Basic Electronics
Prof. Mahesh Patil
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 08
RC/RL circuits in time domain (continued)

Welcome back to Basic Electronics. In this lecture we will look at a series RC circuit with a single resistor a single capacitor and a step input voltage. It is a simple circuit and the solution is well known, we will not simply state the solution, but derive it using what we already know and when we do that, we will learn some useful ideas which would certainly help in the long run. So, let us begin.

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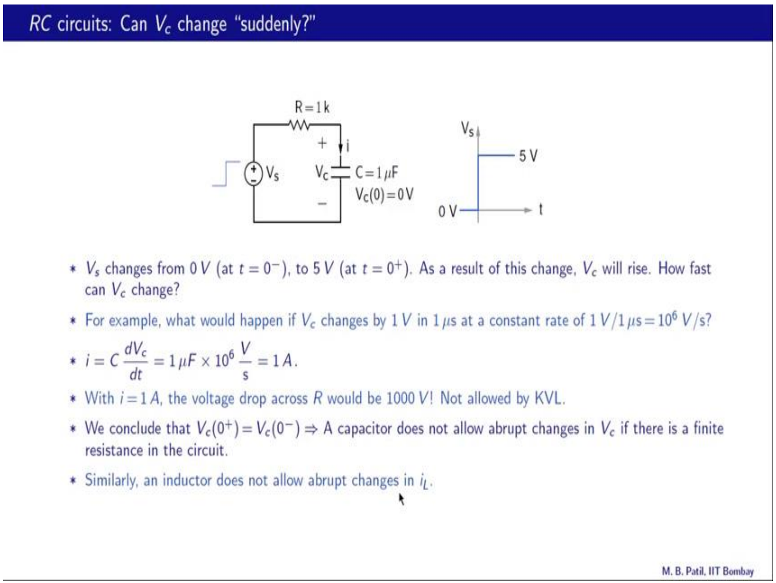

We have looked at the general solution for a current or a voltage in an RC circuit with d C sources; let us now answer this important question, given an RC circuit we want to find out if the capacitor voltage can change suddenly or abruptly and we will answer this question with the help of this example here, what do we have here? We have a series RC circuit R is 1 k, C is 1 micro farad and this RC circuit is given by a step voltage source up to t equal to 0 the voltage is 0 volts, and at t equal to 0 it changes from 0 volts to 5 volts.

The capacitor voltage is given to be 0 volts at t equal to 0. So, we see 0 up to this point, and now as a result of V s changing from 0 volts to 5 volts, we expect this capacitor voltage to change. So, this is our question V s changes from 0 volts at t equal to 0, minus to 5 volts at t equal to 0 plus that is just after t equal to 0, and as a result of this change V c will rise the question is how fast can V c change?

So, for example, what would happen if V c changes by 1 volt in 1 microsecond, and let us assume that this is happening at a constant rate that is 1 volt divided by 1 microsecond or 10 raise to 6 volts per second. Because of this d V c dt, a capacitor current will flow which is given by i equal to C d V c dt C is 1 micro farad, and d V c dt is 10 raise to 6 volts per second. So, this product turns out to be 1 ampere. With i equal to 1 ampere let us calculate the voltage drop across this resistance down, this R is 1 k or 1000 ohms and R times i is the voltage drop, so that would be 1000 ohms times 1 ampere.

So, that is 1000 volts that is a very large drop, and this would simply not be allowed by KVL because we have only 5 volts here we cannot have 1000 volts over here and therefore, we conclude that V c at t equal to 0 plus must be equal to V c at t equal to 0 minus. If this were not equal that would lead to a very large capacitor current and the KVL equation would not be satisfied.

So, therefore, a capacitor does not allow abrupt changes in V c if there is a finite resistance in the circuit. As long as there is a finite resistance in the circuit, which is usually the case we must have the capacitor voltage to be continuous. Now in an R L circuit we must have continuity of the inductor current, because any abrupt change in inductor current would lead to a very large l d i, l dt that is a very large voltage across the inductor and that again would not be allowed by KVL.

So, the take home point from this slide is that the capacitor voltage must be continuous and the inductor current also must be continuous, as long as there is a finite resistance in the circuit.

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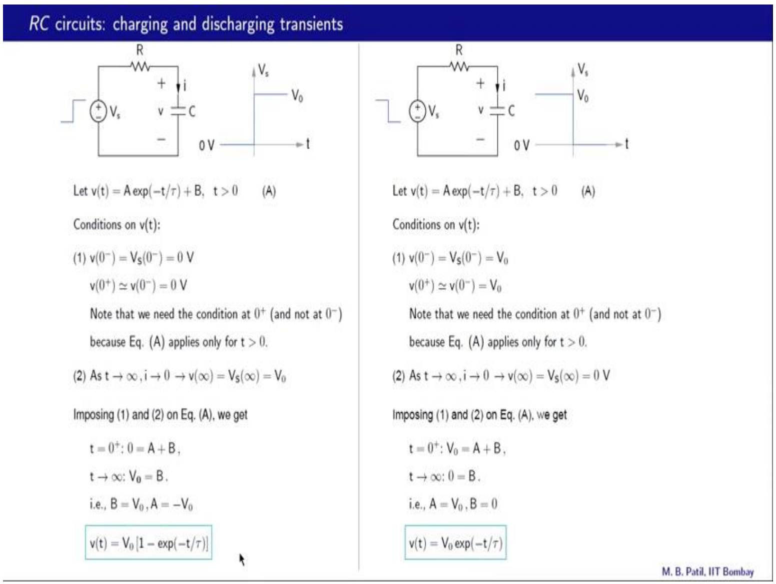

As our first example let us consider this simple RC series circuit. So, we have R and C in series and this combination is driven by a voltage source with a step voltage. So, this V s is 0 volts up to t equal to 0, and then it abruptly changes to V 0 and it remains constant after that point and we are interested in finding this capacitor voltage and the capacitor current as a function of time.

Let us begin by writing v of t the capacitor voltage as A e raised to minus t by tau plus B where A and B are constants to be determined. Now this expression if you recall is valid only as long as the independent sources in the circuit are constant. Now in our case we should use this equation only for t greater than 0 because our voltage is constant only for that region, we cannot use this same equation to describe this region for example, what is this time constant in this equation? It is simply the product of C and thevenin resistance as seen by the capacitor.

Now, this circuit is already in the terminate form and R Th as seen by the capacitor is simply R. Therefore, this time constant is R times C. Let us now obtain these constants A and B and first let us look at this circuit at t equal to 0 minus, what is the situation that we have at t equal to 0 minus, the source voltage has been 0 volts for a long time and; that means, that we have a steady state the voltages and currents have settled down to some constant values.

Now, if this V is constant then this current i which is C d V dt must be 0, and if that is the case there is no voltage drop across this resistance and V is then equal to V s and V s is 0 at t equal to 0 minus therefore, v is also 0. So, we have v at 0 minus equal to 0 volts; now as we have seen earlier the capacitor voltage cannot change abruptly and therefore, v at 0 plus must be the same as v at 0 minus therefore, v at 0 plus is also equal to 0 volts.

Now, why are we concerned about v at 0 plus? That is because this equation applies only for t greater than 0, it does not apply at 0 minus. So, therefore, we can use this condition to get A and B, but not this condition. Let us now look at the circuit at t equal to infinity, what is the situation? At t equal to infinity we have V s equal to V 0 and sufficient time has passed to reach steady state and therefore, v and i and all other variables in the circuit would have become constant. And once again this i which is C d V dt would be 0, because v is a constant. And then we have no voltage drop across this resistance and the capacitor voltage V is the same as v s at t equal to infinity and V s at t equal to infinity is the same as v 0 and therefore, we have v at infinity equal to V s at infinity equal to V 0.

So, that is our second condition; we can now use these two conditions V at 0 plus equal to 0 volts and v at infinity equal to V 0 to obtain these constants A and B. So, at 0 plus we have V equal to 0. So, we put 0 over here this e raised to minus t by tau is 1. So, therefore, we have 0 equal to A plus B like that. At t equal to infinity, we have V equal to V 0. So, we put V 0 over here, e raised to minus t by tau a 0 as t tends to infinity therefore, we have V 0 equal to b like that. So, our b is V 0 and therefore, A must be minus B 0 like that.

We now substitute these values for A and B back into this equation and then that gives us the final result for v of t. So, v of t is V 0 times 1 minus e raised to minus t by tau. So, this equation describes the charging transient. Let us now look at the discharging transient we now have V s going from V 0, to 0 at t equal to 0 and we are interested in finding V and i as a function of time as a result of this change.

Let us begin with v of t equal to A e raised to minus t by tau plus B, where A and B are constants and tau is the time constant given by R times C. Because the thevenin resistance seen by the capacitor is once again R as in this circuit here; now this equation is valid for t greater than 0 for t greater than 0, we have only a D C source in the circuit namely V s equal to 0 and therefore, this equation is valid. It is not valid in this region where V s is changing.

Now, like we did in the charging transient let us find the conditions on v of t and then calculate A and B, we need two conditions 1 condition is v at 0 plus and the other condition is v at infinity. So, let us first get this condition and to do that let us look at the circuit at t equal to 0 minus, what is the situation? At t equal to 0 minus V s has been V 0 for a long time; that means, all the variables in the circuit have become constants this V has become constant and therefore, i which is C d V by dt is 0 at t equal to 0 minus. If i is 0 there is no voltage drop across R and therefore, this v is the same as V s that is V 0. So, that tells us that we at 0 minus must be equal to V 0 and because the capacitor voltage must be continuous, we must have v at 0 plus equal to v at 0 minus therefore, we get v at 0 plus equal to V 0.

What about t equal to infinity? At t equal to infinity again the various quantities in the circuit would have become constants, because we have steady state therefore, this v is a constant i which is C d V by dt is 0 this voltage drop is 0 and this v is equal to V s. Now V s at t equal to infinity is simply 0 and therefore, we have V at infinity equal to 0 volts.

Now, using these two conditions we can calculate A and B, let us see how to do that. At t equal to 0 plus we have v 0 plus equal to v 0 therefore, we substitute for v 0 and then for t 0 here. So, e raised to minus t by tau is 1 for t equal to 0 therefore, we get V 0 equal to a plus b this equation here. For t equal to infinity we have v equal to 0 so we substitute v equal to 0 here t equal to infinity this term now becomes 0. So, therefore, we have 0 equal to B. So, our constant B is 0 and therefore, a is equal to V 0 like that.

Now, putting this back over here we get the final expression for v of t for the discharging transient. So, that is V of t equal to V 0 e raised to minus t by tau. At this point it is a good idea to stop the video and derive these equations for the charging transient and the discharging transient it is of course, always possible to memorize these equations, but that is not such a good idea because if we memorize things they do not stay with us for a long time they tend to evaporate after some time whereas, if we derive the same equations, they will stay with us for a much longer time. So, that is always advisable whenever possible.

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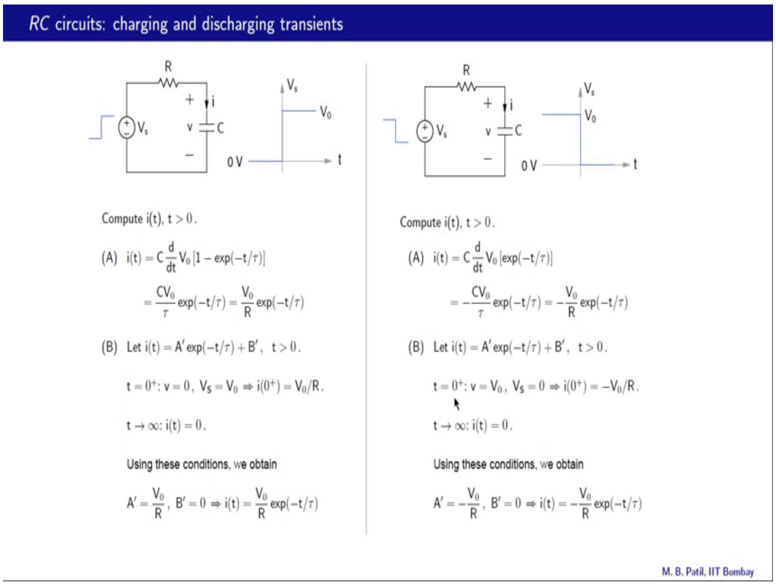

Let us now find the current as a function of time for t greater than 0, for the charging transient; there are two ways of doing this one we already know how this voltage varies with time. So, what we can do is simply calculate i as C d V dt that is C d dt of V 0 times 1 minus e raised to minus t by tau. So, we differentiate this entire expression, this one of course, is a constant. So, that drops out then we get C V 0 by tau, that tau coming from this term here times e raised to minus t by tau that is V 0 by r, e raised to minus t by tau, because this tau is equal to R times C and that C will cancel out and we get V 0 over R here. The other way to find i of t is to start from scratch, that is we pretend that we do not really know what V of t looks like, we begin with an expression for i of t and then find the constants involved in that expression.

So, let i of t be a prime, e raised to minus t by tau plus B prime, for t greater than 0 for which region we have D C conditions, because our V s is simply a constant equal to V 0. Now we can do that because as we have said earlier any current or any voltage in the circuit has this form and therefore, we can write i of t like this. What is the next step? The next step is to find conditions on i of t, which will enable us to find a prime and b prime.

So, let us do that now; as we have seen earlier at t equal to 0 plus the voltage across the capacitor is 0, and that tells us what the current should be. If we have V s equal to V 0 at 0 plus, V equal to 0 the voltage difference across this resistance is V 0 minus 0 and therefore, the current is V 0 minus 0 divided by R that is V 0 by R. What about t equal to infinity? As t tends to infinity we have steady state and all quantities including this voltage become constant and therefore, i which is C d V by dt becomes 0. So, at t equal to infinity we have i equal to 0.

Using these conditions we obtain these constants now, we substitute t equal to 0 here 0 plus i equal to V 0 by R here, that gives us an equation for A prime plus B prime then we substitute t equal to infinity here. So, this term drops out i equal to 0 here. So, that tells us that B prime must be 0. So, that gives us a prime equal to V 0 by R and B prime equal to 0. And finally, putting these back over here we get i of t equal to V 0 by R e raised to minus t by tau and that of course, is exactly the same as what we got in the first case that of course, is expected.

Let us consider the discharge in case now and find i of t for t greater than 0. There are two ways to do this again, 1 we already know V of t for t greater than 0 that we calculated in the last slide. Since V of t is known the current is given by C d V dt. So, therefore, i is C d dt of V 0 e raised to minus t over tau, that is V of t as we obtained in the last slide. When we differentiate this quantity we get minus C V 0 by tau times e raised to minus t by tau and since tau is equal to RC, we get minus V 0 over R e raised to minus t by tau.

In the second method we start from scratch assume some form for i of t, and then find the constants involved in that expression using conditions found for i of t. So, let us do that let i of t be a prime e raised to minus t by tau plus B prime, and this constants A prime and B prime need to be determined using conditions on i of t. Now this expression holds for t greater than 0, for which we have D C conditions namely the source voltage is a constant that is 0 volts. At t equal to 0 plus as we found in the last slide we are V equal to V 0 across the capacitor, and that gives us i at 0 plus. What is i at 0 plus? It is this voltage minus this voltage divided by R, this voltage for 0 plus is 0 this voltage is V 0. So, the current is 0 minus V 0 divided by R that is minus V 0 over R.

Next let us consider t equal to infinity what is the situation as t tends to infinity we have steady state all currents and voltages in the circuit have become constant therefore, this voltage has also become constant and i which is C d V dt is 0. So, for t equal to infinity we have i equal to 0 now using these two conditions i at 0 plus equal to minus V 0 over

R and i at infinity equal to 0 we can obtain these constants a prime and b prime a prime turns out to be minus V 0 by R V prime turns out to be 0 and that gives us i of t equal to minus V 0 over R times e raised to minus t over tau.

Let us now compare the charging and discharging transients in particular let us look at the currents in the charging case the current is given by V 0 by R e raised to minus t by tau and in the discharging case the current is given by minus V 0 by R e raised to minus t by tau.

So, in the charging case this current is always positive for t greater than 0 and in the discharging case the current is always negative for t greater than 0 and that is exactly what we would expect if the current is positive; that means, this capacitor voltage will go on increasing and that corresponds to the charging process for the capacitor on the other hand when the current is negative that is when this i is negative the actual current is flowing in that direction and that will decrease the capacitor voltage. So, therefore, the capacitor is actually discharging.

Let us now look at the plots for V of t as well as i of t both in the charging case and the discharging case.

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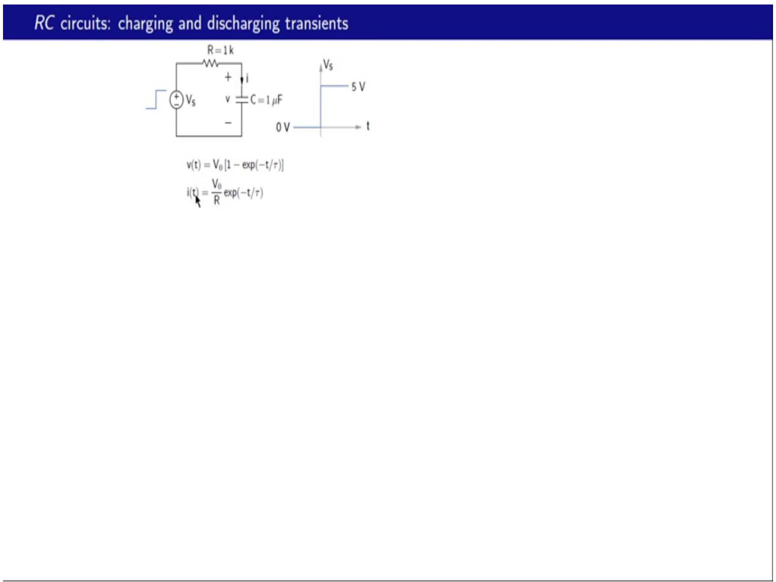

Here is the charging case the source voltage is going from 0 volts to 5 volts at t equal to 0 and these are the expressions which we already obtained earlier for the voltage and the current this is the capacitor voltage and that is the current through the capacitor. So, V of t is V 0 1 minus e raised to minus t by tau and i of t is plus V 0 by R e raised to minus t over tau.

To summarize we have derived analytic expressions for the capacitor voltage and current for a series RC circuit when a step input voltage is applied in the next lecture we will look at graphs showing these quantities as a function of time until then goodbye.