Antennas
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Module - 04
Lecture - 16
Linear Arrays-II

Hello, in the last lecture we discussed about linear array. And we actually started with a very simple concept of 2 isotropic elements. And we had seen that when these 2 isotropic elements are fed with equal amplitude and different phases, we can get the beam in different direction. For example, if the phases between the 2 elements is 0 degree then the beam is in the broadside direction; that means, if I have an array axis like this. So, beam will be maximum in the perpendicular direction. And then we had seen that as we changed the phase between the different elements. So, for example, when the phase difference was 180 degrees for spacing of lambda by 2 we saw that the beam instead of being maximum in the broadside it went into the endfire direction. And as we changed the phase from 0 degree to 180 degrees, we can change the beam.

So, we started with 2 isotropic elements, and then we discussed about 2 dipole antennas which are fed with equal amplitude and then different phases, and we had actually seen the principle of pattern multiplication.

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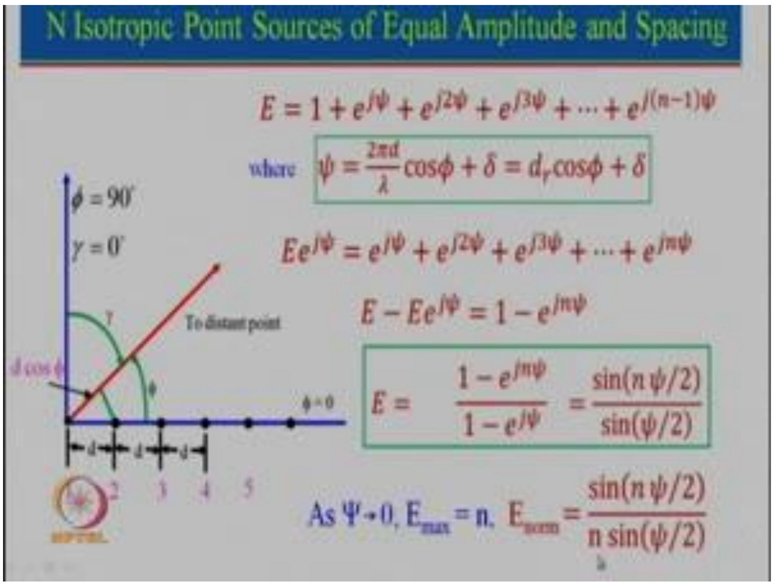

So, how the dipole orientation will change the pattern, after that we did the derivation for linear array of n elements, and what we have found out that for a linear array the array factor is given by sine n psi by 2 divided by sine psi by 2. And the maximum value of this can be obtained when psi tends towards 0. So, when psi tends to towards 0 then sine n psi by 2 will be approximately equal to n psi by 2, as we know that sine x is approximately equal to x. The denominator sine psi by 2 also becomes psi by 2. So, psi by 2, psi by 2 gets canceled we left with the value of n. So, the maximum amplitude of e is nothing but n. So, if we normalize this particular function then we divide it by n so; that means, now this function has a maximum value of 1. Now this entire thing can be plotted for different values of n and different values of psi, and the plot is shown over here.

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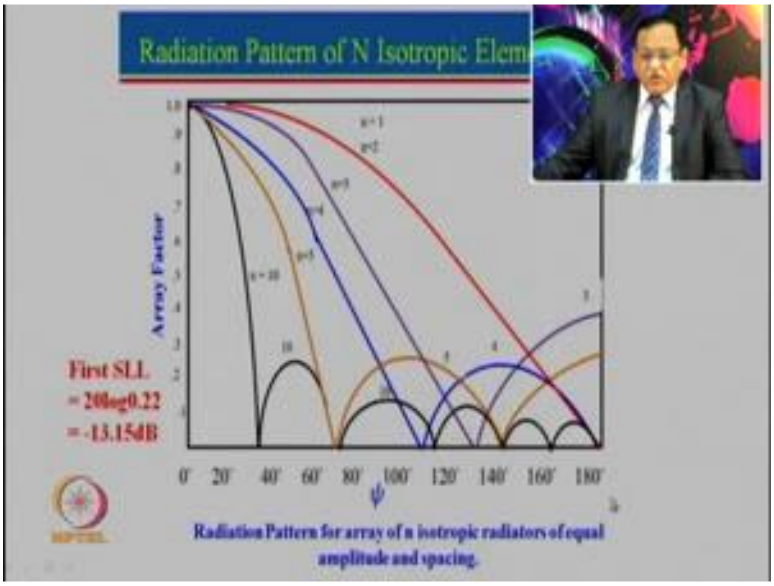

So, along the x axis what we have? Psi, which is varying from 0 degree to 180 degrees, and along the y axis we have a array factor which is the normalized electric field. So, let us see if there is a single element what will happen, array factor will be simply equal to 1, for n equal to 2 the array factor goes from 1 to a low value of 0 at psi equal to 1 by 80. In fact, this can be checked if psi is equal to 1 80 and if n is equal to 2, we can see that this value will go to 0 value and then as n increases. So, n is equal to 3 the curve goes like this here there is a one side lobe level when n is equal to 4. There is a complete side lobe level and if I look at n equal to 10, there are multiple side lobe levels whose amplitude is decreasing progressively.

You can also see that as we are increasing n. So, n equal to 2 3 4 5 10 if we look at the half power beam width, how do we define half power beam width where array factor is reduced from 1 to 1 by square root 2 which is approximately 0.707. So, if you draw the horizontal line from here. So, we can see and this is the half part. In fact, the other part will be actually symmetrical to this which will go like this here. So, we are only showing the one part from the broadside direction.

So, we can see that the half power beam width continuously will decrease as we increase number of element say from starting if we draw our imaginary line then n equal to 2 half power beam width will be large then it will decrease. So, which is we also know that from array factor theory that actually says aperture area is nothing, but equal to directivity is given by 4 pi a by lambda square where a is area. So, we increase number of element area will increase and that is how the directivity will increase. So, half power beam width decreases mean that directivity is increasing.

So, now from here we can actually find out various things that, what will happen where will be different nulls there, but before that let us just look at what happens if we change the phase for number of elements.

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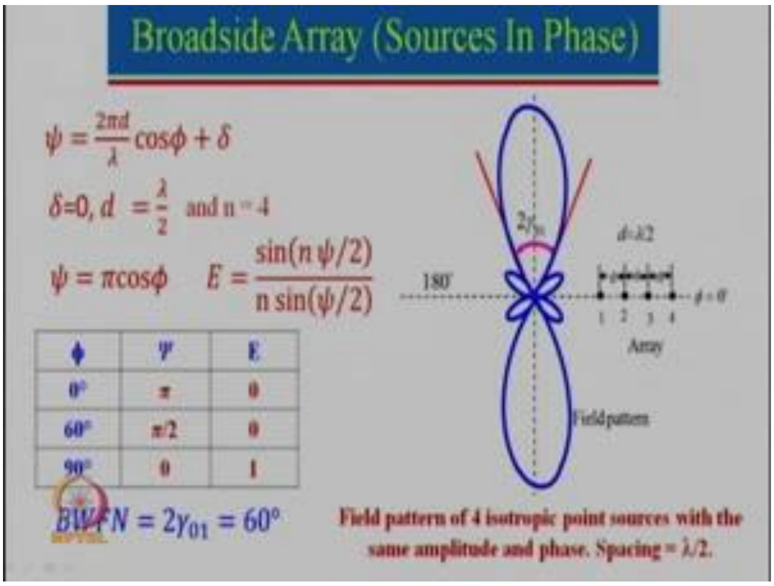

So, we will take a case here where we have 4 different elements. All these elements have a spacing of d in between and we are taking a special case of d equal to lambda by 2. So, we start with the function psi, psi is 2, pi dB lambda cos phi by delta this derivation we did in the last lecture. For broadside delta will be equal to 0, we have taken d equal to lambda by 2 and n equal to 5. So, then if we substitute this value in the function psi, we can see that d is lambda by 2. So, that will reduce to pi, cos phi will come here delta is here. And the array factor is as before sine n psi by 2 divided by n sine psi by 2.

So, let us just try to plot this. In fact, we would like to plot this, radiation pattern and we can of course, start with let us say phi equal to 0 and then 10 degrees 20 degree 30 degree we can rotate for complete phi equal to 0 degree to 360 degrees and that takes lot of time. So, we can do some quick things also and that is suppose if we take phi equal to 0. So, if we take phi equal to 0 cos of 0 will be 1. So, psi will become pi here. And if psi is equal to pi we put the value of here this is pi and we put n equal to 4 here then this function will become equal to 0. So, that is why it is 0.

At phi equal to 60-degree cos phi will be 1 by 2, this will become pi by 2, again we put here pi by 2 and this is 5. So, 4 into pi by 2 into 2 will be equal to pi, and sine pi is 0. So, again this will be 0 now when we take a case of phi equal to 90 degrees which is broadside. So, cos 90 will be 0 psi will become 0 and sine 0 divided by 0 will be nothing, but again we have done that before that. When psi tends towards 0 this whole function will now become one because this is normalized with respect to n. So, we get 1.

So, if we now start plotting here. So, starting from here phi equal to 0. So, phi equal to 0 e is 0. So, that is a 0 value as phi increases this lobe comes and then at 60 degrees this function goes to 0. And then we increase further to 90 degrees it goes to the maximum value of 1. And then this whole trend is repeated because the sine functions repeat itself. So, we can see from here that half power beam width will be given over here, but the beam width between first null is nothing, but in this case 60 degree because this angle is 60. So, this will be 30 degrees and 30 degrees. So, beam width between the first null will be 60 degrees.

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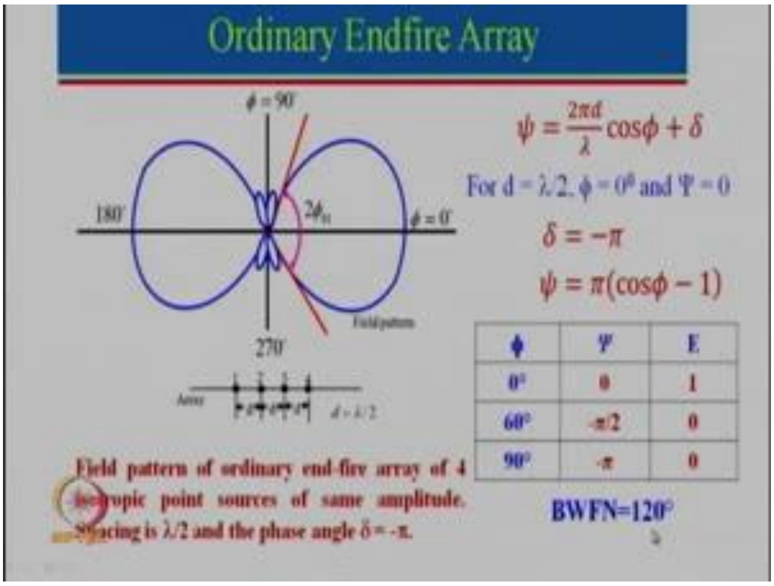

Let's just take another case now that is the case of ordinary endfire. So, we start again with the psi. We are taking d equal to lambda by 2 and this time we want beam maxima in y equal to 0 degree that is what will give us endfire array. And for this particular value we force the conditions psi equal to 0 that is for maximum radiation. We put these values over here. So, delta is calculated from this which comes out to be minus pi or 180 degrees. We substitute the value of this delta over here we get psi equal to this.

Now, again we can look at a different angle. So, at phi equal to 0 cos of 0 will be 1. 1 minus 1 will be 0. So, psi becomes 0 and if psi is equal to 0 array factor gives us value 1 when phi is 60. So, cos of 60 will be 1 by 2 minus 1, so that will be minus pi by 2 and if we substitute this value in the array factor that comes to be 0 and when phi equal to 90 cos 90 will be 0 minus 1. So, that will be minus pi and 0.

So, we can now do the plot same way. So, along phi equal to 0 this is maximum and as phi increases it is changing and it goes to 0 value. Then in between it increases and again comes back to 0 value. So, if we now see the beam width between the first null. So, the beam width between the first null is given by this particular angle here, which happens to be 120 degrees and that actually is much larger if you compare this with the broadside array the beam width between the first null was 60 degrees, now it is 120 degrees. So, this made researcher think why ordinary endfire array has a broader beam width which results to a smaller gain. So, what can be done? So, to do that then this concept of ordinary endfire came into picture, we look into that what is that concept, but let us first look at as an example first.

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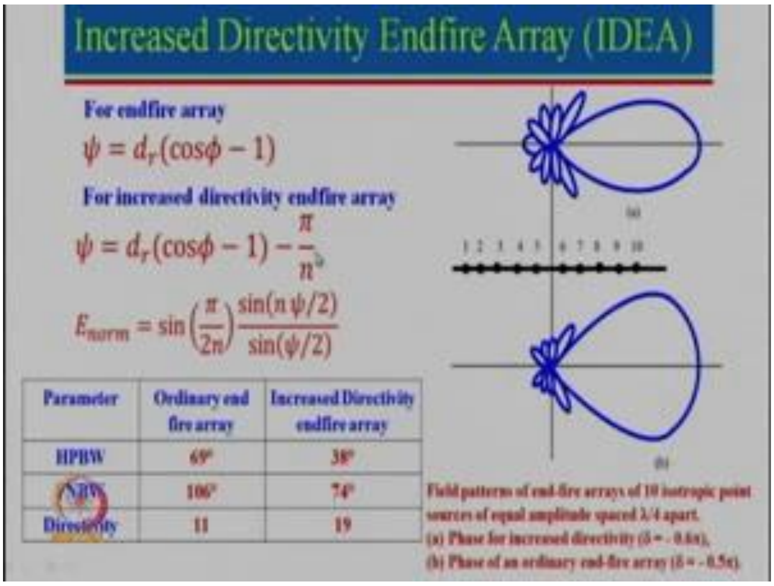

So, here we have actually 10 isotropic elements. In this case the distance is lambda by 4 between these elements. So, there are 2 plots shown over here this is the plot for ordinary end fire array. And this is the plot which is actually increased directivity end fire array, which I have given a nickname as idea an idea can change the life. And here an idea has changed the directivity, and increased the directivity. So, what really is happening how it is happening? First let us see the example quickly. So, in this particular case for ordinary end fire array, half power beam width is 69 degrees whereas for increased it is 38 degrees. So, one can see that there is substantial reduction in the half power beam width and that is why directivity increased from 11 to 19.

So, how this particular thing is realized, so for that we have to actually think about what is the phase for end fire array, we had seen that psi is given by the d r is nothing, but 2 pi dB; lambda that is cos phi minus 1 this accounts for the phase, but for increased directivity we take additional phase delay of pi by n compared to the previous case. And why do we take this additional phase difference. So, that will be clear if we actually look at this plot here.

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So, just look at the now we will not look at the normalized value, but let us just look at this function here. Sine n psi by 2 divided by sine psi by 2, in this case we have plotted 2 plots separately, this is the plot for the denominator which is sine psi by 2 and psi is varying from 0 to pi. So, as psi changes from 0 to pi at pi it will be sine pi by 2 which will be 1. So, this one will vary sinusoidal from 0 to 1.

What about the top case numerator here numerator is sine n psi by 2? And we have taken an example of n equal to 8. So, if we start with 8 here and then as psi changes one can actually see that there are number of sinusoidal variations and the total you can see here the maximum for psi equal to pi this is 8 times pi by 2, and that is that variation will be close to 4 half cycles or 2 complete cycles. So, what is the first maxima here, the first maxima come when psi is equal to pi by n, and that we can check here, this is n and if psi is pi by n. So, n, n will cancel this will become pi by 2 and sine pi by 2 is equal to 1. And that is the maximum value when this term here becomes pi by 2 pi by n then that will be the term corresponding to the 0 value.

Now, if we look at the radiation pattern, let us just look at the pattern. We can see that the first beam maxima come and then there is a null and then there is a side lobe coming into picture. So, now, just remember this now after the null only some maxima come right. So, now, if we just look at this plot here, so that first side lobe level actually corresponds to this point, because it is coming after the null. So, what is this point then why something is maximum over here and if you actually look at this point here at this particular point, the beam is maximum and at this particular one if we just look at the corresponding value. So, here the maximum value is 1. So, sine n pi by 2 maximum value will be equal to 1, and what will be the corresponding value here well we have to put psi equal to pi by n. So, if you put psi equal to pi by n. So, this function becomes now sin pi by 2 n. So, this is the maximum value of this particular function, so if you normalize that with respect to 1 by divided by this.

So, now for increased directivity n phi r. Normalized value is not this function divided by n because n happens over here that is the maximum point. This is the value which is corresponding to this maximum divided by corresponding value over here, if we go back and look at the same curve. So, the point which we are looking at is somewhere here, where this value of the numerator is increasing to the maximum value and divided by denominator value. So, if we now start from that.

So, what is happening? We are starting now instead of starting from here, now we are starting from here. So, if we start from here you can see that my null will come much faster and if the null is coming faster and from here if I look at half power beam width half power beam width will come very fast. So, that is the reason if we provide this additional phase and we start from here my half power null beam width is reduced and we have a more side lobe. And over here if you look at the amplitude of the side lobe is much larger than the amplitude of the side lobe over here. The reason for that is over here all the side lobes were divided by a maximum value of n, whereas all the side lobes are not divided by n, but a lower value and hence side lobe levels increase, in this case that is the reason why adding this additional phase. So, this is the additional phase corresponding to psi which is pi by n. So, if you use this particular thing we need to use this normalized function and that is how we can realize the increased directivity endfire array.

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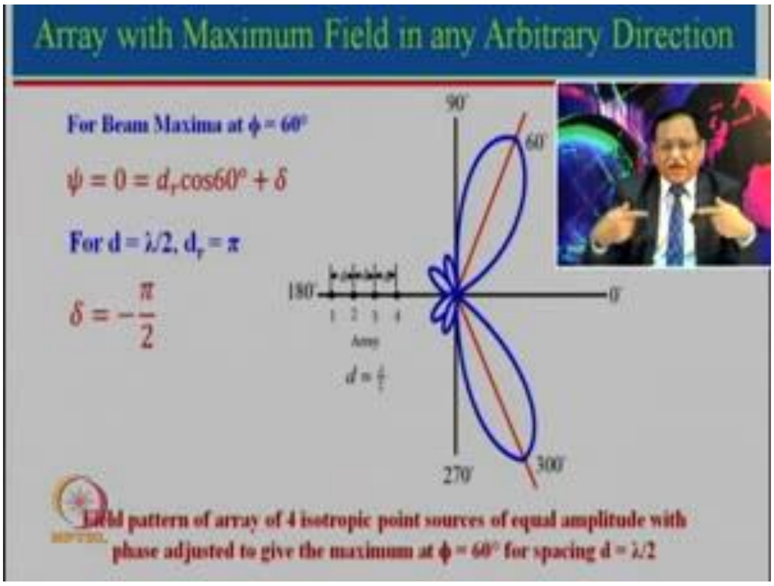

So, let us look at a few other example, let us say we want a beam maximum at phi equal to 60 degrees. So, this is 0-degree phi is 60 degrees here. So, this is our desired beam maxima. So, for that we need to calculate what should be the corresponding phase value. So, psi is equal to 0 we put that condition this is same as before, but phi has been put as 60 degrees plus delta. So, we need to calculate delta and we take a case of d equal to lambda by 2, if you do that d r is pi and delta becomes minus pi by 2 so; that means, if we take a phase delay of minus 90 degree; that means, this should be fed at 0 this should be at minus 90 then this will be another minus 90 which will be minus 180, and this will be minus 270. So, if we feed it like this then the beam maxima will be in this direction which is at phi equal to 60 degrees.

And since the beam is symmetrical with respect to the axis of the array the whole pattern is getting rotated by full conical thing. So, that is the reason we actually see the 2 different cone, but actually this is 2 dimensional, but the whole this cone is getting rotated completely here. So, what you really get is a conical radiation pattern.

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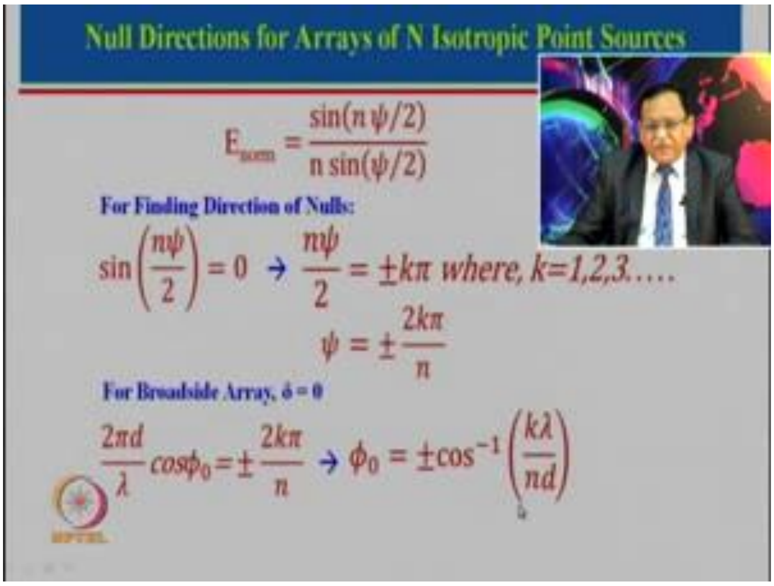

So, now we need to find out where is the direction of null and other parameter for example, here one of the thing let us just go back. So, we can find the null over here and here, but how do we mathematically calculate in which direction null is coming and also we would like to find out in what direction minor lobes are coming and also we want to find out what is the amplitude of these minor lobe or also known as SLL side lobe level. So, how do we calculate these well these can be calculated from the simple this array factor function. So, let us start looking into that one by one.

So, null direction. So, this is our array factor when will be the null whenever the numerator becomes 0 except for the condition that psi should not be 0. Because there it becomes 0 divided by 0 and that is maximum value. Or other values wherever this becomes 0. So, now, we can say we would like sine n psi by 2 to be 0 which will give us the direction of null. So, this value should be then equal to plus minus k pi; that means, k times 180 degree. So, k will be 1 2 3 k cannot be 0 because that will give us a 0 value. So, from here we can find out what is psi. So, psi should be given by 2 k pi by n.

So, let us just take an example. We will take an example of a broadside array. So, delta will be equal to 0. So, we now substitute the value of psi over here, which is 2 pi dB, lambda cos phi 0, and then this term comes over here and from here we can find out what is the value of phi 0.

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So, phi 0 can be calculated and this one will give us the direction of null. And all these thing I have done for just a broadside all these general cases are given over here. So, what we have first case is general case where psi will have a delta term delta can vary anything from 0 to 180 degree, and this one is a null direction for any length this is an approximation which is used for a long array.

Normally what it means long arrays if number of elements are more; that means, if n is large array will become large. So, some approximations have been made here, and then beam width between the first null this expression is given here. So, let us say for broadside. Now if you see here it shows sine inverse whereas what I have written here is cos inverse. So, there is nothing wrong everything is fine the only difference is this phi 0 is measured from the axis of the array whereas, over here gamma 0 is measured angle from the broadside. So, from broadside psi 0 will be nothing, but 90 minus phi 0, cos 90 minus phi 0 will give rise to sine. So, that is why the function sine is coming into picture here. And from here if you just look at it sine inverse x will be approximated to x that is the approximation when this term is very small. And that is obvious if n is large this term will be small. And the same thing is done for ordinary end fire or increased directivity and if the null direction is given here. So, beam width between the first null will be 2 times this. So, that is the expression given for this.

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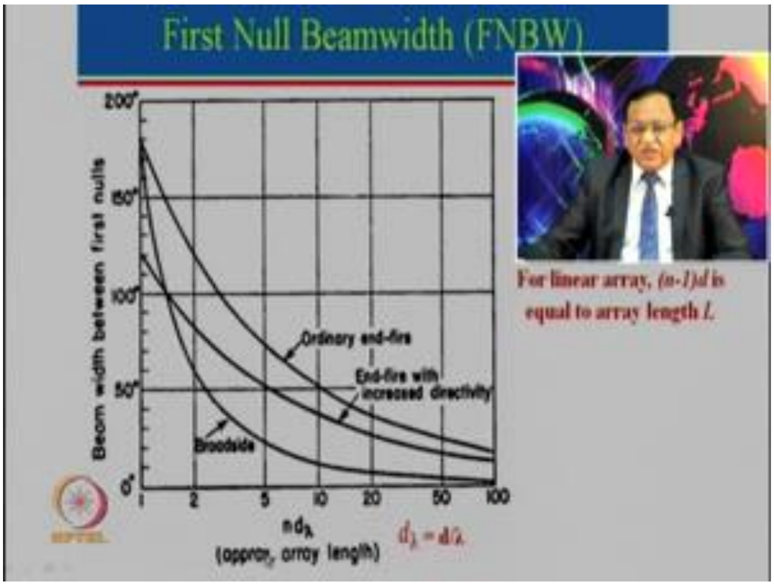

So, now let us just look at the next case examples which we have seen beam width between first null and approximate array length. So, we can see that as array length increases we can see from here as array length increases. So, n increases then null direction will reduce; that means, beam width between the first null direction will reduce. So, you can see that as the so, this is the situation for broadside that is the case for ordinary endfire. And you can see that for increased directivity a beam width between the first null is reduced; that means, correspondingly half power beam width will reduce also and that is why directivity will increase.

Now, there is a just small approximation which I want to highlight. That is n d lambda is considered as approximately array length what is d lambda that is d divided by lambda, but in reality for linear array with uniform spacing d array length is actually equal to n minus 1 times d. So, this is just an approximation over here.

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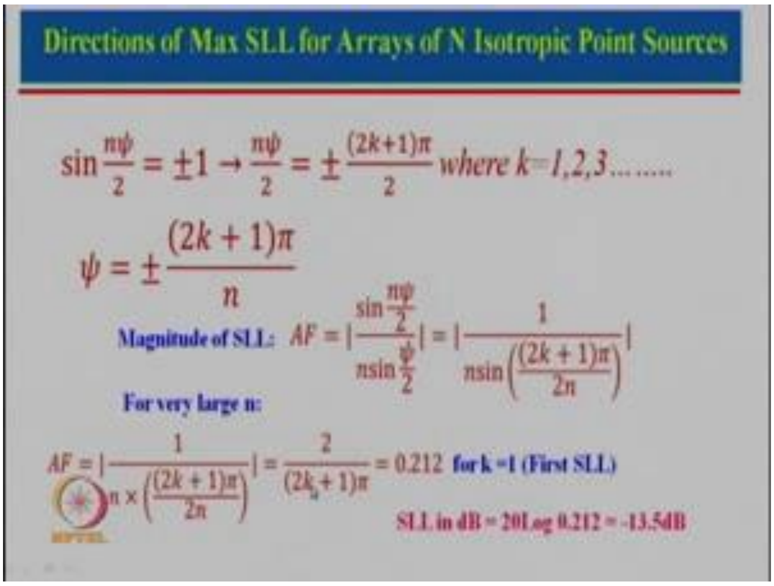

So, now let us see how we can calculate the direction of maximum side lobe level. And it is actually a gain we start with the array factor what we really want is now this time we want the numerator should be equal to 1. And when numerator will be one when n psi by 2 is equal to pi by 2 then sine pi by 2 will be 1. So, if we just put that into here sine n psi should be one or it can be plus minus 1.

Now, just to mention see array factor is always the magnitude of the entire value. So, whether it is plus or minus ultimately we are concerned about only magnitude. So, from here we can say n psi by 2 should be odd multiple of pi by 2. It cannot be even multiple because even multiple will give us a 0 value. So, this is the function how n psi by 2 is related. And from here we can find out the value of psi 2 k plus 1 and once we know what is the value of psi for side lobe level we can put now psi as 2 pi dB, lambda cos phi plus delta and depending upon the value of delta we can calculate the value of maximum radiation, other than the main lobe and that will be minor lobe radiation.

So, this actually gives us the direction of side lobe. How do we calculate the magnitude well let say this is the array factor all we now know that sine n psi by 2 for this case should be equal to 1? So, we have put one over here, and now in the denominator we have n sine and what is psi, psi is given by this value here if we substitute this value. So, this is the magnitude of the SLL function coming into picture here. And from here we can make one more approximation now that is if n is large. So, for very large n we can say sine x will become x. So, we sub make this x over here multiply this here, n, n will get cancelled that is the array factor for large n. In fact, this gives us value for side lobe level for different values of k.

So, the first side lobe will be for k equal to 1 and that comes out to be 0.212. And if we take 20 log then it comes out to be minus 13.5 dB. So, we can actually see that if all the elements are fed with uniform amplitude the best side lobe level we can get is minus 13 point 5 dB. For first side lobe for second side lobe k will become 2 this value will be smaller for third side lobe k will be 3 this value will become even further smaller and this is obvious from the curve that the first side lobe level is larger we go to the next one then we go the next one, we can see that the side lobe levels are decreasing, but the problem is that the first side lobe level is never below minus 13.5 dB.

So, now the next thing is. So, what is the desired side lobe level so in fact, in general think about the side lobe as power going in the undesired direction. So, let us say if the side lobe level is say minus 10 dB. That means 10 minus 10 dB. Corresponds to 10 percent so; that means, 10 percent power is going to the undesired direction. So, if the side lobe level is minus 20 dB. That means, only one percent is going in the undesired direction. So, think about a radar application which is probably transmitting 1 kilo watt of power in the main beam and if side lobe level is only 10 dB. That means, 10 percent of 1 kilo watt 100-watt power is getting directed in some other undesired direction. Even 20 dB, would mean 1 percent of the power. So, of one kilo watt 1 percent is still 10-watt power going in the undesired direction. So, especially for application there we need to transmit higher power we always try to make a lower side lobe level.

Also for many satellite applications we prefer that the side lobe level should be 20 dB, or 30 dB below the main beam. So, the direction of the mind lobe can be obtained just from the same expression here.

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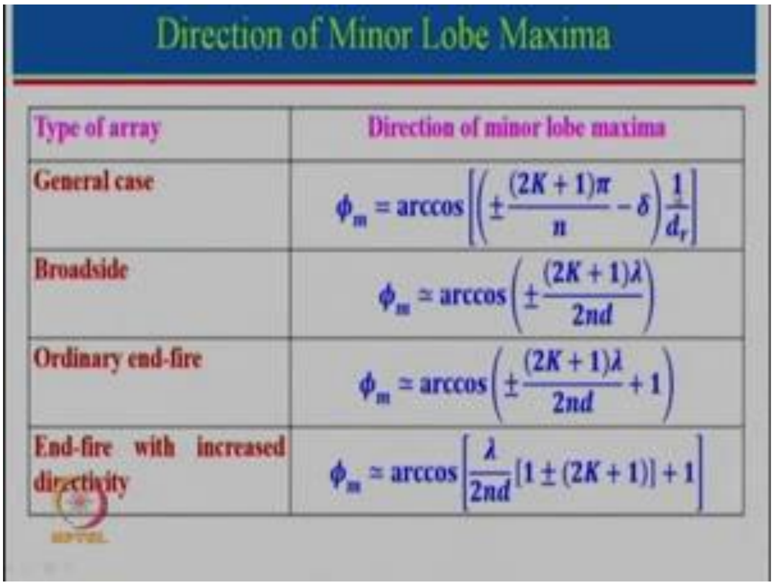

So, substitute the value of psi and we can get the different values of the minor lobe. So, this is the general case here, where this is nothing, but psi special cases are given over here for broadside ordinary endfire. So, by using this concept here you can actually speaking find out the direction of minor lobe.

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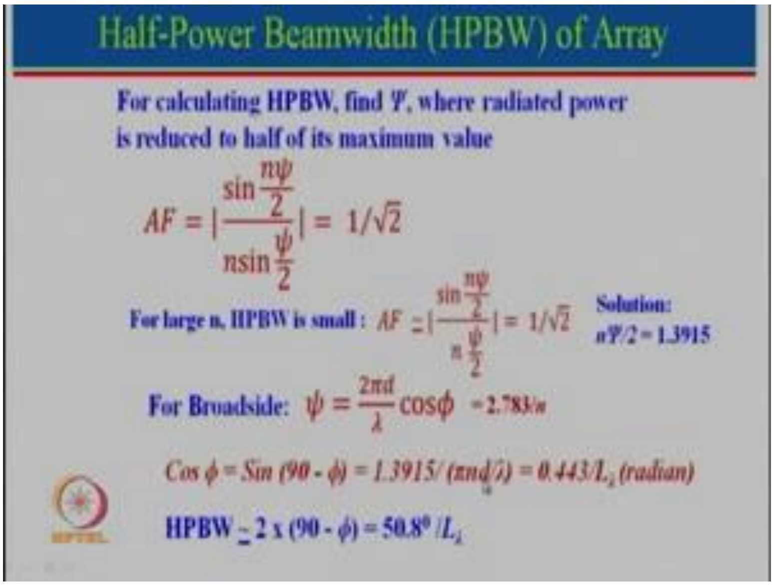

So, how do we calculate half power beam width. So, half power beam width can be actually found in a simple way, wherever array factor becomes one by square root 2 because this maximum value is 1 so; that means, now if we assume again assumption of large n then this function here sine psi by 2 can be made equal to psi by 2. So, this is like sine x by x which is the sync function and we want that 1 by square root 2. So, if we solve this one here n psi by 2 is equal to 1.3915.

And if you want to use your calculator to find out you can actually put this value here except that you put 1.3915 in degree and; that means, you convert 1.3915 multiplied by 180 divided by pi otherwise this value is in radian. So, again for broadside we put the value of psi here delta is 0, and from here we can calculate what is the cos phi and this gives us the expression for half power beam width which is 50.8 degree divided by lambda, what is lambda it is nothing, but n times d which is length divided by lambda.

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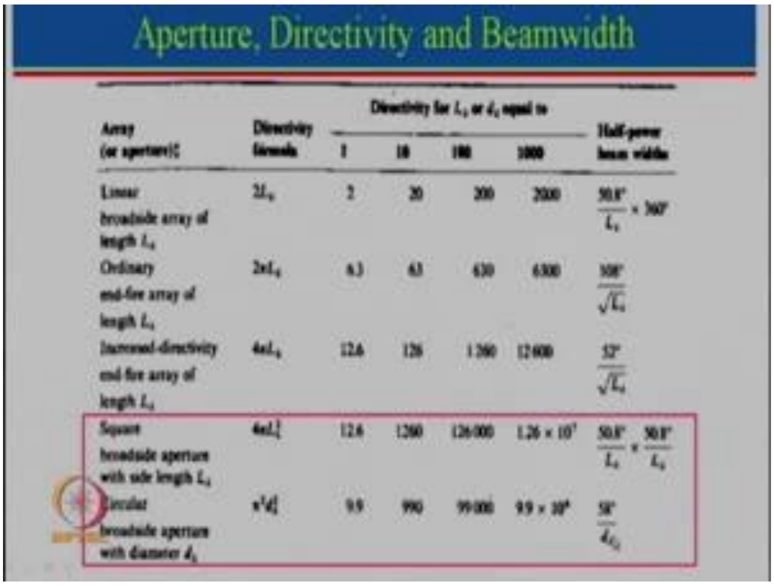

Aperture, Directivity and Beamwidth

Array (or aperture)	Directivity formula	Directivity for L_λ or d_λ equal to 1	10	100	1000	Half-power beam width
Linear broadside array of length L_λ	$2L_\lambda$	2	20	200	2000	$\frac{50.8^\circ}{L_\lambda} \times 360^\circ$
Ordinary end-fire array of length L_λ	$2\pi L_\lambda$	6.3	63	630	6300	$\frac{108^\circ}{\sqrt{L_\lambda}}$
Increased-directivity end fire array of length L_λ	$4\pi L_\lambda$	12.6	126	1 260	12 600	$\frac{52^\circ}{\sqrt{L_\lambda}}$
Square broadside aperture with side length L_λ	$4\pi L_\lambda^2$	12.6	1260	126 000	1.26×10^7	$\frac{50.8^\circ}{L_\lambda} \times \frac{50.8^\circ}{L_\lambda}$
Circular broadside aperture with diameter d_λ	$\pi^2 d_\lambda^2$	9.9	990	99 000	9.9×10^6	$\frac{58^\circ}{d_\lambda}$

And here this expression gives all the different cases here. So, here we have a beam width directivity aperture. First we will start with the linear source which is broadside endfire, increased directivity. Expression for directivities are given here and here half power beam width expression are given. So, one can see that half power beam width in one plane is 50.8 by lambda as we saw, but in other plane it is 360 degrees, but for ordinary endfire half power beam width is same in both orthogonal plane. And here we have a case of square broadside array; that means, an aperture of a square of length total length lambda, and lambda.

So, we know from aperture theory, that the directivity is given by 4 pi a by lambda square what is for square is lambda multiplied by lambda. Similarly, for square we can say what is the aperture theory says 4 pi a by lambda square, and what is a pi r square. So, 4 pi into pi r square reduces to pi square d square lambda and these are the corresponding half power beam width.

So, today what we have seen that different cases we studied. So, how array factor can be simply seen for the broadside array for ordinary end fire and for increased directivity how normalized value of array factor changes and then we actually look at some of the special cases of broadside array ordinary endfire array increased directivity, and we also found out how to calculate the phases for the desired beam direction and then we also derived the expression for how to calculate the null direction, how to calculate beam width between the first null, then how to calculate the direction of side lobe level, and how to calculate the magnitude of side lobe level. And then towards the end we saw how to calculate half power beam width of the antenna and from half power beam width we can calculate the directivity.

Thank you very much.