Antennas

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Module - 04
Lecture - 15
Linear Arrays-I

Hello, today I am going to talk about antenna arrays. Till now we have discussed about basic antenna configurations which are dipole antenna, monopole antenna, slot antenna and loop antenna. Now all these antennas have a very small gain relatively 2 dB or it could be 3 to 4 dB depending upon the length of the say dipole antenna for monopole antenna it depends upon the size of the ground plane. For example, monopole antenna for infinite ground plane directivity is about 5 dB whereas for smaller ground plane monopole directivity is only about 2 dB.

Now, also these antennas have a very broad beam. So, they will be sending the radiation in all the directions for many point-to-point communications we require antennas which are pointing towards a certain direction and for that we can use arrays of these elements. So, today we are going to discuss about various types of arrays.

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So, we will start with the arrays of 2 isotropic sources. So, where we will assume that there are 2 isotropic. As I mentioned earlier there is no isotropic source as such, but for assumption we can use isotropic source which radiates equally in all the direction. So, we will use the arrays of 2 isotropic sources. And we will consider various cases for example, they are fed with certain amplitude and we will vary the phase. And we will see that how the radiation takes place in different direction.

The next topic we will take is principles of pattern multiplication where we will take a real antenna example. In this case we will take a dipole antenna and then we will see how dipole antennas can be used in an array form and we apply the principle of pattern multiplication. After that we will talk about linear arrays of n elements, where n can be anything from 2 to 10, 20, 100, and to start with we will take a case that all these elements are fed with uniform amplitude, but their phases will vary.

So, for example, if all the elements are fed with the same phase, it will radiate in the broadside direction which is perpendicular to the direction of the array. Then we will go to the next topic that is ordinary endfire. And in this case the phases are changing. And then from ordinary endfire we will actually see that there is a problem the directivity of that is not as good as we would expect and also beam was broadened. So, by using the concept of increased directivity end fire area I have given a nickname of idea, an idea can change the life. So, we will see that by using just slightly different phase difference, we can increase the directivity of the antenna. And then we will look into scanning array whereby changing the phase element you can stand the beam in different directions. After that we will talk about non-uniform amplitude and we will actually see why we need it.

First of all, if you use uniform amplitude, what we will see in the case of a uniform, first side lobe level is about 13 point 5 dB below. And we cannot get lower than that. So, by using non-uniform amplitude we can reduce the side lobe level to 20 dB or even 30 dB. And after that we will take examples of planar array in which we will talk about rectangular array, circular array, triangular array, hexagonal array and so on.

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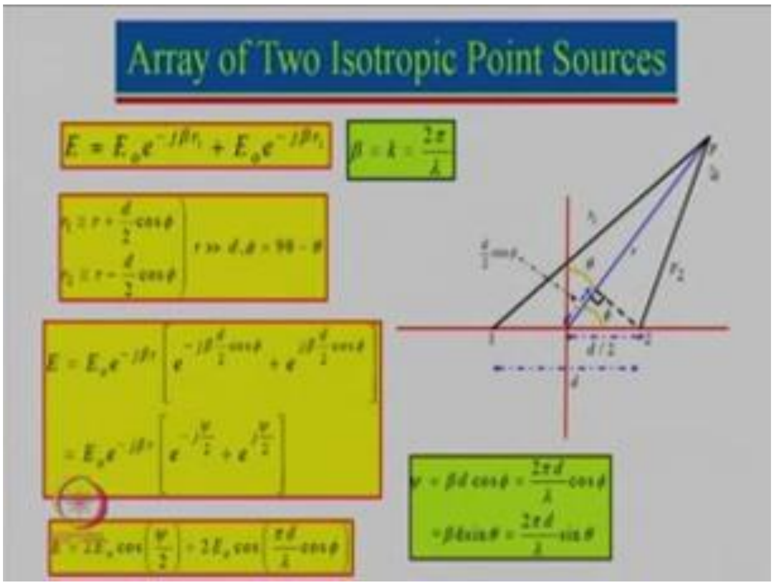

So, let us example of array of 2 isotropic point sources. So, let us say we have 2 sources here, which are source 1 and 2. Both of these sources are isotropic elements, and we want to find out what is the E field at point P. Now few assumptions we will do it here, first is we are actually going to a faraway point. So, we are going to calculate what is the far field radiation. So, the condition over here is that far field distance is much greater than the d, which is the distance between the 2 elements. So, now, assuming that since the distance is very large we can assume that the field over here will be relatively constant as far as the amplitude is concerned, but the phase will be different. So, we can write total E field as E 0 e to the power minus j beta r 1 where r 1 is the distance from source one and beta r 2 from source 2.

Now, since r is much larger than d, we can use approximation. And that approximation is that r 1 can be said approximately equal to r plus d by 2 cos phi and r 2 will be r minus. So, just to look at here this is r 2 here and r is the distance from the origin. So, r 2 can be drawn over here, and this distance will be half of the total distance which is d by 2. This angle is phi. So, we can say that d by 2 times cos phi will be this particular distance. So, r 2 can be approximately written as r minus d by 2 cos phi and r 1 will be r plus d by 2 cos phi. And here we are just measuring angle theta if it is measured from the perpendicular axis. So, then we can say 5 plus theta is equal to 90 degrees. So, we can also write phi as 90 minus theta.

So, now we substitute the value of r 1 and r 2 in this case in this equation here. So, that equation gets modified. What you can see here that r is constant. So, we can write e to the power minus j beta r outside. And the inside term will be corresponding to this term here which will come e to the power minus j beta, and since there is a minus sign and another minus sign this becomes plus. Now here we are just denoting psi. So, psi is beta d cos phi which is written over here. And what is beta is equal to 2 pi by lambda. In fact, many a times we write beta or k. So, both the symbols are acceptable. If you read different books you will see they use different symbol.

So, basically we can write now beta as 2 pi by lambda multiplied by. So, this is the expression for psi instead of measuring angle phi from the axis of the antenna, if we do measure from perpendicular axis then we can write in the form of psi theta, now, when we write here e to the power minus j psi by 2 plus e to the power j psi by 2. This is very similar to writing something like e to the power minus j x, plus e to the power j x. And that will be equivalent to 2 times cos psi by 2. So, we can replace this function by a very simple cos function and now we can substitute the value of psi. So, this is the array factor for 2 elements which are from isotropic elements.

So, now let us see a case here. So, we have just noticed that e is equal to 2 E 0. Now this is a constant factor. So, we can normalize this factor and say it is 1.

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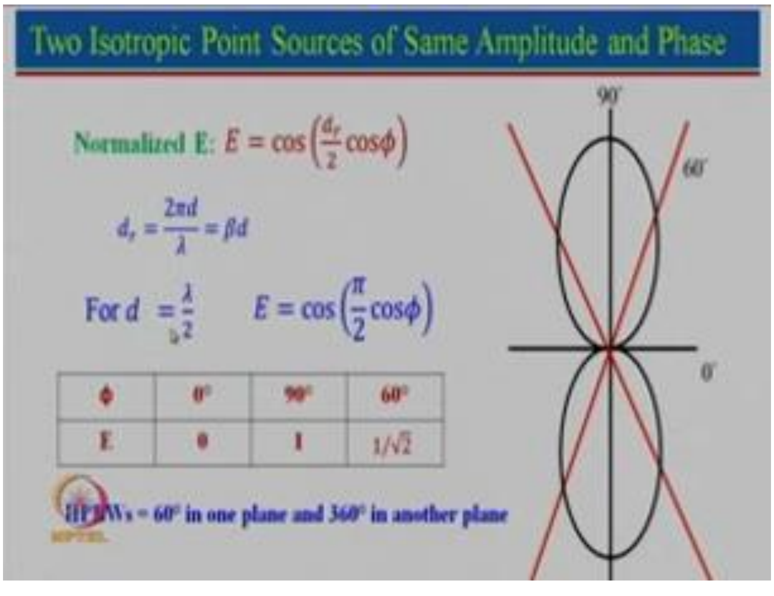

So, we can write normalized E field as cos d r by 2, where d r is defined in terms of beta d, which is equivalent to 2 pi d by lambda. So, one can actually use this simple expression. So, let us take an example where spacing is equal to lambda by 2. So, if we take spacing as lambda by 2, substitute the value over here and using this this gets simplified to cos pi by 2 cos phi.

So, now here phi will actually vary from 0 to full 360 degrees to calculate the overall radiation pattern, but we can just take a few quick numbers and try to estimate the radiation pattern. So, let us say if I take a case of phi equal to 0. Now cos 0 is equal to 1, this inside the bracket will be pi by 2 cos pi by 2 is 0. So, E field will be 0. Now for phi equal to 90, cos 90 will be equal to 0, cos 0 will be 1, and at angle 60-degree cos 60 degree will be 1 by 2, so this will be pi by 4, cos pi by 4 is equal to 1 by square root 2. So, based on just this information we can do the plot. So, this is phi equal to 0 and along phi equal to 0 we can see field is 0. And now this is increasing at phi equal to 60 this value is 1 by square root 2, and then it becomes maximum value which is 1. And then it is getting repeated over here. Now this one is just showing the radiation pattern in this plane, but there is a also pattern which is along that this entire pattern will be rotated along this particular axle.

So, what you are going to see that this lobe is here and that lobe is completely rotated and it will be full 360 degrees. So, we can actually now mention, that half power beam width in this particular plane is nothing, but 60 degrees, we can say that this is the half power point 60 degree is from here. So, this is 30 plus 30 this is 60 degrees. So, half power beam width is 60 degrees in this plane, but half power beam width is 360 degrees in another plane. And these 2 half power beam width can be used to estimate the value of the directivity, and for that we have given you formula earlier which is 4 pi by theta e theta h. So, here in one plane theta e is nothing, but 60 degrees, which is equal to pi by 3 and in the other plane theta is equal to 2 pi, from that you can calculate the directivity of the antenna.

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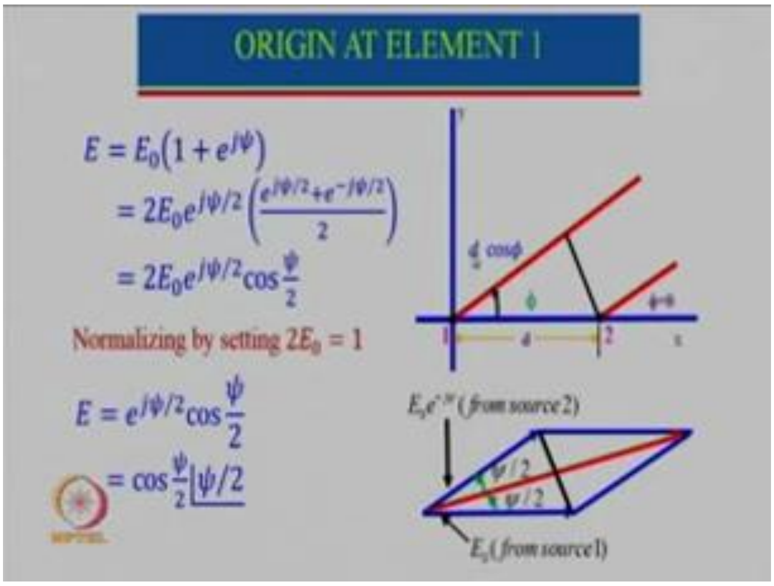

Now, instead of taking the 2 dipoles which were symmetrical with respect to origin, we can actually take one antenna at the origin and we can take second antenna at a distance of d. Now it does not matter how you place it E field should remain same, but let us just look at the derivation now, for one which is at the origin. So, far field will be nothing, but e to the power minus j beta r. So, there will be no r 1, r 1 is equal to r here. So, that term corresponds to 1 and this distance is d. So, now, the term will be e to the power j psi. So, this psi here you should remember that, if the distance is d by 2 this will become psi by 2. If the distance later on we will take example when it becomes 2 d, then this expression will be 2 psi.

So, now this 1 plus e to the power j psi can be simplified, if we take e to the power j psi by 2 outside. What we are left with is the plus and minus half terms. And if you write this whole thing this is same as before which is cos psi by 2. Of course, there is a additional phase term, but the additional phase term is really speaking we are not concerned about when we are looking at a far field. See we are concerned about the phase term only, when you were trying to do vectorial sum of different element. But once you have done the vectorial sum after that we are only interested in the magnitude of the whole system. So, now, by setting E 0 equal to 1, we can write exactly the same expression as before of course, there is a phase term which we are going to neglect. And this entire equation can be also looked into the graphical form.

So, one will be that will be the horizontal here and plus e to the power j psi which will be somewhere here. And if we take the effective or the addition of these 2 vector component, and that will be maximum will be along the direction of psi by 2. So, it does not matter whether we take antennas symmetrical to the origin or one antenna at origin or some other place. Radiation pattern amplitude will remain same.

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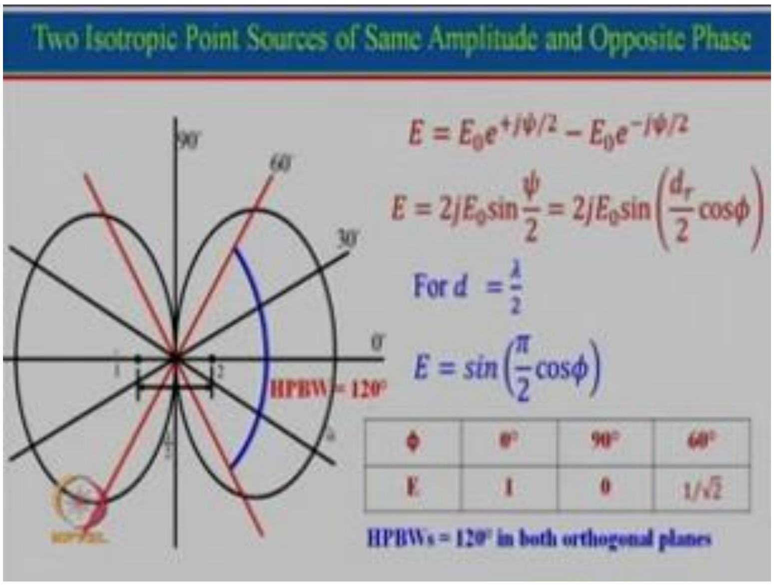

So, now let us just take another case where the 2 antennas are of the same type, but they are fed with opposite phase. So; that means, now this antenna is fed with let us say amplitude a then this is fed by minus a, which is 180-degree phase shift. And they are again symmetrical with respect to origin. So, for the first element the element will be value will be e to the E 0 plus j psi by 2 which is the distance. Here now the second component since it has a negative amplitude. So, we can say that E field has a negative term. So, now, we can combine these 2. So, instead of cos psi by 2 now the term will be sine psi by 2. And this can be further expanded psi we can put the value which is d r by 2 cos phi. Again let us take a case for d equal to lambda by 2.

Now, the question comes all the time why we take d equal to lambda by 2, we can always take d as lambda by 10 or lambda by 4 or lambda or 2 lambda or 3 lambdas. Now actually speaking when you take the spacing as lambda by 2, the mutual impedance between the 2 is actually close to 0. So; that means, this antenna can be individually optimized and another one can be individually optimized and the impedance loading of this antenna to this will not be there. So, basically what happens if you excite one antenna, some field will be induced on the second antenna. And if the second antenna is excited some field will get induced on the first antenna. So, if we take a distance as lambda by 2, the impedance effect is almost negligible. That is why we take as lambda by 2, but later on we will see what happens. If you change the spacing what is the effect on the gain radiation pattern and so on.

So, right now we will take a special case of d equal to lambda by 2. We substitute this value over here and we get the total pattern as sine pi by 2 cos phi. So, let us try to plot again. So, we will take again a few cases here. So, let us take we take phi equal to 0. So, cos of 0 will be equal to 1. So, this expression becomes pi by 2 sine pi by 2 is 1, when pi is equal to 90-degree cos 90 will be 0. So, sine of 0 will be 0 when pi is 60-degree cos 60 will be half this will become pi by 4. So, sine pi by 4 is equal to 1 by square root 2. So, if we now start plotting it over here. So, along pi equal to 0 field is 1. So, field is maximum. As we move along field is decreasing it becomes 1 by square root 2 here and then it decreased to 0. And then this whole thing is repeated in 0 degree 90 degree to full pi plane here. And now we can actually see the half power beam width in this case is equal to 120 degrees.

Now in the previous case we had seen half power beam width was 60 degrees, over here we see the half power beam width is 120 degrees. So, that made people think why we cannot improve it, and this thought process only led to the increased directivity endfire array which we will see later on.

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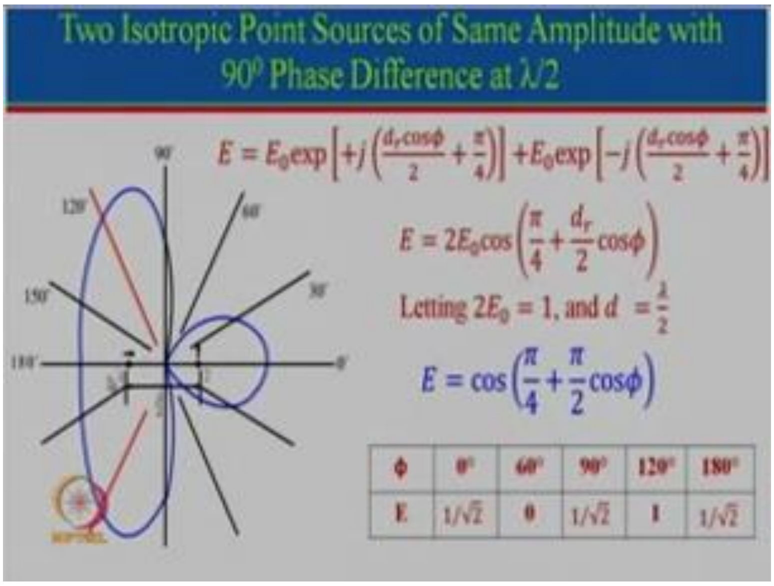

ϕ	0°	60°	90°	120°	180°
E	$1/\sqrt{2}$	0	$1/\sqrt{2}$	1	$1/\sqrt{2}$

So, now let us take a next example where the 2 antennas are fed with equal amplitude, but the phase difference is 90 degrees. Here we are going to take 2 different cases of distance. First case we will take when the distance is lambda by 2 and since the phase difference is 90 degrees which is equal to pi by 2. So, we can actually take 0 pi by 2 or we can take minus pi by 4 and plus pi by 4. So, that is the value which has been put here, by substituting the value of that phase difference. So, that equation gets modified slightly, and this is the combined effect where we are putting pi by 4 plus the rest of the term. This term is same as what we had seen earlier when the phase difference was 0, just because of this additional 45 degrees on one side we are getting the term.

So, again we normalize. So, let us say 2 E 0 is normalized to 1. We are taking a case of d equal to lambda by 2. So, we substitute the value. So, this is the total field for these 2 antennas which are of equal amplitude, but 90-degree phase difference. Again we do the same thing. So, we put pi equal to 0 and then cos 0 will become 1, this is pi by 2 plus pi by 4 that comes here, and then when pi 60 cos 60 will be half. So, that will become pi by 4 pi by 4, plus pi by 4 will be pi by 2 cos pi by 2 is 0. And that is how you fill the entire axis starting from 0 degree to 180 degrees and that will be repeated for 360 degrees.

See again we start plotting this whole thing. So, along 0-degree field is 1 by square root 2, which is now going to 0 at 60 degrees. And then it increases to 1 by square root 2 and then it goes here at 120 degrees it is maximum. Then at 180 it becomes 1 by square root

2. So, if you really see here in this particular half the half power beam width is complete, one 180 degrees. Because it starts from 1 by square root 2 here, and then it goes to one then 1 by square root 2 goes to one and 1 by square root 2 and there is a back radiation here.

However, this situation changes slightly if I instead of taking lambda by 2 distance.

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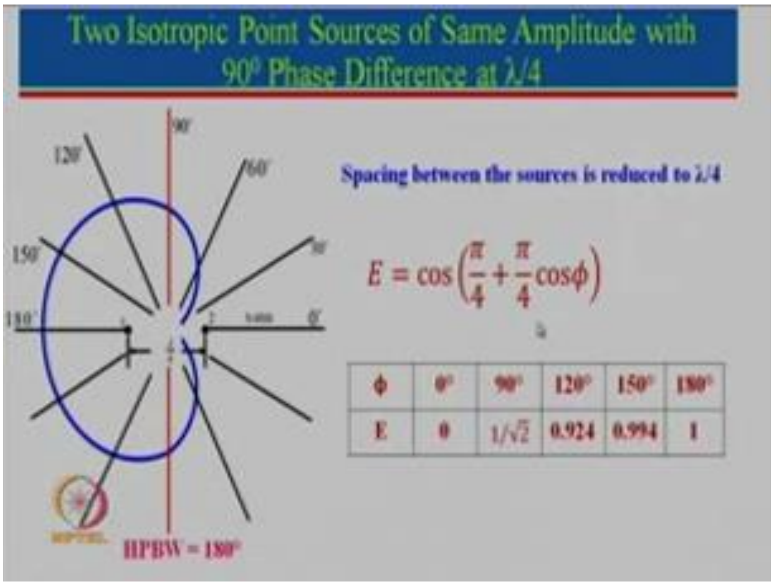

If I take lambda by 4 distance; if you put instead of lambda by d equal to lambda by 2 over here, we put lambda by 4 this equation gets modified over here. And now if you substitute different values of phi just to take one example. So, cos 0 if we put here, cos 0 is equal to 1. So, this will be pi by 4 plus pi by 4 will be pi by 2 this value will be equal to 0. And then we can fill all the other rows here, now, coming to this one here. So, we start plotting. So, this is 0 degree and at 60 degrees it is increasing, at over here it becomes 1 by square root 2 and the beam maxima is over here, and then it is coming back to this. Now if you look into here that the back radiation is almost negligible.

Now, you can actually think about for example, if the antenna is put near the coastal area. So, if you put omnidirectional antenna at the near the coastal area then it will be radiating lot of power in the sea or along the ocean, which is not really desired, but if you put 2 elements of equal amplitude with the lambda by 4 distance and pi by 4 phase difference, then we can actually realize a radiation pattern which is radiating only in the one side and the backside radiation will be almost negligible. Of course, the same thing can also be obtained by using a reflector also which will reflect the signal from the backside.

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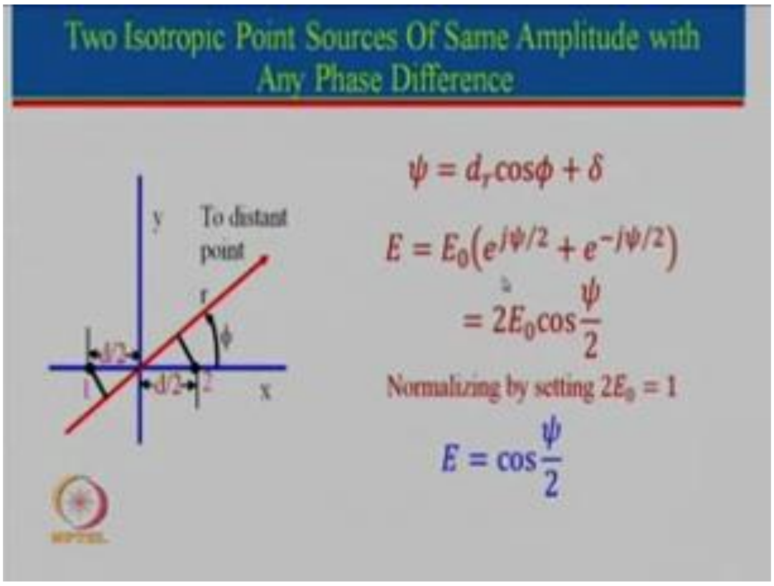

So, let us just take now the next case which is a 2 isotropic sources of same amplitude with any arbitrary phase difference. So, that any arbitrary phase difference is defined by delta here. So, we can again do the derivation that this is minus delta by 2. This is plus delta by 2. So, expression is similar, except that now psi is total will be including plus delta. So, the expression is exactly same as before which is cos psi by 2, except that now psi consists of the distance as well as consists of the phase. And by changing the value of the phase we can actually change the beam direction.

As we had seen if delta is equal to 0 beam maxima was on the broad size direction, and when delta was changed from 0 to let us say 90 degrees the beam maxima was 120 degrees for length spacing equal to lambda by 2. And when delta was 180 degrees the beam maxima was along the endfire. So, you can change the radiation from broad’s broadside to the end fire direction, simply by changing the phase difference between the 2 element.

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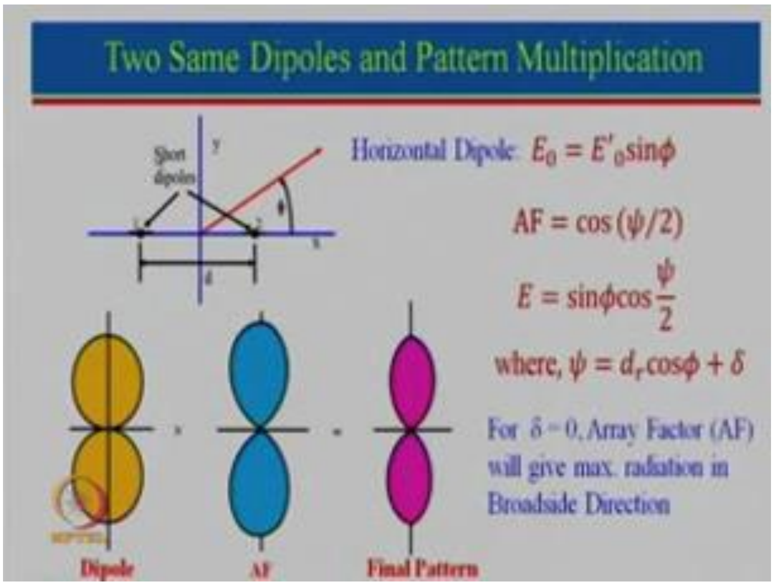

Now, let us just take a real example, where instead of taking isotropic elements. So, we have taken 2 same dipoles. And these are small short dipole and these are kept in the horizontal direction. And for this horizontal direction they are fed with equal amplitude and equal phase that is what means same. So, we have 2 same antennas. Now these 2 antennas are actually kept horizontal a dipole is kept in the horizontal direction. So, where will be the beam maxima? Beam maxima will be in the perpendicular direction. Recall what I mentioned when we were talking about a dipole antenna. So, dipole antenna if you see from the front side, you will actually see the maximum intensity. And as you move from the maximum towards the tip, you will see the minimum radiation, but in this case dipole is not vertical it is horizontal.

So, for horizontal, if you look at the dipole you will see the maximum radiation in the broadside direction and minimum radiation in this particular direction which is along the x axis. Now that is the dipole pattern.

Now, array factor since these 2 amplitudes are same phase is same. So, array factor will be having in the maximum in the broadside direction. So now the total pattern will be multiplication of these 2 pattern and that is known as principle of pattern multiplication. So, in the form of equation we can say for the dipole we can write the equation. Now this is sine phi it is important how phi is mentioned and how you are doing. So, in this case if you see phi is measured from the x axis. So, in this direction phi is equal to 0. So, sine 0 will be equal to 0, and that gives the minimum radiation. And that phi equal to 90-degree sine 90 will be 1, and that gives the maximum radiation here.

Now, the array factor is nothing, but cos psi by 2. So, what will be the total radiation pattern? It will be multiplication of these 2. So, that is the total field. And what is psi? Psi is over here. And since it is broad size delta is equal to 0. So, now, we need to multiply these 2. So, if you just look at here. So, this is 1, multiplied by 1 will be equal to 1. So, let us say if this is 0.7 here, and let us say this is also approximately 0.7. Then 0.7 multiplied by 0.7 will be 0.49, which will make it much narrower so. In fact, you can think in the lighter vein that 2 fat people multiplied together will give rise to a slim person. So, if we can apply this concept in the real life, we can actually solve the problem of obesity from the entire world, where 2 fat people multiplied together will lead to the narrow people. Or thin person well that does not happen in the human being, but over here the simpler wires or dipoles here can multiply and this is giving a narrow beam, and if the beam is narrow naturally it will have a higher directivity.

Now, it is important how you place the antenna. Instead of putting the antenna in the horizontal in this case if we put the dipoles in the vertical orientation, which is right over here which is a vertical.

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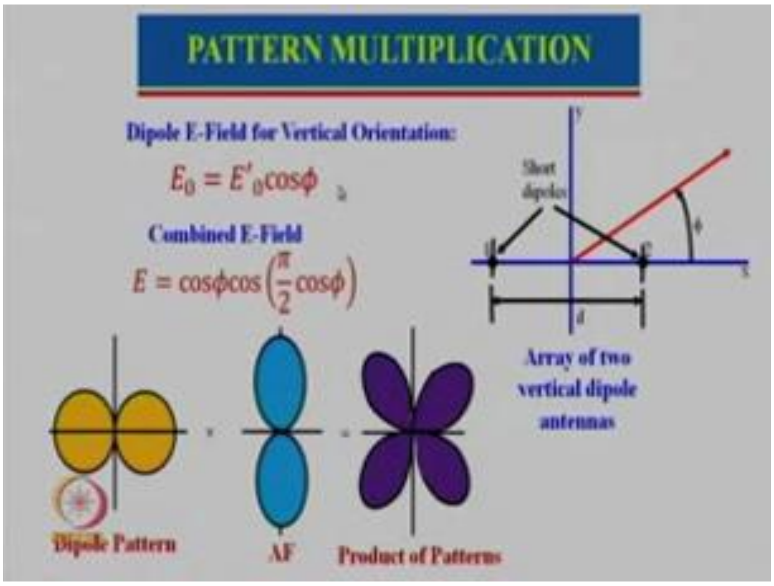

So, now even though the dipole is same, but we are now writing instead of previous case we had written sine phi, but now it becomes cos phi. So, it is important that from where phi is measured. So, just look again here this is the phi equal to 0 axis. So, cos 0 will be 1, that will give me maximum radiation here and when phi is equal to 90 degrees that will get to 0. So, this is what is the dipole pattern.

But the array factor remains the same because the 2 dipoles are fed with equal amplitude and equal phase. So, array factor is in the broadside direction. So, now, if we multiply the 2 what we get we get kind of a butterfly pattern. Why let us see this is 1 here. 1 multiplied by 0 will be 0 here, over here this is 0, but this is 1. So, one multiplied by 0 will be again 0 here. So, in between along this here this has a finite value, this also has a finite value. So, that gives me a maximum thing, but now suppose you just thought this as a dipole antenna, and you are going to put the thing, but orientation is very important. So, if you just put horizontal then the beam maxima will be this, but if you made the mistake of putting it vertical, and you are trying to make the measurement in the broadside direction you will actually get a 0 value. And that actually creates butterflies in your stomach that is what is the pattern about.

So, please ensure that how you place the antenna what is the orientation of the antenna and how do you feed the antenna. Of course, this vertical thing will be perfect case if you are designing an endfire array, if it was endfire array, then what happened this is the dipole pattern which is maximum here. Endfire array then the array factor will be also maximum in this. And then what will you get you will get narrower beam in this thing that would mean higher directivity in the endfire direction. So, it all depends upon what is the application you are looking at.

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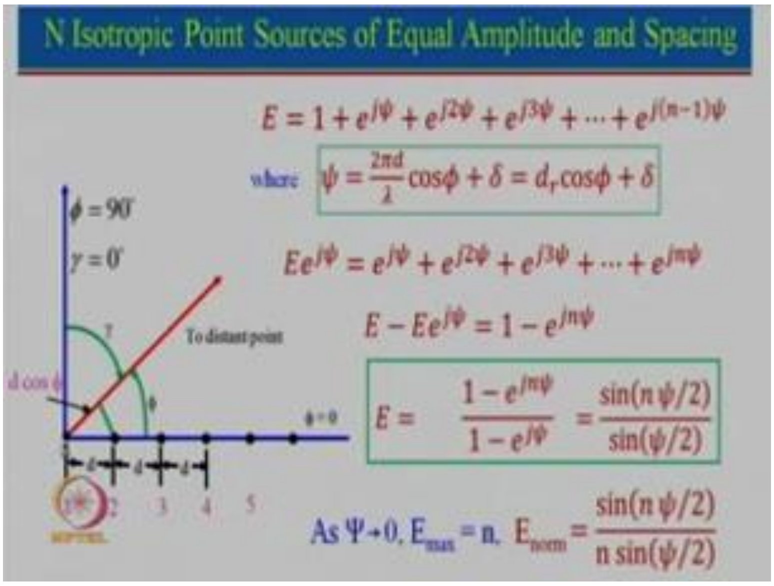

So, now we can actually go through the next point n-isotropic point sources of equal amplitude and spacing. So, here what we have? There are n number of elements are there and they are started with 1 2 3 4 5 and so on. We are starting from origin. So, that will be the first point here axis is same this is phi equal to 0. So, this will be phi equal to 90 degrees over here. So, we can actually now find out what is the total field at a faraway point. So, it is relatively simple if you follow the principle which we did for 2 elements and since this is origin, origin the distance will be r 1, but r 1 is equal to r. So, this will be 1.

The next term will be e to the power j psi. Then the second will be next will be 2 psi because the distance is 2 d, then 3 d is the distance of 3 psi and so on. Now where psi is as before, there is a no change. Now we need to solve this particular equation. The easier way to solve this equation is that you multiply this entire equation with the e to the power j psi. So, it is multiplied on both left side and right side. So, that is the term over here. Now all it is we take the difference of these 2. And if we take the difference of the 2 this term will be left which is 1, and the last term over here. All the other terms will get cancelled over here. So, this is the left side and that is the right side. From here we can find what is e and that is given by this expression.

And this expression now can be simplified to this in a very simple way. What you do you take e to the power j n psi by 2 outside here, and from here e to the power j psi by 2 outside here. And then what is left is e to the power minus j n psi by 2 minus e to the power plus j n psi by 2 which is nothing but sine n psi by 2 and down below it will be sine psi by 2. So, this is the array factor. And what is the maximum value of this array factor? The maximum value is when psi is ending towards 0. Because when psi is ending towards 0, see if you just look precisely 0, it will give you something different. Like say sine 0 is 0 divided by sine psi by 2 that is 0. So, 0 divided by 0 will be indeterminate.

But you have to apply de-hospital rule and that says when it is tending to 0. So, sine x tending to 0 where z is small will be equal to x. So, this term becomes x and this n psi by 2 and down below is psi by 2. So, the maximum value will be equal to n. So, we can now write e normalized is nothing, but sine n sine by 2 divided by this here. Now this is an important thing and in the next lecture we will talk about more detail about this particular thing here. And remember all the pattern all the calculations can be derived from just this one very simple equation. I will just show you the plot, but we will continue in the next lecture.

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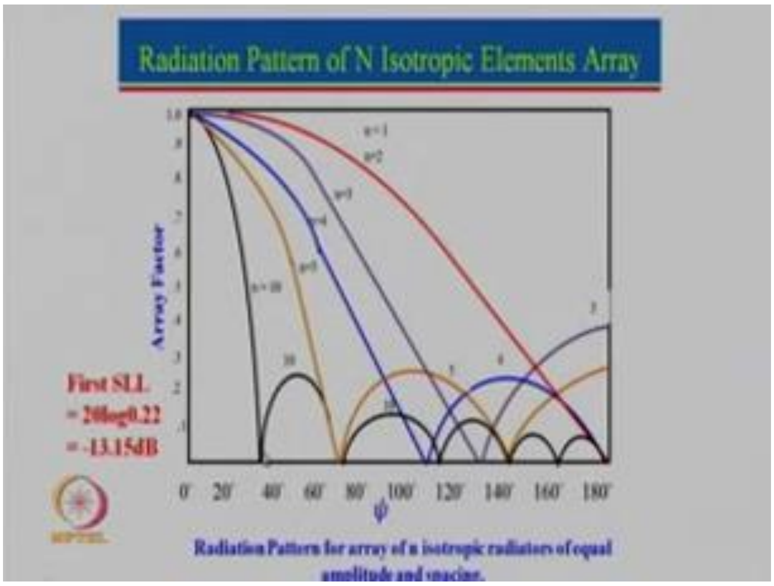

So, this plot which has been plotted for n equal to 1 2 3 4 5 and 10, and if you can actually see the number of side lobes here. By using this equation, we can find out what is the half power beam width, we can find out what is the direction of the null, we can find out in which direction there will be side lobe, what will be the amplitude of the side lobe. So, in the next lecture we will do the larger array and we will also see different cases.

But just to summarize. So, we started with the 2 isotropic elements, which are fed with equal amplitude different phases, and we also saw for 90-degree phase difference, for 2 different cases lambda by 4 and lambda by 2. And for lambda by 4 we could actually realize the radiation which is negligible in the back direction. And then we looked into the pattern multiplication. And how the dipoles can be placed and how the radiation pattern cumulative can be obtained. And we just looked at the derivation of the n element. So, please go through it and in the next lecture. We will talk about more about how to calculate directivity, how to calculate null positions and so on.

Thank you.