, that looks somewhat similar to that dolph Thschebyscheff function here. Or we can use a triangular distribution whether pedestal, and if we use triangular distribution whether pedestal it almost looks like a close to Taylor distribution. So, we can reduce the side lobe level by using this pedestal and also since this element here is actually being fed 0 power you. So, that is why it actually array in effect reduces from 17 elements to 15 elements. So, if we provide the pedestal; that means, some power is fed to the last 2 elements. And hence you will get a little better directivity compared to these numbers here. So, by providing a pedestal to cosine we can increase this factor from 0.81 to may be close to 0.9 or may be 0.7 can be increased to 0.8 or 0.85. So, that will reduce the slide lobe level slightly, but also decrease the directivity also slightly compared to the uniform array.

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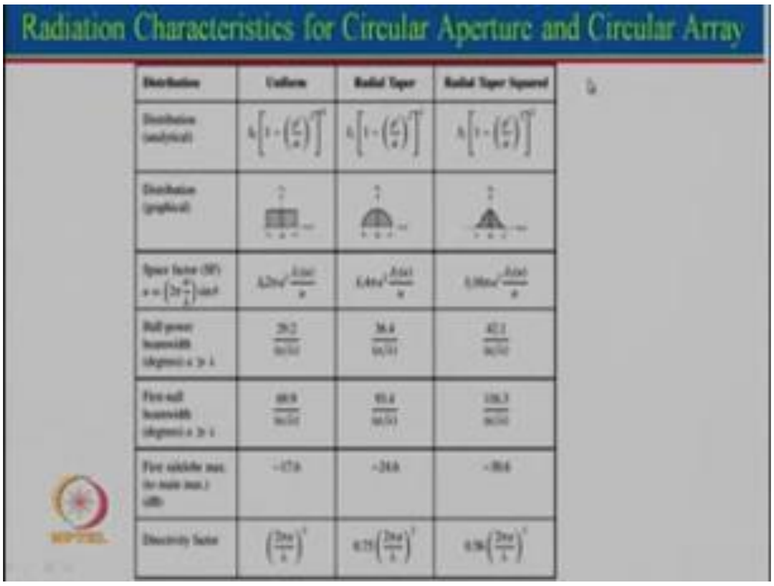

So, now instead of a here we had taken a simple space factor for the aperture. The proper aperture lime source that can be extended to the square, but if we use a circular aperture or circular array, what we see now here if the distribution is uniform a radial taper it is going from 0 to the outward or the radial taper square. So, if we use circular aperture what we see here, that for uniform distribution also side lobe level is minus 17.6, whereas for uniform square aperture it was 13.2. So, from 13.2 it has reduced to 17.6 what is the reason. So, you have think about a square aperture as a complete thing which is fed over here whereas, circular aperture is in between. So, if you think about a square aperture super impose on a circle. What we are seeing is that the end element is not being fed. And also the next to the end element now are actually getting less excited. So, in a reality what we have done by using a circular aperture and there are no end elements only so; that means, there is no power going to the end element and hence it is kind of natural amplitude taper.

So, since there is natural amplitude taper here, side lobe level is reduced for all the direction. So, it is actually sometimes good to use a circular aperture or circular array. And here I just want to tell that these arrays theories are very important when we going to look at all the other cases for example, when we discuss next topic which is a micro strip antenna, we will see that it is an array of 2 slot antenna. When we discuss about let us say horn antenna. For horn antenna we will see that in one plane we have a uniform distribution and in another plane we have we will see that it has a cosine distribution. So, in both the planes we have a you can use the array theory or you can apply the space factor concept and simply by using this concept we can actually find out what will be the radiation pattern let us say in this plane if it is uniform or we can find out what is the radiation pattern in this plane if it is non uniform.

So, this array theory will be very useful. The same thing will also applied let us say when we talk about a reflector antenna. So, again a reflector antenna has a circular aperture it may be a parabolic, but the outer here is a circular we can actually apply again the circular amplitude concept and find out what is the radiation pattern in the 2 planes. So, array theory becomes then space factor and it can be applied by even for a helical antenna when we look into the helical antenna actual mode. We will see that helical antenna normally follows increased directivity area concept. The same array theory will also applicable to log periodic antenna and yagi uda antenna. So, I want that you people should study this array theory very thoroughly.

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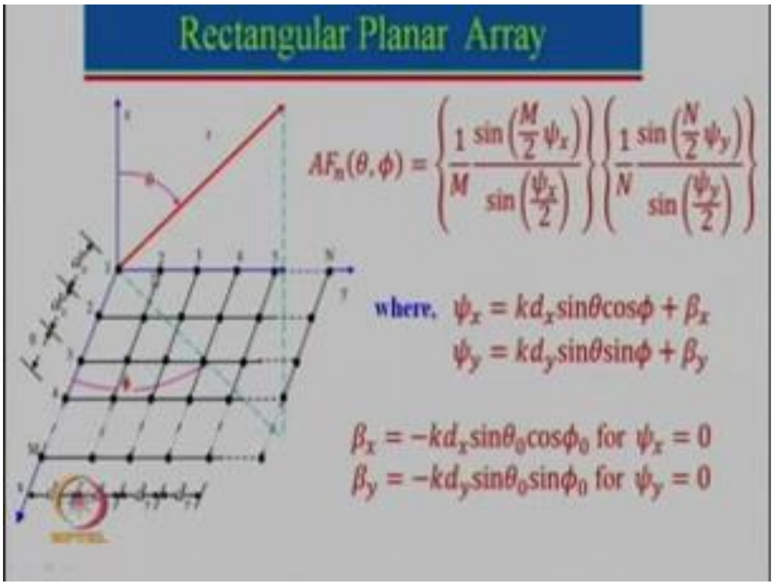

Now, we will just quickly look into the planer array. So, if you are going to the planer array what we have now array elements are there in both x and y plane. So, it is a combination of linear array, but now is linear array if you think about this is a x direction this is a y direction. So, think about there is a linear array over here of m element. And then this linear array is getting repeated by n times. So, the total number of elements are m multiplied by n. So, finding array factor is very simple. All we do it is first we find out the array factor for this linear array. And that will be nothing by given by this term here, 1 by m sin m by 2 psi x. What is psi x here, it is corresponding to the element going towards x direction.

Now, we can apply principle of pattern multiplication. So, this entire linear array can be thought of a single element, and then that single element is getting repeated n time. So, we have now an linear array of this combined thing over here and we can now apply n element array which is over here. So, now, the difference compared to the previous cases that in the previous case we had only one side term, but now we will have a psi x term. So, psi x is nothing, but given by this expression here. How do we get this expression let us say we are trying to find out the radiation factor at far away distance r? What we do it is we take a projection that on the x y plane. Now since this angle is theta the projection of that in this one plane here will be multiplied by sin theta. So, you can see that sin theta is coming at both the places.

Now, when this is projected down here, if we take this particular thing along x axis, then we need to multiply by cos phi. And that is what is done over here and if we multiply this along the y axis now it should be multiplied by sin phi and that is the term over here. And what is beta x and beta y well it is same as delta earlier. So, that is a phase difference in direction phase difference in y direction. And we can find the phase values in x and y direction simply by making psi x psi y equal to 0. So, if we make that 0 we can find the value of the phase shifts. So, for any given value of theta 0 and phi 0 we can find out. So, let us say we want the beam maxima at theta 0 to 10 degree. And we want another one phi maxima at phi 0. So, our angle is given see remember theta 0 phi 0 both of them are 0 that will be broad side. So, now, if theta 0 is equal to let us say 10 degrees, that will be from here 10 degrees. And phi is say 20 degrees that will be from here. So, that is how you look at.

So, I want that you do little practice now. So, find out as a problem find out what is the phase shift for let us say a theta 0 equal to 10, and phi 0 equal to 20. You can take the spacing to be equal to lambda by 2 for both the cases. So, we will continue from here in the next lecture.

Ah just to summarize. So, today what we have seen we started with the grating lobe, how the grating lobes can be avoided. Then we looked into it that even if one of the element is not fed array will still perform may be not to the optimum level, but still connectivity will be maintained. And after that we looked into the different cases of an non uniform amplitude. So, we saw that. So, we can use a triangular distribution co sinusoidal distribution or cos squared distribution or in parallel we can use binomial distribution or Thschebyscheff or Taylor distribution by using non uniform distribution what we have achieved is we can reduce the side lobe level, but at the expense of decrease in the gain of the antenna. And then we looked into how to calculate the array factor for planer array. So, in the next lecture we will look into different examples of planer array, we will look into rectangular array square array triangular array hexagonal array circular array and so on.

So, meanwhile please do some practice and do some design. You can even look into a simple design let us say that we want to design an antenna array. For example, a rectangular array which should give me a beam width of let say a 30 degree in one plane and in orthogonal plane I want beam width to be 20 degrees. So, look into that how to design that particular array.

Thank you and will see you.