Basic Electronics
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Lecture – 56
Boolean algebra

Welcome back to Basic Electronics. In this lecture, we will continue with logical functions and see how they can be simplified. We will also look at two standard forms used to express logical functions namely the sum of products form and the product of sums form. Finally, we will look at the do not care condition seen frequently in truth tables for logical functions. Let us begin.

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Let us look at A few more useful theorems; here is one A plus A B equal to A. Now to prove this theorem we can follow two approaches A as we have been doing we can construct truth tables for the left-hand side and the right-hand side for all possible input combinations and show that they are the same. Approach B we can use identities and theorems which we already know and then show that the left-hand side and right-hand side are the same.

So, let us follow this second approach for this case, this is our left-hand side A plus A B we can write A as A dot 1 then we can combine these two terms like that. What is 1 plus B, it is just 1, and then A dot 1 is A and that is the right-hand side that proves this theorem. Here is another similar theorem A dot A plus B equal to A. Let us start with the left-hand side, and we can write this as A dot A plus A dot B, what is A dot A is A, and that gives us A plus A B. And we have already seen that A plus A B is the same as A. So, therefore, we get A.

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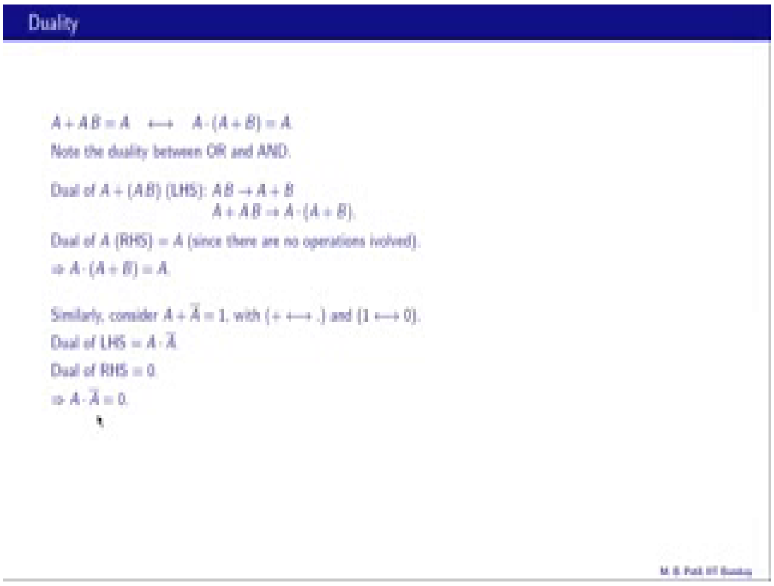

In the last slide, we have proved this theorem and also this theorem, but we do not actually need to do that we can obtain this second theorem from the first theorem simply by using the principle of duality between OR and AND. Let us see how that can be done. Let us look at the left-hand side this A and B by duality becomes A plus B or A OR B and then this A plus this becomes A dot the dual of that which is A plus B. And therefore, our left-hand side has now become A dot A plus B; and the right-hand side has only A and the dual of A is A, since there are no operations involved. And therefore, we can obtain A dot A plus B equal to A that is this theorem.

Similarly, let us consider A plus A bar equal to 1. And let us see; what is the dual of this theorem knowing that plus and dot dual of each other and 1 and 0 are also dual of each other. So, what is the dual of the left-hand side, we replace this plus with dot there and get A dot A bar. What is the dual of the right-hand side dual of 1 is 0. So, therefore, we get 0 here, and therefore we can write A dot A bar equal to 0. So, once we know this theorem, we can obtain this one simply by the principle of duality without proving it again.

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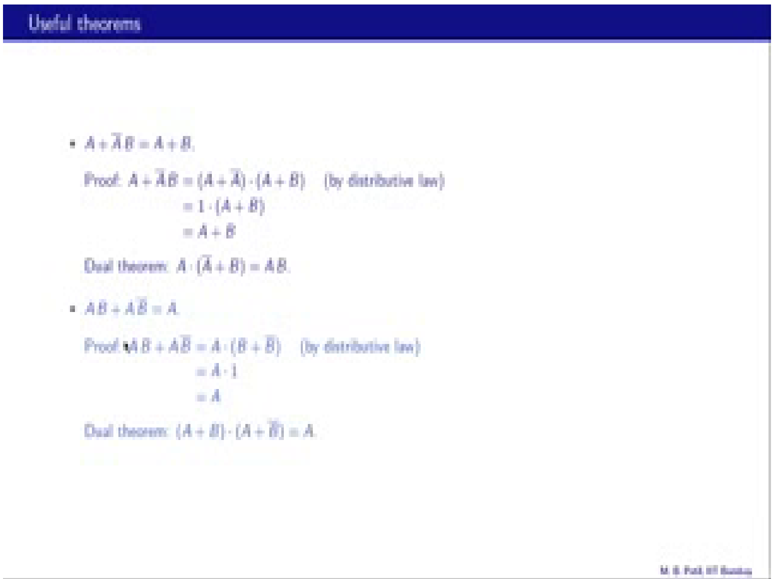

Let us continue here is another theorem A plus A bar B equal to A plus B, let us prove this theorem. We can use the distributive law to write A plus A bar B as A plus A bar and A plus B. What is A plus A bar it is 1, so it is 1 dot A plus B and that is simply A plus B. So, that proves this theorem. There is a dual theorem this 1, and you should figure out how this follows from this original theorem. Another theorem A B plus A B bar equal to A. How do you prove this, A B plus A B bar is A dot B plus B bar again by distributive law and B plus B bar is 1, so that is A dot 1 which gives us A. Once again, there is A corresponding dual theorem, and you should check out that this follows from this one.

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Let us consider this statement. In an India-Australia match, India will win if one or more of the following conditions are met. And what are the conditions A - Tendulkar scores a century, B Tendulkar does not score a century, and Warne fails Shane Warne, fails means he fails to get wickets, C Tendulkar does not score a century and Sehwag scores a century. So, these are the conditions. And clearly this statement is fairly complex; it is not very easy to comprehend. And let us see if we can use our identities and theorems to simplify this statement.

Let us define T a logical variable to stand for Tendulkar's scores a century, and what does it mean that means, this variable is 1 if Tendulkar indeed scores a century, otherwise it is 0. Similarly S stands for Sehwag scores a century, W stands for 1 fails to get wickets and I stands for India wins. And now we can rewrite this statement as I equal to T plus T bar W plus T bar S. And why do we have this OR operations here that is because the statement says that India will win if 1 or more of the following conditions are met and that is exactly the meaning of this logical OR operation.

And what are those three conditions, one is Tendulkar scores a century that is the same as this logical variable T, second is Tendulkar does not score a century and 1 fails, so it is AND of two things 1 is T bar and the other 1 is W and so on. And we can now simplify this expression. Let us write T as T plus T. Then we can combine this T and this T bar W this T and this T bar is like that this T plus T bar W. We can write as T plus T bar dot T plus W. Similarly, T plus T bar S is T plus T bar dot T plus S T plus T bar is 1, so that gives us T plus W again T plus T bar here is 1 that gives us T plus s. So, we get T plus W plus T plus S that is the same as T plus W plus S because this T plus T is the same as t. So, now we have A much simpler statement and that says India will win if 1 or more of the following conditions is true; one Tendulkar strikes that is Tendulkar scores a century, B Warne fails to get wickets, and C Sehwag strikes that is Sehwag scores a century.

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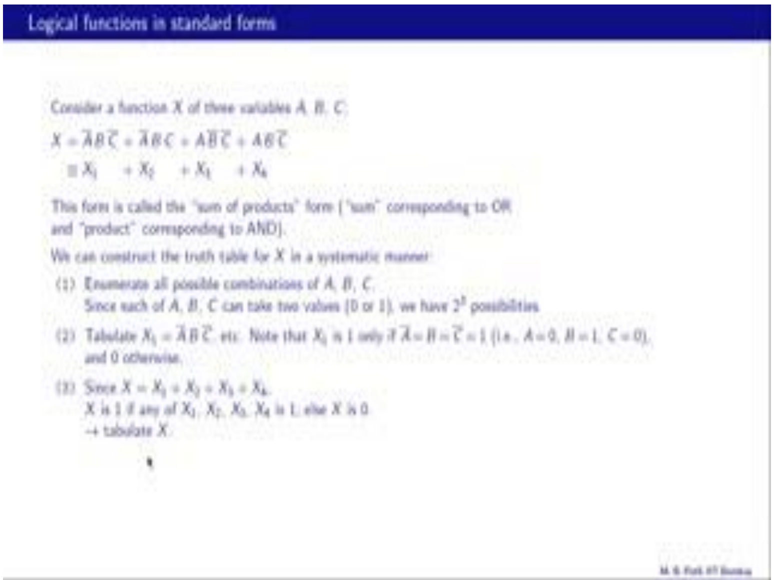

Let us now discuss logical functions in standard forms take this as an example it is A function of three variables A, B and C and we will call this function as X 1 this function as X 2 and so on. So, this X is the same as X 1 plus X 2 plus X 3 plus X 4. And this form is called the sum of products form, because it looks like a sum in reality of course, these are the OR operations. So, it is a sum of these terms and each term looks like a product in reality of course, these are AND operations here. So, this form is called the sum of products form sum corresponding to OR and product corresponding to AND.

We can construct the truth table for X in a systematic manner knowing that we have this sum of products form. How do we do that, one enumerate all possible combinations of A B and C, since each of A B C can take two values 0 or 1. We have 2 raise to 3 possibilities and we have seen that before. Second, tabulate X 1, which is A bar B C bar etcetera. And how do we do that we know that X 1 is 1 only if A bar is 1 B is 1 and C bar is 1 that is A - 0, B is 1 and C is 0 otherwise X 1 is 0. So, we can tabulate X 1, X 2, X 3, X 4 in this manner. And finally, since X is X 1 plus X 2 plus X 3 plus X 4. We know that X is 1 if any of these four terms is 1; otherwise X is 0 and in that manner we can tabulate X. Let us take an example and do this.

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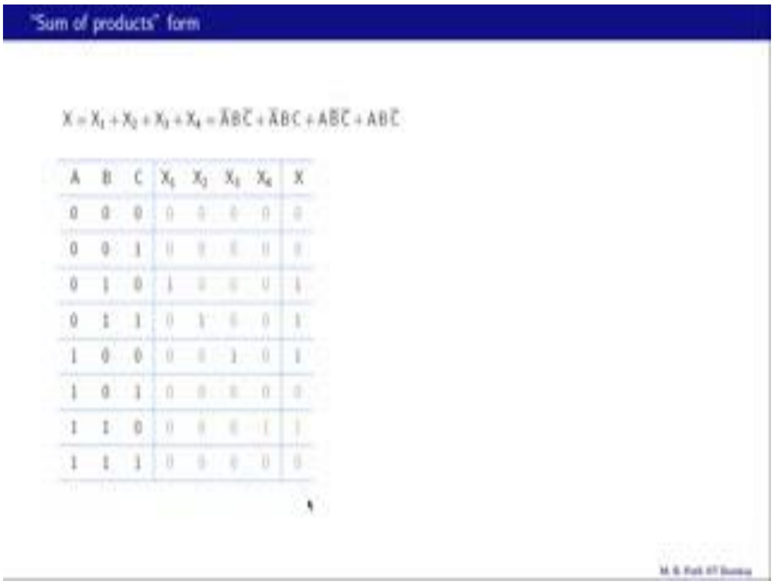

So, let us now construct the truth table for this function X, which we saw in the last slide. And step number one as we said is to enumerate all the possibilities, all the different combinations of A, B and C. And let us just go through this once again because it is so important. So, we start with 0 0 0 and then we allow C to change every single entry. So, 0 to 1, 1 to 0 and so on we allow be to change every two entries 0 0 then followed by 1 1 then 0 0 and then followed by 1 1. And we allow A to change every four entries, so four 0s followed by four 1. And if we had one more variable for example, then that variable would change every eight entries and so on and that is a way to exhaust all possible input combinations.

Let us now look at X 1, and X 1 is the same as A bar B C bar and this term is 1 if A bar is 1 B is 1 and C bar is 1 and A bar is 1 when A is 0. So, what we need is A equal to 0, B equal to 1, C equal to 0, so 0 1 0. So, let us look at that in this table, it is here. So, for this combination X 1 would be 1 and for all other combinations X 1 would be 0 like that. What about X 2, we need A equal to 0, B equal to 1, C equal to 1, so 0 1 1 that is here. So, X 2 is 1 for this combination and it is 0 otherwise. X 3, we need A equal to 1, B equal to 0, C equal to 0, so 1 0 zero that is here and so on. X 4 minute A equal to 1 B equal to 1 and C equal to 0. So, 1 1 0 here, so that is 1 and all others 0.

Now, we have got X 1, X 2, X 3, X 4 and now we can construct the truth table for X. And since this is an OR operation here X is 1 if any of these is 1. So, therefore, X will be 1 here, here, here and here, and otherwise it will be 0. So, four 1s and otherwise all 0s, so that is what X looks like.

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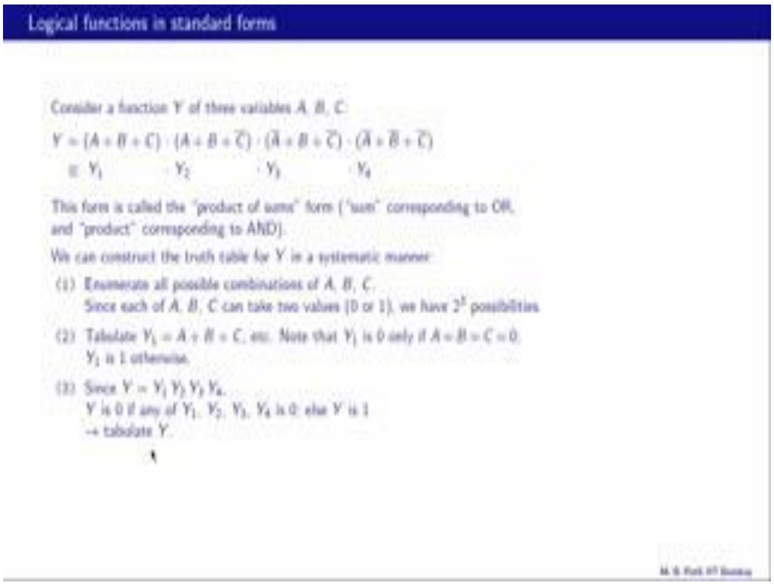

Let us now look at an alternative form for writing a logical function. So, let us consider a function Y again of three variables A B C and that is what Y looks like it is Y 1 and Y 2 and Y 3 and Y 4; Y 1 is A plus B plus C A OR B OR C; Y 2 is A plus B plus C bar and so on. Now, this form is called the product of sums form because this looks like a product although this is an AND operation really and each of these looks like a sum because of this OR operation here. So, that is why it is called the product or sums form. And we can construct the truth table for Y in a systematic manner just like we did for the sum of products form. And let us see how we can go about that.

Step number 1; we enumerate all possible combinations of A B C and this we have seen already more than once. Step number 2, we tabulate Y 1 which is A plus B plus C and noting that Y 1 is 0, only if each 1 of these is 0 that means, A is 0, B is 0, and C is 0 and in the same manner we tabulate Y 2 Y 3 and Y 4. And finally, we use the fact that Y is and of Y 1, Y 2, Y 3 and Y 4. And therefore, Y is 0 if any of these is 0 otherwise Y is 1 and that we can use to tabulate Y as A function of A B C.

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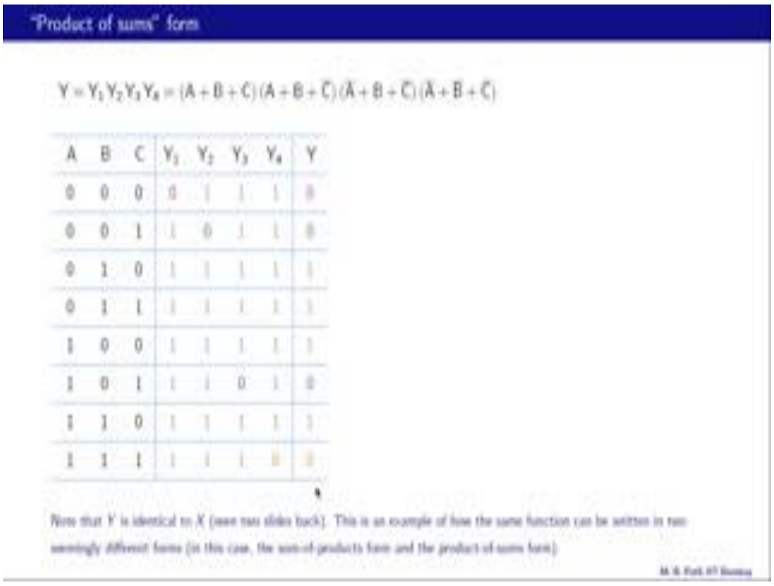

So, let us work this out. So, here is our table. And we will not go through this part again of listing all the input combinations we have seen that before. Let us start with Y 1 which is A plus B plus C and Y 1 is 0 only if each of these is 0. So, A is 0, B is 0 and C is 0 and that happens over here. So, for that entry, Y 1 is 0, otherwise Y 1 is 1 like that. What about Y 2, Y 2 is 0, if A is 0 B is 0 and C bar is 0 so that means, C is 1. So, we have 0 0 1 that is write here. So, Y 2 is 0 for that combination and otherwise it is 1 like that. What about Y 3, Y 3 is A bar plus B plus C bar. So, what we need is A equal to 1, B equal to 0, C equal to 1; so, 1 0 1 that is right here. So, Y 3 is 0 there and otherwise it is 1.

What about Y 4 A bar plus B bar plus C bar. So, we need A equal to 1, B equal to 1, and C equal to 1 right here; 0 there and 1 wherever else. And now we can get our Y as A function of A, B, C. Y is the AND of all of these. So, Y is 0 if any of these is 0 so that means, Y would be 0 here, here, here and here, so four zeroes and the rest all ones, so that is our overall function.

And let us make a comment about this. If we compare this table with the table that we have seen for the function X, two slides earlier, you will find that these entries are identical, so that means this Y is identical to X which we saw over there. But that X looked like a very different function and that only goes to say that this is an example of how the same function can be written in two seemingly different forms, apparently different forms. In this case, X was in the sum of products form and Y was in the product of sums form. So, they look completely different, but they are actually identical functions. So, that can happen and we should remember this point.

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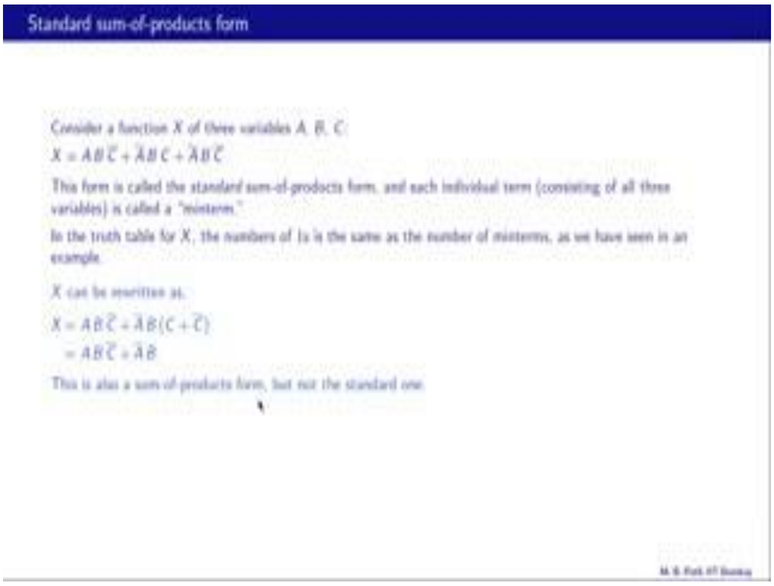

We will consider some definitions now. Let us consider this function X of three variables A, B, C, A B C bar plus A bar B C plus A bar B C bar. Now, this form is called the standard sum of products form and each individual term this one or this one or this one which consists of all three variables is called a minterm. So, there are three minterms in this expression. And if we prepare truth table for X, we will find that the number of ones for X is the same as the number of minterms here. So, we will find three 1s in the truth table for X. Now, X can be rewritten as A B C bar, the first term as it is plus A bar B and we can combine these two terms because this A bar B is common here and we get A bar B and C plus C bar, C plus C bar is 1.

So, what we get is A bar B ended with 1 which is just A bar B. So, now, we have another sum of products form, but notice that this term here the product term does not have all three variables, and therefore although this is also A sum of products form it is not the standard one.

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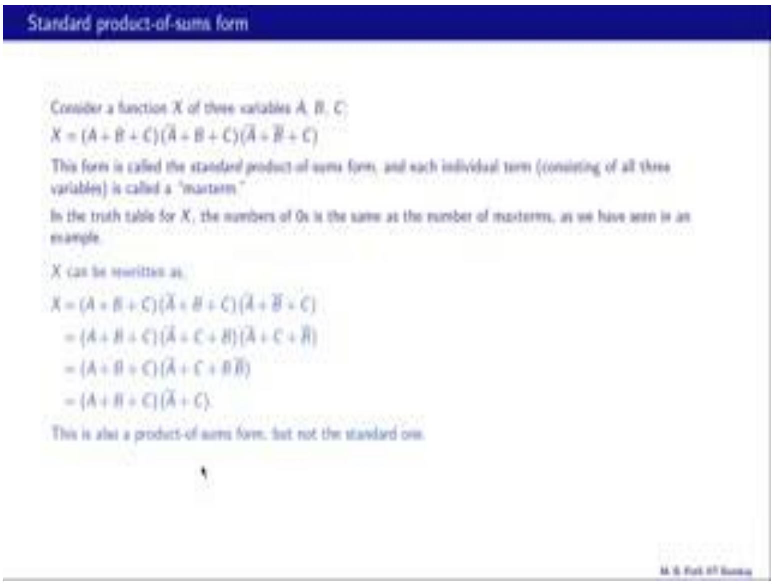

In a completely analogous manner, we can also talk about the standard product of sums form, and let us take this as an example here X equal to A plus B plus C A bar plus B plus C A bar plus B bar plus C. And of course, let us not forget that there is this and operation here as well as here, although we do not explicitly show it sometimes. Now this form is called the standard product of sums form and each individual term which consists of all three variables. Notice that there are three variables here, here as well as here each individual term is called A maxterm. So, this is a maxterm that is a maxterm and that is also a maxterm. So, there are three maxterms in this expression. And if we prepare a truth table for X, the number of zeroes in that table would be the same as the number of maxterms. So, in this case, there would be three zeroes because we have three maxterms

Now, X can be rewritten as follows. This is our original expression and we note that there are some things common in this second bracket and the third bracket A bar plus C here, A bar plus C here, so that is common. So, let us rewrite this whole expression as this one. So, we have written that as A bar plus C plus B, and A bar plus C plus B bar. Now, we can use the distributive law that we have looked at earlier to be precise the second distributive law that we have seen, and rewrite this combine these two brackets then we get A bar plus C plus B and B bar like that and B and B bar of course is 0. And then we get A bar plus C here and of course, this bracket stays as it is. So, finally, we have A plus B plus C and A bar plus C.

Now this is also A product of sums form this is a sum here this is a sum here and there is a product or the AND operation here, but this is not the standard form because here we do not have all three variables.

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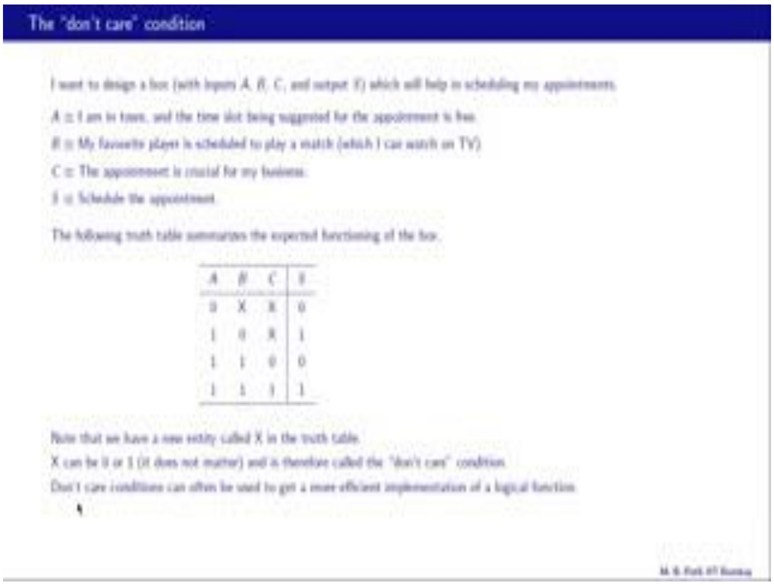

We have seen so far that digital variable or a logical variable can take only two values 0 and 1, but there is a third possibility and that is called the do not care condition. So, let us illustrate what does means with an example. Let us say I am a businessman very busy of course, and I do not have time to think about mundane matters like scheduling appointments and such, I would rather spend that time to make more money. So, I want to design A box which has three inputs A, B, C and an output S which will help me in scheduling my appointments.

So, let us define our variables logical variables A, B, C and S. A stands for I am in town; and the time slot being suggested for the appointment is free that means, if this whole condition is true then A is 1; otherwise A is 0. B stands for my favorite player is scheduled to play A match which I can watch on T v, for example, Tendulkar or whoever your favorite player is. C stands for the appointment is crucial for my business; and S is the output variable, it stands for schedule the appointment. Let us now prepare the truth table with these conditions.

So, here is the table A, B, C are the input variables and S the output variable. And let us now try to understand each of these entries A equal to 0, B equal to X, C equal to X, S is

0. Now, what is the meaning of A equal to 0; that means, this condition is false and that means, that the time slot being suggested is not available. So, the question of my favorite player playing or the appointment being crucial simply does not arise. So, therefore, it does not matter whether B is 0 or 1 or C is 0 or 1 there is simply no scope of scheduling an appointment and therefore, S is 0. Now, these conditions are called do not care conditions. So, what it means is it does not really matter, whether that is 0 or 1 the function is not going to change in other case.

So, let us note that we have A new entity called X in the truth table do not care condition which we have seen here. Now, the next entry A equal to 1, B equal to 0, C equal to X and S equal to 1, let us see how this works A is one. So, the time slot is available B is 0s, so my favorite player is not going to play A match on that day. C is do not care which means it does not really matter whether this appointment is crucial for my business or not, and either way I am going to go ahead and schedule this appointment. So, therefore, S is equal to 1.

Next, we have 1 1 0, S is equal to 0. So, the time slot is available my favorite player is playing A match on that day which I would like to watch if I have A choice and it turns out that disappointment is not crucial for my business. So, therefore, I will go ahead and watch the match and not schedule the apartment. So, therefore, S is 0. Next, the time slot is available, my favorite player is playing A match on that day, but now I do not want to watch the match because the appointment is crucial for my business. Therefore, I will not worry about the match, I will worry about my business, and therefore I will schedule disappointment, so therefore S is 1.

To summarize, we have seen A new value for logical variables and that is denoted by X; it stands for the do not care condition. X can be 0 or 1, it does not matter, it does not change the value of the function that we are interested in and it is therefore, called the do not care condition. And do not care conditions can often be used to get a more efficient implementation of a logical function. And we will see several examples of how that can be done.

In summary, we have seen how a logical function can be expressed in the sum of products form or the product of sums form. We also looked at a third possible value for a logical variables that is the do not care condition denoted by X. In the next class, we will look at how to minimize logical function. See you next time.