Basic Electronics
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Lecture - 09
RC/ RL circuits in time domain (continued)

Welcome back to Basic Electronics. In the last lecture we obtained analytic expressions for the capacitor of voltage and current, for a series RC circuit with the step input voltage; we considered both charging and discharging transients. In this lecture we will look at the graphs for these quantities, and learn a lot more about these transient. We will then look at how to handle an RC or RL circuit with the piece wise constant source and follow that up with an example. Let us start.

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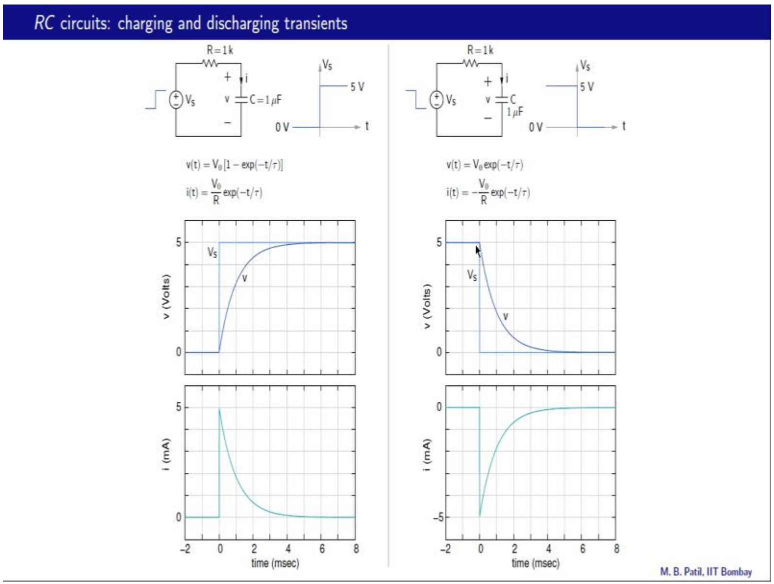

These are the plots the input voltage is given by this step V form the light blue curve, and the capacitor of voltage is given by this curve here the dark blue curve; and let us check whether this does correspond to the expression for the capacitor voltage that we have over here; what is v of t at t equal to 0? At t equal to 0 this term is 1. So, therefore, we have V 0 times 1 minus 1 that is 0. So, at t equal to 0 we expect v of t to be equal to 0. What about t equal to infinity? At t equal to infinity this term is 0 and therefore, we get v equal to V 0 and that is what we see over here and how long does the transient take? As we are seen earlier the transient takes 5 10 constant.

Let us check whether that is happening, we have R equal to 1 k, C equal to 1 micro farad. So, the time constant is 1 k times 1 micro that is 1milliseconds. So, we expect the transient to last for 5 times the time constant that is 5 milliseconds; 5 milliseconds is over here and we observed that the transient does vanish at about that time and there after the circuit teaches a steady state, and there is no perceptual variation in either the capacitor voltage or the capacitor current beyond that point.

What about the current the current is given by this expression, at t equal to 0 plus this term is 1 and therefore, the current V 0 by R, our V 0 is 5 volts, R is 1 k. So, this number is 5 volts divided by 1 k or 5 million and that is what we observe over here. At 0 minus the current is 0; at 0 plus it is 5 millions. As t to needs to infinity this term goes to 0 and therefore, the current is expected to go to 0 and that is indeed what we observe over here.

Let us know make a 5 observation, first the capacitor of voltage is continuous and that is of course, expected as we explained earlier, because if there was a discontinuity in the capacitor voltage that would need to very large currents, which would not satisfy our circuit equations. On the other hand there is no subscription on the capacitor current, and we do observe discontinuity at t equal to 0. Second observation the capacitor current is always positive as is also obvious from this equation, this term e raised to minus t by tau e is always positive, and V 0 and R are positive constants and therefore, the capacitor current is always positive.

Now, since the capacitor current is c d V by dt, a positive capacitor current means a positive d V dt and therefore, we expect the capacitor voltage to always keep rising like that. Eventually of course, the capacitor current becomes 0 and therefore, the capacitor voltage becomes constant. If the current is 0 ; that means, d V dt is 0 ; that means, V becomes constant as you know over here in this graph. Since the current is large in the beginning, we have a larger slope d V dt here; now the current goes on decreasing so this slope also goes on decreasing and eventually of course, the current becomes 0 and the slope becomes 0 ; that means, V becomes constant.

Finally notice that the shape of v of t or i of t is one of the 2 shapes we saw earlier in the graph of F of t equal to A e raised to minus t by tau plus B; if you do not remember that you can go back a few slides and make sure that these 2 shapes indeed follow what we predicted in that slide over there.

Let us now look at the discharging transient this is the circuit V s goes from 5 volts to 0 volts, at t equal to 0 and the expressions for v of t and i of t which we derived earlier are reproduced over here; v of t is V 0 e raised to minus t by tau, i of t is minus V 0 by R, e raised to minus t by tau. Here are the plots this is our source voltage V s it starts at 5 volts it has been 5 volts for a long time, and then at t equal to 0 it abruptly changes to 0 and then remains 0 thereafter. This dark blue curve is the capacitor voltage as a function of time and this graph here is the capacitor current, notice that the capacitor current is negative.

Let us check whether these results the plots here correspond to the equations that we have derived. Let us look at the capacitor voltage first. At t equal to 0 this term is 1 and therefore, v of t is V 0, V 0 is 5 volts and therefore, the capacitor voltage is also 5 volts. As t tends to infinity this term becomes 0 and therefore, the capacitor voltage becomes 0 as in this graph. And how long does this take? We expect that to take 5 time constants R is 1 k here, C is 1 micro farad as in the previous case and therefore, the time constant is 1 millisecond. 5 tau is 5 milliseconds therefore, at 5 milliseconds we expect the transient to vanish and that is what we observe over here V becomes constant after about 5 milliseconds.

Let us look at the current plot now, here is the expression for the current and remember that this is valid for t greater than 0. Now at t equal to 0 plus this exponential factor becomes 1 and we have I of t equal to minus V 0 by R, V 0 is 5 volts R is 1 k and therefore, we have I of t equal to minus 5 volts by 1 k that is minus 5 millions and that is indeed what we observe in this graph here. As t increases this exponential factor decreases and therefore, we expect the capacitor current to decrease in magnitude, it will of course, remain negative because of this minus sign.

Eventually as t tends to infinity this factor will become 0 and therefore, the current will become 0 and that is what we observe in this graph; as t increases the current decreases in magnitude and eventually it becomes 0. As we would expect the capacitor voltage is continuous, but the current is discontinuous, there is a discontinuity at t equal to 0 and this observation is similar to what we saw in the charging case.

Now, the capacitor current in this case is always negative eventually of course, it becomes 0, and what does that mean? Since the capacitor current i is c d V dt negative current means a negative d V dt; that means, the slope of the capacitor voltage versus time graph would always be negative and that is what we observed over here.

So, therefore, the capacitor voltage keeps decreasing eventually of course, the current becomes 0, d V dt becomes 0 therefore, the voltage becomes constant. So, there is a lot to learn from graphs and you would have heard that a picture is worth a thousand words, and that is also true about graphs. A graph is packed with information we only need to look for it, and there are several features that we should look for and we should make sure that they are compatible with the analytical understanding or the equations that we have.

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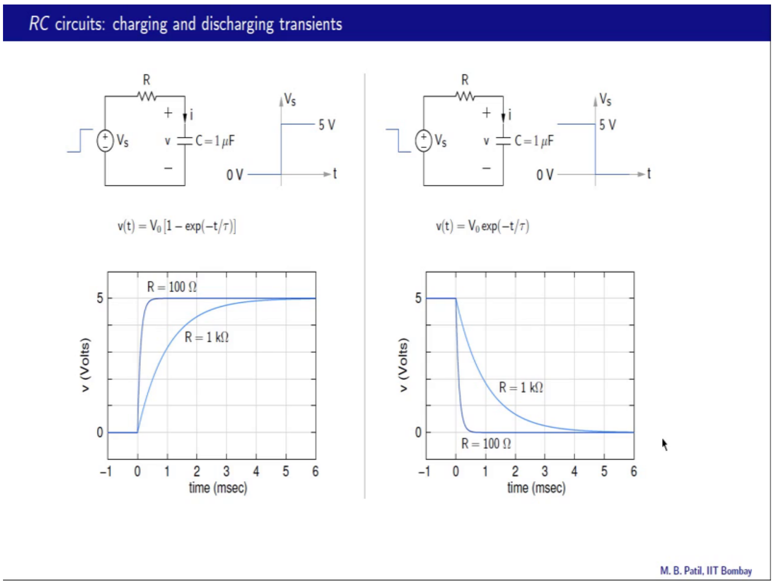

Let us now look at the effect of the time constant on the capacitor voltage transient; first let us consider the charging case in which the source voltage changes from 0 volts to 5 volts abruptly at t equal to 0, and as a result the capacitor voltage also changes it follows this equation which we are derived earlier. Now with R equal to 1 k, the time constant is 1 k times 1 micro that is 1 millisecond and in about 5 time constants that is in 5 milliseconds we expect the capacitor voltage transient to vanish and a steady state to be established; and that is what we observe in this plot, this sky blue curve corresponds to R equal to 1 k, in the beginning the capacitor voltage has been 0 volts for a long time. at t equal to 0 it starts rising towards 5 volts and in about 5 milliseconds this time point here it reaches almost 5 volts and does not change after that.

Now, let us look at the second case namely R equal to 100 ohms that is 0.1 k, what is the time constant in this case? It is 0.1 k times 1 micro that is 0.1 milliseconds. Now in this case we expect the transient to last for 5 times 0.1 milliseconds, that is 0.5 milliseconds and that is indeed what we observe in this plot. This is 1 millisecond 0.5 millisecond is somewhere here and we observe that for R equal to 100 ohms, we reach the steady state in about 0.5 milliseconds and after that point the capacitor voltage does not change.

Let us now consider the discharging case. So, V s changes from 5 volts to 0 volts at t equal to 0, and as a result the capacitor voltage also changes and it follows this equation which we have derived earlier. Now with R equal to 1 k the time constant is 1 millisecond and in about 5 time constants, the transient vanishes as seen in this graph this is 5 volts, and t equal to 5 milliseconds we reach steady state and the capacitor voltage does not change thereafter. For R equal to100 ohms, the time constant is 0.1 milliseconds and in 5 tau that is in 0.5 milliseconds, we reach steady state here and the capacitor voltage becomes constant.

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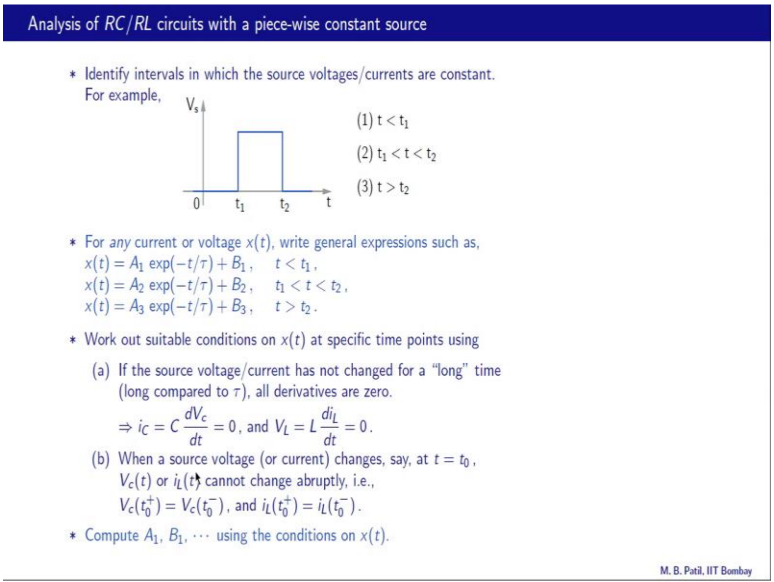

So, far we have considered RC and RL circuits with the constant source; a voltage source or a current source. Let us now look at RC and RL circuits with a piece wise constant source, here is an example there is a voltage source here which varies with time like that, in this interval t less than t 1 this voltage is 0, between t 1 and t 2 it is non 0 let us say 5 volts, and then for t greater than t 2 it is 0. Again let us see how we can handle such a situation; clearly this is not a d c source or a constant source, but it is piece wise constant we have one piece here which is constant we have another piece here which is constant and we have a third piece here which is constant.

And in each of these intervals, we can use the expressions that we have derived that is for any current or voltage x of t in the circuit, we can write a general expression such as x of t equal to A 1, e raise to minus t by tau plus B 1 for the first piece, that is t less than t 1. Similarly for the second interval from t 1 to t 2, we can write x of t as A 2 e raised to minus t by tau plus B 2. Now this A 2 and B 2 are also constants and generally they would be different from A 1 and B 1; and finally, in this third interval that is t greater than t 2 we can write x of t as A 3 e raised to minus t by tau plus B 3. In order to get a complete picture, what we need to do next is to find this tau, the circuit time constant and also these constants A 1 B 1, A 2 B 2 etcetera.

Now, the time constant is given by R times c for an RC circuit, where R is the Thevenin resistance seen by the capacitor; for an RL circuit tau is given by L divided by R where R is the Thevenin resistance seen by the inductor. We have already seen these formulas earlier; how do we obtain these constants? A 1 B 1 A 2 B 2 and so, on what we need to do is to work out suitable conditions on x of , which could be a current or a voltage at specific time points for example, in this case we might look at x of t at t 1, t 2 minus infinity and plus infinity.

And in order to do that in order to find these conditions, we can use the following considerations; a if the source voltage or current has not changed for a long time long compared to the circuit time constant tau, then we know that all derivatives must be 0 because all quantities in the circuit, all voltages and currents must have become constant, the circuit must be in a steady state. For example, take this piece if V s has been 0 for a long time, then we know that when we come to t 1 minus that is just a little bit before t 1, then the circuit is in steady state and all voltages and currents must be constant; and once we know that we know that the capacitor current which is given by C d V c dt must be equal to 0.

So, the capacitor can be treated as an open circuit and the inductor voltage which is given by L di L dt must also be 0 because i L would have become constant; that means, the inductor can be replaced with a short circuit since the voltage across the inductor is 0; b when a source voltage or current changes say at t equal to t 0, in this example t 0 would be t 1 or t 2 then the capacitor voltage or the inductor current cannot change abruptly as we have seen earlier; that means, we see at t 0 plus the capacitor voltage at t 0 plus, must be equal to V c at t 0 minus and in an RL circuit the inductor current i L at t 0 plus must be equal to the inductor current at t 0 minus.

So, using these considerations, we can work out suitable conditions on the current or voltage at specific time points and then use those to compute these constants A 1, B 1, A 2, B 2 and A 3, B 3 in this example.

So, once we have tau and once we have these constants, we have a complete description for the quantity of interest.

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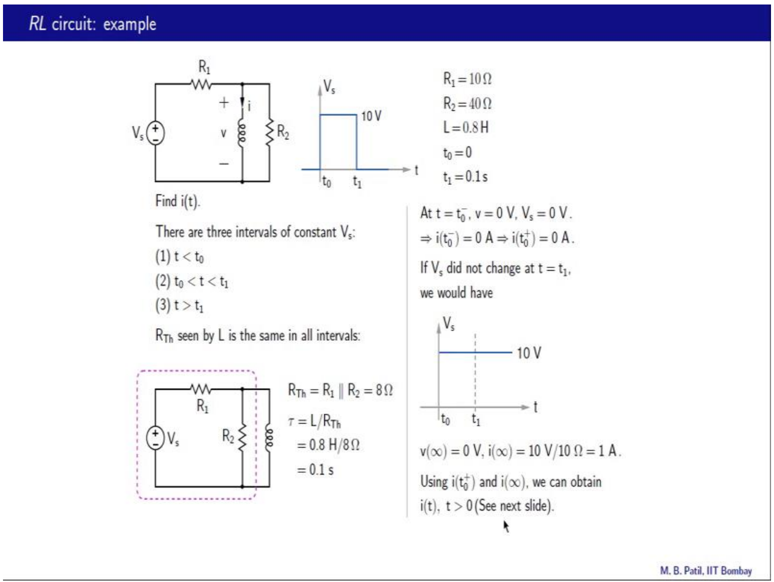

Let us now apply the ideas that we have just learnt in the last slide to this particular example; we have an RL circuit here with 2 registers R 1 and R 2 and a single inductor. R 1 is 10 ohms, R 2 is 40 ohms and the inductance is 0.8 henry. The input voltage is a pulse given by this graph here; t 0 is 0, and t 1 is 0.1 second. Up to t 0 the source voltage V s is 0 volts, between t 0 and t 1 it is 10 volts, and after t 1 it is again 0 volts and we are interested in finding this current through the inductor as a function of time.

Step number one is to identify intervals in which the input voltage is constant and from this graph you can see that there are 3 such intervals, interval 1 t less than t 0 where V s is 0 volts; interval 2 t 0 less than t less than t 1 in which V s is 10 volts and interval 3 that is t greater than t 1, in which V s is 0 volts. Like we mentioned in the last slide what we need to do next is to consider each of these intervals identify the circuit time constant in each interval, write a general expression for i of t in each of these intervals and then find suitable conditions on i of t at suitable time points, which will enable us to calculate the coefficients, the constants involved in those expressions.

So, let us begin with the circuit time constant; now we figure that in all of these intervals the time constant does not change, because the circuit topology is the same in all 3 cases. So, let us proceed with that calculation now. What is the circuit time constant tau? It is L by R Th, where R Th is the Thevenin resistance as seen by the inductor. To make this calculation a little easier let us redraw the circuit as shown here, what we have done is we have taken this inductor on the other side that of course, does not change the circuit and now we look at the rest of the circuit this circuit here from the inductor; and note that the circuit topology is the same in all 3 intervals, the only thing that is changing is the value of V s; in the first interval V s is 0, in the second interval it is 10, and in the third interval it is 0 .

So, the Thevenin resistance seen by the inductor is the same in all 3 cases and how do we find that? We deactivate the voltage source; that means, we short this voltage source and then what happens is R 1 and R 2 come in parallel so therefore, R Th is simply R 1 parallel R 2. So, that turns out to be 10 parallel 40 or 8 ohms, then the time constant is given by L divided by R Th over L is 0.8 n d, R Th is 8 ohms. So, therefore, the time constant is 0.1 second. Next let us find suitable conditions on i of t, which will eventually enable us to find the constants involved in the expressions for i of t in these 3 intervals. Let us consider t 0 minus; that means, just before t 0, what is the situation at t equal to t 0 minus? The source voltage has been 0 for a long time, we have a steady state all currents and voltages have become constants and therefore, this V which is L di by dt is 0.

If V is 0 the voltage across R 2 is 0 that means no current can flow through R 2. So, the current path that we have is like this, and since this voltage drop is 0 the current I must be V s divided by R 1. Now V s is 0 in this interval up to t 0 minus and therefore, the current i at t 0 minus must be 0 like that. Now since this i is an inductor current, we know that it must be continuous; that means, i at t 0 plus must be equal to i at t 0 minus and therefore, we get i at t 0 plus equal to 0 ampere.

Let us now obtain another condition on the current, and we will do that by assuming that this V s is did not change; in real life of course, V s has changed at t 1, but let us pretend that it has remained constant.

The question is does this make sense, how can we assume that V s is 10 volts in this interval when it is actually 0 volts? The answer to that question is that it does make sense, as long as we are in this region up to t 1. So, what we will do is we will assume this condition, but we will consider the solution to be valid only up to this point t 1. To put it a bit informally, the inductor or the circuit does not really know that a change is going to take place add this point, and therefore the response of the circuit would be the same in this interval.

Whether or not V s state constant or it went back to 0 at this point; with this situation let us now find this current as t tends to infinity, what do we have at t equal to infinity? The circuit would have reached steady state all currents and voltages would have become constant therefore, this V which is L di dt would have become 0; so we have a short circuit here, and the current i would then be V s divided by R 1.

The current through R 2 of course, would be 0 because this voltage drop is 0, at t equal to infinity V s would be 10 volts. So, therefore, the current I would be 10 divided by R 1 that is 10 volts divided by 10 ohms that is 1 ampere. Let us now use these 2 conditions namely i at t 0 plus and i at infinity to obtain the solution for i of t in this interval, and we will do that in the next slide.

In conclusion we have started looking at RC and RL circuits with a piece wise constant source, in the next lecture we will continue with the RL circuit example and obtain the complete solution until then goodbye.