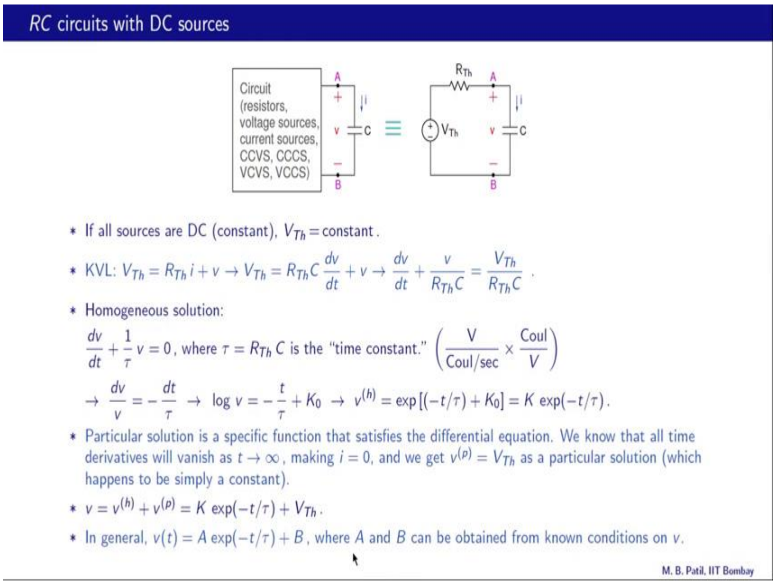
Basic Electronics
Prof. Mahesh Patil
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 07
RC/RL circuits in time domain

Welcome back to Basic Electronics. We will come across a few circuits in this course for which we will need to find the time taken by a capacitor to charge or discharge from an initial voltage V 1 to a final voltage V 2. In this lecture we will make a beginning in that direction, we will first look at the general solution for n RC or RL circuit see how to plot the solution as a function of time, in subsequent lectures we will consider some specific examples so let us begin.

(Refer Slide Time: 00:54)



Let us now start our discussion of RC circuits with DC sources, and this situation as we will find out is very common in electronic circuits and once we have this background, we will be able to calculate for example, pulse width or oscillation frequency in a circuit. Here is the circuit a linear circuit which consists of resistors, independent voltage sources which are DC, independent DC current sources and then dependent sources such as current controlled voltage source, current controlled current source etc. To this circuit we connect capacitor and now let us see how this capacitor voltage or current evolves with time.

Since this circuit is a linear circuit, we can represent it with this thevenin equivalent consisting of a voltage source V Th in series with a resistance R Th, and this capacitor of course, appears as it was earlier over here. Let us now make a few important points if all sources are DC or constant, then V Th is a constant and it is important to bear in mind this assumption if for example, we have a sinusoidal voltage source here or a triangular current source here then our analysis is not going to be valid.

Let us begin by writing the KVL equation for this loop, and what is that? That says that this V Th must be equal to this voltage drop R Th times i plus V like that. Now this current i that is flowing through this resistor is also the capacitor current and therefore, i is equal to c, dv dt. So, we substitute for i C dv dt and then we get V Th equal to R Th C dv dt plus v. We can rewrite this equation as dv dt plus v divided by R Th times C equal to V Th divided by R Th times C, all we have done here is divided both sides by this vector here.

Notice that our right hand side here is a constant because our sources are DC sources and therefore, V Th is a constant. Now how do we solve this ordinary differential equation? We first obtained the homogenous solution and we do that by replacing this right hand side the forcing function with 0, and then we get this equation here; we have defined tau which is called the time constant as R Th times C over here; this time constant tau has units of time, and let us quickly verify that what are the units of R Th? It is a resistance. So, the units are ohms, ohms is volts by ampere and ampere is coulombs per second. What about C? C has units of charge by voltage that is coulombs per volts. So, this volts cancels out, this coulombs cancels out leaving behind seconds that is the unit of time.

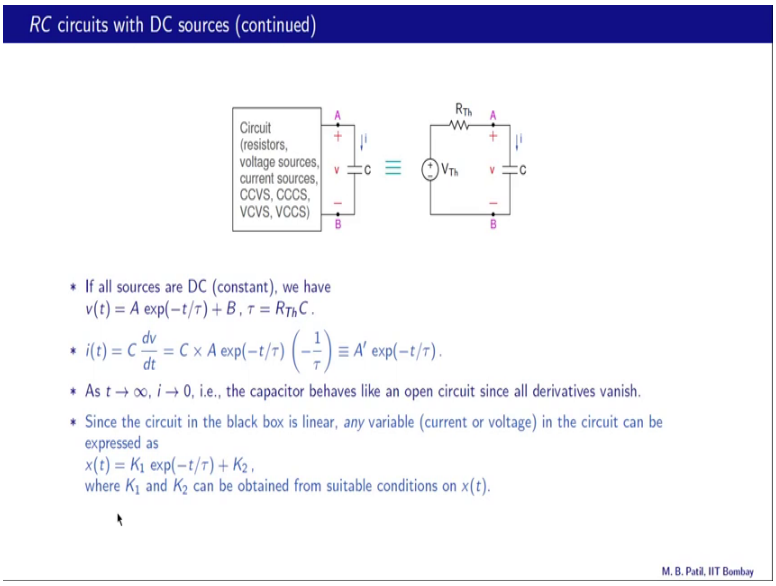
Let us now get the homogeneous solution; we rewrite this equation as dv by v equal to minus dt by tau. Now we can integrate both sides, the integral of this left hand side will give us log of v this log of course, is to the base e and the integral of this part the right hand side will give us minus t by tau and there is an integration constant here which is k 0. So, this equation can be rewritten as v equal to e raised to minus t by tau plus k 0 and then we can define e raised to k 0 as another constant k and therefore, we have V the homogenous part indicated here with this superscript h equal to k times exponential minus t by tau.

The second part of the solution is the particular solution, what is the particular solution? It is a specific function that satisfies the differential equation. In this case this is our differential equation, so we are looking for some solution, which satisfies this equation. And in RC circuits also RL circuits it is very convenient to think of the steady state and that can be used as a particular solution and we do that as follows. We know that all time derivatives vanish as t tends to infinity, that makes the current equal to 0 for a capacitor because the current is i equal to C dv dt if V becomes constant then i becomes 0 and then we get v particular indicated here with this superscript equal to V Th for this specific case.

How do we get this? This current has become 0. So therefore, no voltage dropped here and therefore, we have V equal to V Th. So, that is how we have got this particular solution and let us check that this solution does indeed satisfy the differential equation, let us substitute V equal to V Th in this equation here, since V Th is a constant dv Th dt would be 0 and then we have on the left hand side V Th divided by R Th times C and that is precisely the same as the right hand side. So, therefore, this particular solution does satisfy our ordinary differential equation.

Now finally, how about net solution is given by the addition of the homogenous part which is here, and the particular part which is here; so we have v equal to K exponential minus t by tau plus V Th, and in general v of t can be written as some constant times c raised to minus t by tau, plus some other constant where A and B can be obtained from known conditions on v and we will very soon see how that can be done.

(Refer Slide Time: 07:40)

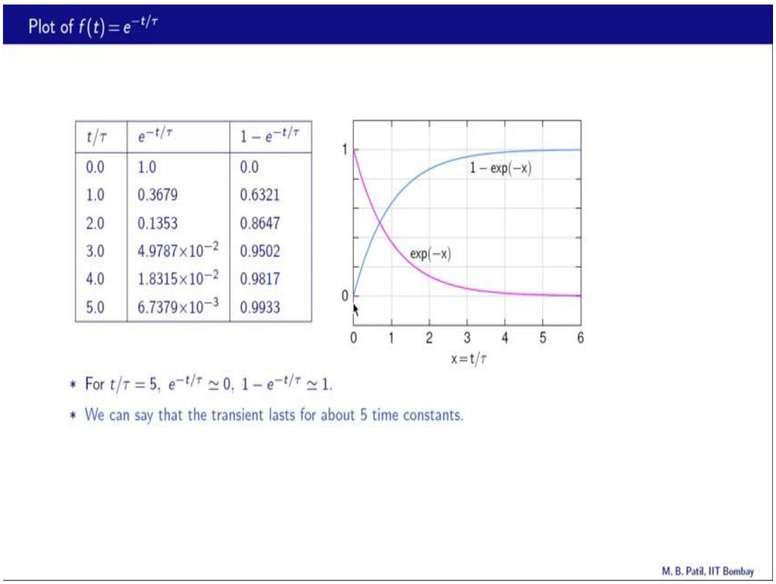


So, here is the summary of what we studied in the last slide, if all sources all independent sources these are DC or constant, then we have v of t equal to A e raised to minus t by tau, plus B where A and B are constants and tau is called a time constant and its given by r thevenin times C, where r thevenin is the thevenin resistance as seen from the capacitor. What about the current? The current can be computed once we know the voltage, because we know that i if C dv dt. So, all we need to do now is to differentiate this part and then multiply that by C. So, then that gives us C times A raised to minus t by tau times minus 1 by tau. So, that can be rewritten as some other constant A prime times e raised to minus t by tau.

One observation that we can make from this equation all of t equal to a prime e raised to minus t by tau is the following. As t times to infinity, this e raised to minus t by tau becomes 0 and therefore, the current becomes 0 and this observation also agrees with our intuition because as t tends to infinity, we reach a steady state and in steady state all quantities tend to be constant. So, therefore, this v will also become a constant and since the current here is C dv dt; if v is constant that current is going to become 0.

So, therefore, the capacitor behaves like an open circuit in steady state as t tends to infinity, what we have looked at so far is the voltage across the capacitor and the current through over capacitor, but because this circuit is a linear circuit, we can actually comment on all variables inside the circuit that is all voltages and all currents. Since the circuit in the black box each linear any variable, that is any voltage or any current in the circuit can be expressed as X of t equal to K 1 a raised to minus t by tau plus K 2, where K 1 and K 2 can be obtained from suitable conditions on that particular variable X of t and as we look at some examples this procedure will become very clear.

(Refer Slide Time: 10:04)



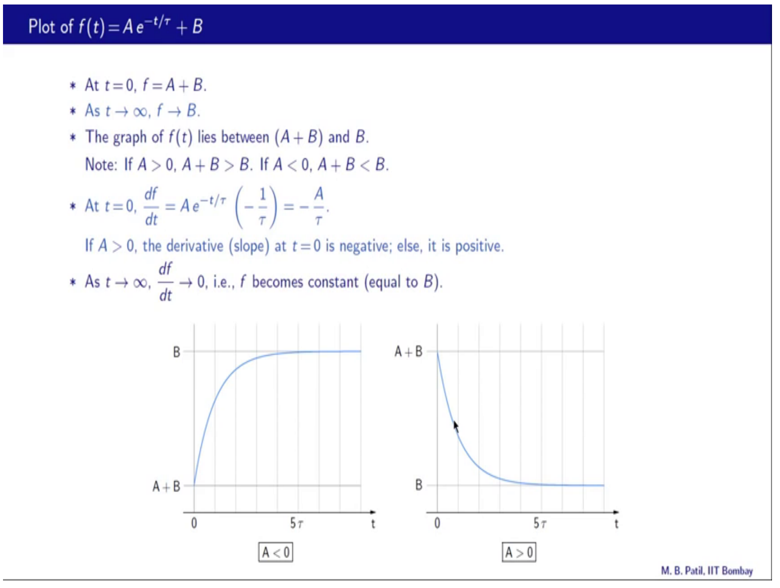
Let us now look at this function f of t equal to e raised to minus t by tau, which appears in our solution as we saw in the last slide here is a table, which shows how this function varies with time, the first column here is t by tau. So, this is t equal to 0, this is t equal to 1 times tau that is t equal to tau, this is t equal to 2 tau, this is t equal to 3 tau and so on. This column here is our f of t that is e raised to minus t by tau and this column here is 1 minus f of t. For t equal to 0 e raised to minus t by tau is 1, for t equal to tau it is 0.37, for 2 tau it is 0.13, for 3 tau 0.05, 4 tau 0.01, 8 and 5 times tau it is 0.0067.

So, we seen that as t increases, this function f of t gets closer and closer to 0; what about 1 minus f of t? To begin with it is 1 minus 1 equal to 0 at t equal to 0, and then as f of t goes on decreasing, 1 minus f of t goes on increasing and eventually of course, as e raised to minus t by tau approaches 0, 1 minus e raised to minus t by tau approaches 1.

So, by the time we come to 5 times the time constant this function has almost become equal to 1. So, for t by tau equal to 5, e raised to minus t by tau is nearly equal to 0, this number here and 1 minus e raised to minus t by tau is nearly equal to 1 this number here and therefore, we can say that the transient lasts for about 5 time constants after that things become constant. Here is the figure showing e raised to minus t by tau as a function of t by tau, this curve here and 1 minus e raised to minus t by tau as a function of t by tau, the blue curve here and as we have seen in this table at t equal to 0, we have e raised to minus t by tau equal to 1 and as t increases, this function goes to 0 and after about 5 time constants that is after t by tau equal to 5, we see that this function becomes approximately equal to 0 and will stay at 0 thereafter.

Similarly, 1 minus f of t that is 1 minus e raised to minus t by tau starts off at 0 and eventually it becomes equal to 1, after about 5 time constants.

(Refer Slide Time: 13:20)



Let us now take a look at the plot of f of t equal to A e raised to minus t by tau plus B; where A and B are constants and why do you want to do that? That is because as we have seen in the earlier slides this is the general form for our solution, and this will apply to any quantity, any current or any voltage in an RC circuit or RL circuit with DC sources. So, let us first figure out some constraints on this function and then we will look at the plot. What about t equal to 0? If we substitute t equal to 0 here e raised to minus t by tau becomes 1 and therefore, we have f equal to A plus B. What about t equal to infinity? As t tends to infinity e raised to minus t by tau becomes 0 and therefore, we have f tending to B.

From these 2 observations we conclude that the graph of f of t will always lie between A plus B and B. If A is positive then A plus B is going to be greater than B and otherwise it is going to be less than B. Let us now look at the derivative of f, what is df dt? We need to differentiate this part that is A e raised to minus t by tau times minus 1 by tau and the derivative of B of course, is 0 because that is a constant. So, therefore, we get df dt equal to A e raised to minus t by tau times minus 1 by tau. At t equal to 0 e raise to minus t by tau is 1 and therefore, we get df dt equal to minus A over tau.

Now, if A is positive then this derivative at t equal to 0 is going to be negative, because of this minus sign here; and what is the derivative? This derivative is nothing, but the slope of the function at t equal to 0. If A is less than 0 then this derivative is going to be positive; what about t tending to infinity? As t tends to infinity, this term becomes 0 and therefore, df dt will tend to 0; that means, f becomes constant and what is the constant? We have already seen that that constant is B. So, as t tends to infinity, d of dt tends to 0 irrespective of the sign of A, because A simply does not come into the picture it gets multiplied by this 0 here.

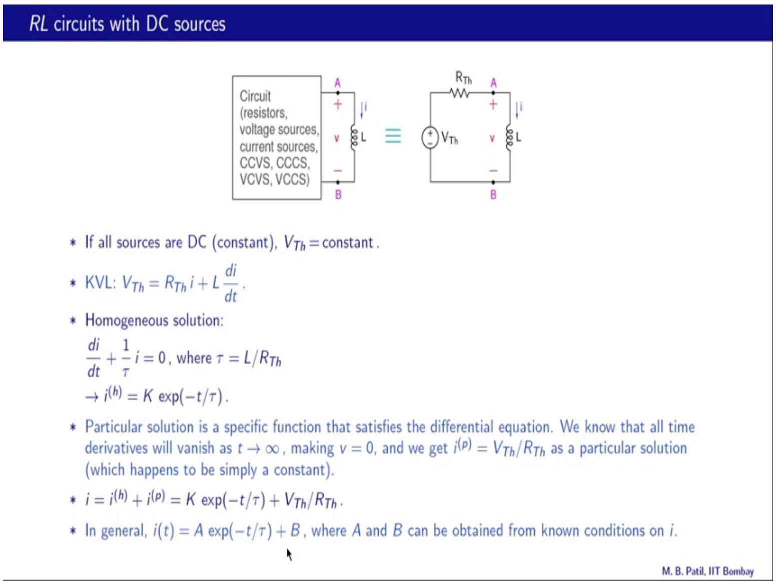
Let us now look at these 2 cases namely A greater than 0, A positive and A negative. Let us start with A less than 0 in this case A plus B is less than B. So, our initial value which is A plus B is going to be smaller than our final value, which is B. What about the slope? If A is less than 0 then the slope at t equal to 0 is positive like that and as t tends to infinity the slope anyway goes to 0 irrespective of the sign of A and we notice that also over here and notice also that things stop wearing after 5 times tau as we have already seen in the last slide. After 5 tau, this function f of t becomes approximately constant, does not change thereafter.

Now, let us look at the second case that is A positive; if A is positive A plus B is greater than B; that means, our initial value is greater than our final value and that is what we see over here, this is the initial value that is the final value, what about the slope at t equal to 0? If A is positive then the derivative at t equal to 0 is negative and that is indeed what we observe over here, there is a negative slope here at t equal to 0.

And after 5 time constants the function becomes constant and that is equal to d as we expect and of course, df dt becomes equal to 0. So, these are the only 2 possibilities we have if a is negative; then f of t starts off with a positive slope at t equal to 0 there is some variation in f of t up to 5 tau and then it approaches the constant that is B and the slope of course, becomes 0. In the other case when A is positive f of t starts with a negative slope at t equal to 0, then there is some variation up to 5 tau and after that f of t becomes constant; that means, the derivative becomes 0.

Let us notice also that these functions are continuous, the derivatives are also continuous and therefore, we do not see any abrupt changes anywhere either in this case or in this case and all of these are very important points and they will help us when we want to plot a current as a function of time or a voltage as a function of time in RC or RL circuits with B C sources. Now very soon we are going to work out some examples, and the plots that we have going to obtain let us say for a current or a voltage would be either of this type or of this type. So, whenever you see a plot you should really come back to this slide and confirm that what we see over there is one of these forms.

(Refer Slide Time: 19:56)



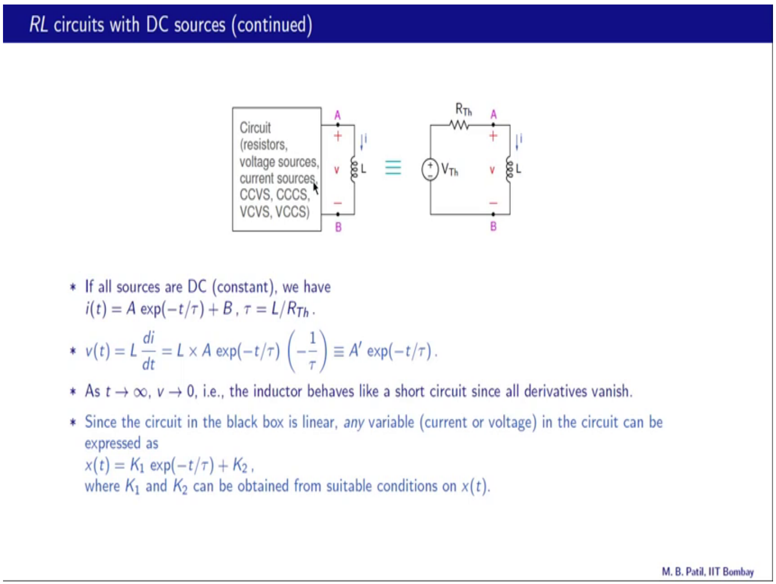
Let us now look at circuits with DC sources. So, here is a linear circuit once again consisting of resistors independent voltage sources independent current sources and dependent sources. To this circuit we connect an inductor and our interest is to find the voltage across the inductor as a function of time, and the current through the inductor as a function of time step number one. We replace this original circuit with its thevenin equivalent consisting of a voltage source V Th, in series with the resistance R Th and the inductor of course, appears as it is. As in the RC circuit case we will assume that all our sources these independent voltage sources and independent current sources are DC; that means, they are constant and therefore, this V Th is going to be a constant.

Let us now write the KVL equation for this loop and what is that? This V Th must be equal to this voltage drop plus this voltage drop. Now this voltage drop across the resistance is R Th times i and the voltage drop across the inductor is given by L di by dt. To solve this differential equation we first obtain the homogenous solution and we do that by replacing this forcing function with 0 like that and then we divide both sides by L. So, we get di dt plus 1 over tau times i equal to 0, where tau is L by R Th; this tau has units of time and that is the circuit time constant.

The homogenous solution indicated here with the superscript h is then given by K times exponential minus t by tau, where this K is a constant. To obtain a particular solution once again we will use information from the steady state condition like we did in the RC circuit case, that is as t tends to infinity we expect all quantities to become constant therefore, this current will also become constant and v which is L times di dt well then become 0. When V become 0 this current is equal to V Th divided by R Th. So, that is a particular solution that we can use and finally, we can add the homogenous and particular solutions to obtain the net current as a function of time. So, i is i h plus i p, i h is given by K e raised to minus t by tau, i p is given by V Th by R Th. So, therefore, our net or total current is K e raised to minus t by tau plus V Th by R Th.

And we do not need to really go through this derivation every time, the moment we see a linear circuit with a single inductor connected to it, and when the sources are DC sources we can write the current as A e raised to minus t by tau plus b where A and B are constants to be obtained from known conditions on the current.

(Refer Slide Time: 23:51)



To summarize the discussion from our last slide, if we have a linear circuit with DC sources then we can write this inductor current as A e raise to minus t by tau plus B where A and B are constants, and tau the time constant is given by L divided by R Th. What about the voltage across the inductor? Once we know the current we can obtain the voltage as L di dt. So, all we need to do now is to differentiate this current and then multiply that by L and that gives us L times A e raised to minus t by tau times, minus 1 over tau.

Now, all of this L times, A times minus 1 over tau can be clubbed together into another constant called A prime. So therefore, the voltage is given by A prime e raised to minus t by tau. What happens as t tends to infinity? As t tends to infinity this e raised to minus t by tau tends to 0 and therefore, the voltage across the inductor becomes 0 and what is the meaning of that? When the voltage here is 0; that means, it is just like a wire with no voltage drop across it, so the inductor behaves like a short circuit.

Now, we have already looked at the voltage across the inductor and the current through the inductor, but we can make a much more general comment, because this circuit is a linear circuit and that is since the circuit in the black box is linear any variable that is any current or any voltage inside the circuit, can be expressed as x of t equal to K 1 e raised to minus t by tau plus K 2; where K 1 and K 2 are constants and they can be obtained using suitable conditions on x of t. To summarize if we have a linear circuit with DC voltage sources or DC current sources connected to a capacitor or an inductor, then any quantity in the circuit that is any current or any voltage is given by this expression here, x of t equal to K 1 e raised to minus t by tau plus K 2.

If we have a capacitor connected to this linear circuit then this time constant tau is given by R Th times C, where R Th is the thevenin resistance seen by the capacitor if we have an inductor then this tau is given by L by R Th; where R Th is the thevenin resistance seen by the inductor.

In summary we have looked at the general form of the solution in n RC or RL circuit with DC sources, we have also seen how to plot the solution as a function of time. These observations will be very useful when we look at some examples in the next lecture that is all for now.

See you next time.