Basic Electronics
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Lecture – 53
Sinusoidal oscillators

Welcome back to Basic Electronics. In the previous class, we have looked at the principle of operation of sinusoidal oscillators. We will now look at two specific sinusoidal oscillators - the Wien bridge oscillator and the phase shift oscillator. Let us begin.

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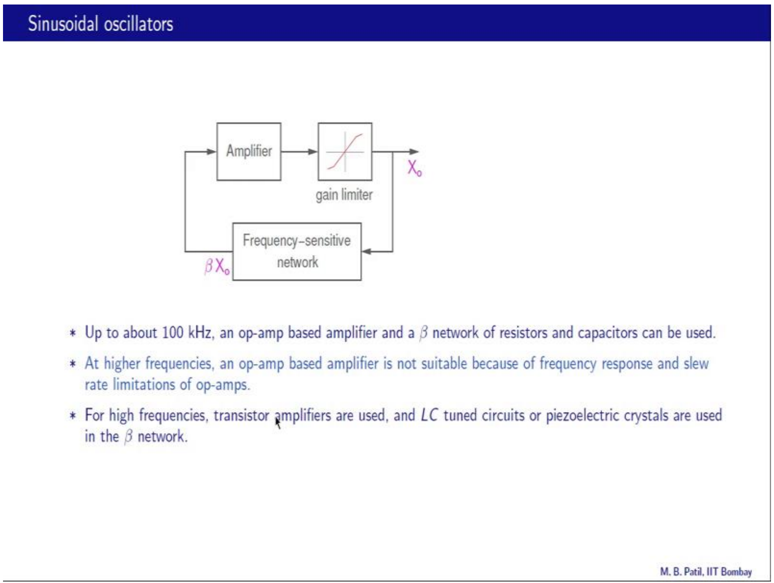

Coming back to sinusoidal oscillators, let us make a few important points; before we actually start looking at some specific oscillator examples. Point number one up to about 100 kilo hertz, we can use an op-amp based amplifier here and a beta network consisting of resistors and capacitors. At higher frequencies, frequencies higher than say 100 kilo hertz and op-amp based amplifier is not suitable, because op-amps have limitations in terms of frequency response, they cannot operate at arbitrarily high frequencies.

And also as we have seen op-amps have slew rate limitations that means, the output of an op-amp cannot raise or fall at arbitrarily high rates. So, because of that we need to find some other alternative at higher frequencies. So, for higher frequencies transistor amplifiers are used. So, instead of op-amp amplifier, we use a transistor amplifier and L C tuned circuits or piezoelectric crystals are used in the beta network.

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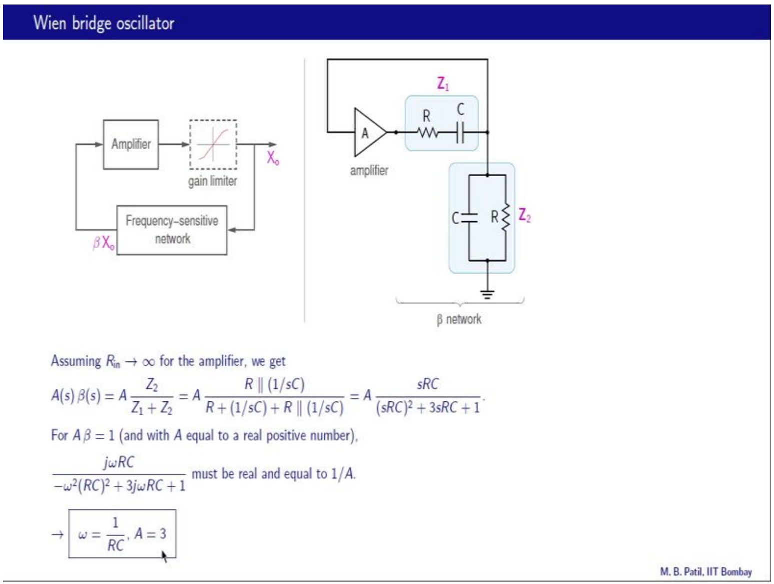

Let us start with the Wien bridge oscillator and that has this particular beta network. This is our sinusoidal oscillator block; and right now we are not showing this gain limiter in this circuit, but when we look at a practical implementation, we will see how that can be handled. In this slide, let us focus on the beta network and then we will see how it can be all combined with the amplifier and the gain limiter. So, here is our beta network it has got this R and C in parallel and the same R same C in series here. So, this we will call as our impedance Z 2 and this as impedance Z 1. And now we need to calculate beta for this network.

And before we proceed let us specifically mention that this assumption is crucial here. This amplifier is assumed to have infinite input resistance, and therefore this current is equal to be 0, and therefore whatever we calculate as beta will actually hold when we connect that network to a real amplifier. Let us now calculate beta. Instead of beta, we will actually calculate A times beta because if you recall the condition for oscillation is a times beta equal to 1. So, it makes sense to calculate this product. And what we will assume is that the frequency of oscillation falls in the range in which this amplifier has a flat frequency response that means it is gain is just a constant and that constant we will denote by the serial number A.

And now let us see what beta should be, what is beta, beta is given by X f which is beta X o divided by the output here. X f is the quantity getting feedback to the amplifier and X o is the actual output voltage. Coming back to our V n base oscillator, where is our V o, V o is is this; what is our X f or V f - the quantity that is getting feedback that is here, that voltage is getting feedback to the amplifier. And therefore, our beta is the ratio of V f and V o and that by a voltage division is Z 2 by Z 1 plus Z 2 like that. So, now it is a matter of substituting for Z 2 and Z 1. What is Z 2, it is R parallel 1 over s C that; and what is Z 1 it is R plus 1 over s C. So, this is Z 1, this is Z 2; Z 2 by Z 1 plus Z 2. And we can simplify this and obtain this expression here s R C in the numerator divided by s R C whole squared plus 3 s R C plus 1.

Now, please do not take my word, sit down with your pen and paper, start with this expression and make sure you actually arrive at this result. So, what do we do next, next we use the condition that A beta equal to 1, and A we will take as a real positive number that is the gain of this amplifier. So, what it means is that this part Z 2 times Z 1 plus Z 2 which is the same as this part here should be equal to 1 over A, so that the product of these two terms is 1.

Let us substitute s equal to j omega in this expression. And then this is what we get we have j omega R C in the numerator, which is purely imaginary number; and we have this expression here in the denominator. The denominator has two parts the real part which is 1 minus omega squared R C squared; and an imaginary part which is 3 j omega R C. Now if this ratio has to be real clearly the denominator also must be purely imaginary, and therefore 1 minus omega squared R C squared must be 0.

So, that gives us the condition for oscillation the frequency required for oscillation that is omega equal to 1 over R C. So, when the circuit oscillates at this particular frequency this cancels and then we get j omega R C divided by 3 j omega R C or just 1 by 3 that 1 by 3 should be equal to 1 by A and that gives us the gain of the amplifier which is 3. So, if we hook up this circuit and make sure that the amplifier has a gain of 3, then it will oscillate at omega equal to 1 over R C.

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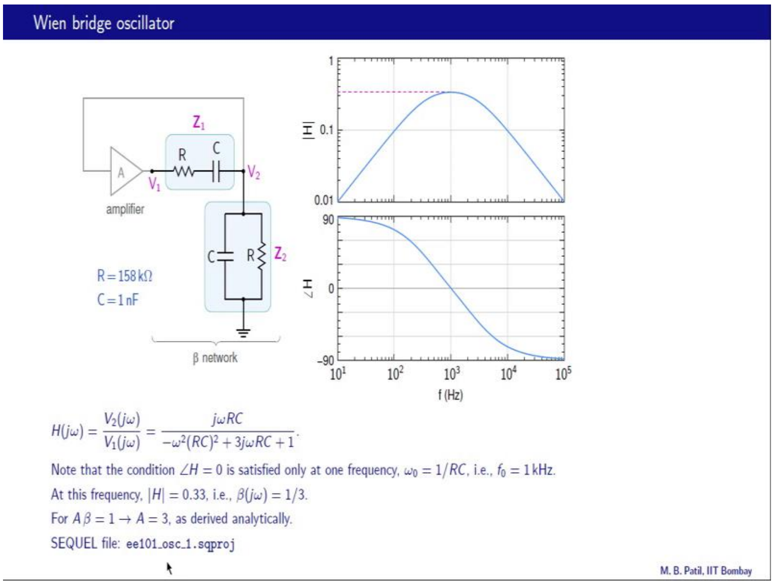

Let us now look at the magnitude and angle plots for the transfer function of this beta network and that is V 2 by V 1, the same as our beta of j omega. So, this H of j omega is identical to what we have called beta of j omega earlier. Here is the angle plot, at low frequencies the angle tends to 90 degrees; and at higher frequencies, the angle tends to minus 90 degrees. And you should be able to figure that out also from the transfer function here. Take that as homework. And at some point in between angle H becomes 0; this is our 0 and that frequency in this case happens to be 10 raised to 3 or 1 kilohertz.

So, the condition that angle H equal to 0 is satisfied only at one frequency and that frequency as we are seen earlier is 1 over R C; the angular frequency. And in this specific example with r equal to 158 k, and C equal to 1 nano farad that frequency turns out to be 1 kilo hertz. Now, let us see what the magnitude of H of j omega is at this frequency, this is going to be our frequency of oscillation. So, let us see what the magnitude of H is or magnitude of beta is. So, this is 10 raised to 3 hertz and that is our magnitude.

This is 0.1, 0.2, 0.3, so that turns out to be 0.33 or beta of j omega then is 1 over 3, it has angle 0. So, it just a real number with magnitude 1 by 3. And for A beta equal to 1, what do we need, we need A equal to 1 over beta which is simply 3; and in fact, this is the same result that we derived analytically. Here is the sequel file for this circuit.

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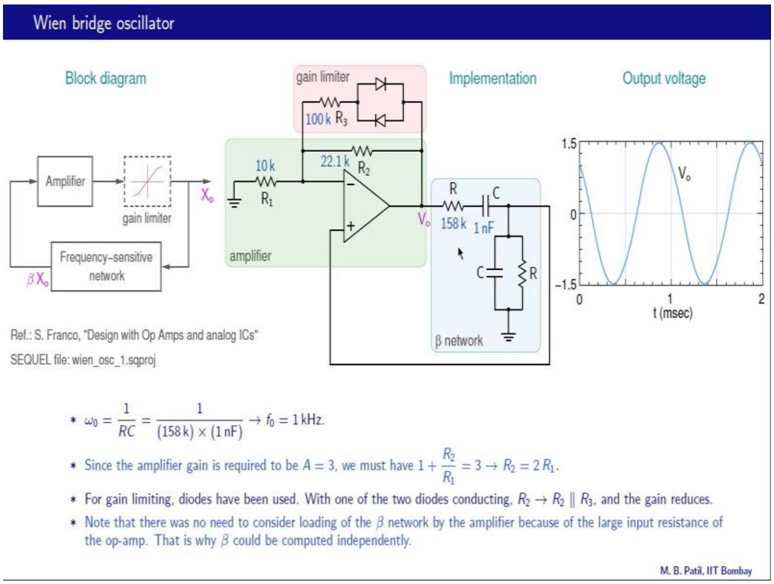

So, here is the complete oscillator circuit based on the beta network that we have been talking about and that is called the Wien bridge oscillator. Now, let us relate the circuit with the block diagram we have looked at. We have an amplifier here. And in this circuit diagram, the amplifier is here. What kind of amplifier is it, it is essentially a non-inverting amplifier and the gain would be 1 plus R 2 by R 1 then there is this gain limiter block in the block diagram and that is right here.

In fact, this gain limiter may be considered to be a part of this amplifier here; although here we have shown that separately. And if you recall we have looked at this register diode network and we have seen that as this voltage changes the resistance or the d V dI for this network also changes, so that is the gain limiter. What about the frequency sensitive network that is this beta network and we have looked at that in detail.

The oscillation frequency as we have seen earlier is 1 over R C in gradients per second. And when we substitute these values which are the same as what we saw in the last slide. The frequency of oscillation in hertz turns out to be 1 kilohertz and that is very clear in this output voltage plot. We have 2 milliseconds here and we have exactly 2 cycles in 2 milliseconds. So, therefore, the period is 1 millisecond that means, the frequency is 1 kilohertz. Now, since the amplifier gain is required to be A equal to 3 as we saw in the last slide we must have 1 plus R 2 by R 1 equal to 3 and that tells us that R 2 must be equal to 2 times R 1 and that is what we have over here. This is 10 k, so this should be twenty k in practice it is made slightly larger, so that the oscillations get started at power up condition; and you can read more about that in this book.

Let us see how gain limiting is happening in this circuit. This is our gain limiter network, and as we have seen when this voltage increases either in the positive or negative direction, one of these diodes conducts and then R 2 gets replaced by R 2 parallel R 3. As that happens this one plus R 2 by R 1 will go down because R 2 has now got replaced with a smaller resistance R 2 parallel R 3 and therefore, the gain will go down, so that is how gain limiting is happening here.

Note that there was no need to consider loading of the beta network by the amplifier because of the large input resistance of the op-amp. So, this is our non inverting configuration and we have calculated the input resistance of this amplifier and we have seen that it is in fact, even larger than the op-amp input resistance. Therefore, we can just ignore this current completely. And what it means is that whatever beta we calculated for this network in isolation is also valid when we connect the beta network and the amplifier together.

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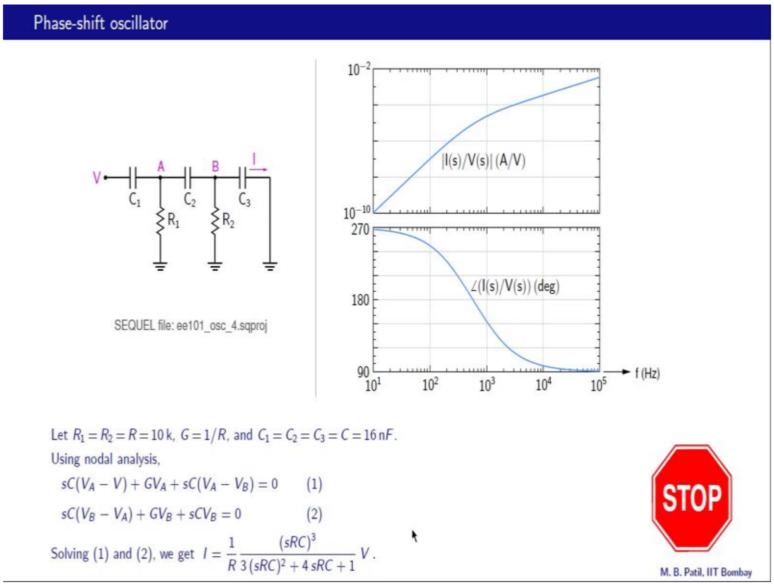

Here is another commonly used beta network and that forms the basis of this circuit called the phase shift oscillator. We will look at the entire circuit late; for now let us just focus on this beta network. This is going to be the output of our oscillator which we have called X o earlier and this current is going to be feedback into the amplifier. So, this current is our X f and the ratio of X f by X o, which is our beta is in this case this current divided by this voltage and of course, both I and V are phasor.

So, let us calculate that first. We will assume that R 1 and R 2 are equal and we will denote that by R that is 10 k in this example. And we will use g to denote 1 by R that makes our equations a little easier to look at. Also the capacitance is C 1, C 2, C 3 are all equal and each one of them is 16 nanofarads. So, to get I by V, let us use nodal analysis. So, at node A, we have three currents that, that and that. What is this current, it is V A minus V times is C 1, C 1 is the same as C that is this term here. What is this current, it is 1 over R 1 times V A that is the same as G times V A, where G is 1 over R 1 which is 1 over R. And this current that is V a minus V b times s C 2 which is the same as s C, so that is the third term over here.

Similarly, let us write KCL at this node these three currents now. What is this current, it is V B minus V A times s C 2 which is the same as s C. This current is G times V b second term; and this current is V b minus 0 times s C 3, so s C times V b. So, these are the two equations that we have and we have two unknowns here V A and V B, we can solve for these two unknowns and then obtain I; once we get I, we can get I by V. Solving equations one and two, we get I equal to 1 over R times this expression times V. So, I by V is all of this. And of course, it has got units of conductance given by this 1 over R, s R C is dimensionless.

Let us now take a look at the magnitude as well as angle of I by V as a function of frequency. And these are computed with these specific numbers here R equal to 10 k and C equal to 16 nanofarads. So, this is the frequency axis frequency in hertz. This is the magnitude of I by V; and this is the angle of I by V.

Let us look at the angle plot at low frequencies the angle approaches 270 degrees and at higher frequencies the angle approaches 90 degrees and that should be also obvious from this expression here. What happens at low frequencies, this term is small, and this term is also small. So, in the denominator we have only one. In the numerator, we have s times s times s, so that is j times j times j which gives us minus j, minus j as a phase of minus 90 degrees which is the same as 270 degrees. What about higher frequencies at higher frequencies we can ignore this one here, we can also ignore this 4 s R C, and then we end up with s cubed divided by s squared, so that is s, s is the same as j omega and that is a phase of 90 degrees. So, that is why we have a phase of 90 degrees as f tends to infinity.

Now we are not really interested in these points. So, much we are interested in the value of the frequency at which the phase is either 180 degrees or 0 degrees, because at those points our I by V is going to be a real number. And assuming our amplifier gain is real that is the only possibility we have of making A times beta equal to 1. Here is 180 degrees and let us see what frequency that corresponds to. We go down and that is the frequency that we are interested in this is 100, 200, 300, 400 maybe about 500, so that is approximately the frequency of oscillation if we connect this beta network to a suitable amplifier.

And what is going to be the gain of that amplifier to get an idea of that we need to look at the magnitude of I by V at this frequency, so that is there. So, somewhere here this is 10 raised to minus 10, minus 9, minus 8 minus 7, minus 6, minus 5. So, the magnitude of I by V and the frequency of oscillation is about 10 raised to minus 5 amps by volts. So, let us remember these numbers frequency of oscillation about 500 hertz and magnitude of I by V at that frequency about 10 raised to minus 5 amps by volts. Our stop sign has come that means, we must have left something incomplete. So, this expression you need to really sit down and derive from these two equations is going to be one or two pages of algebra, but it is definitely worthwhile.

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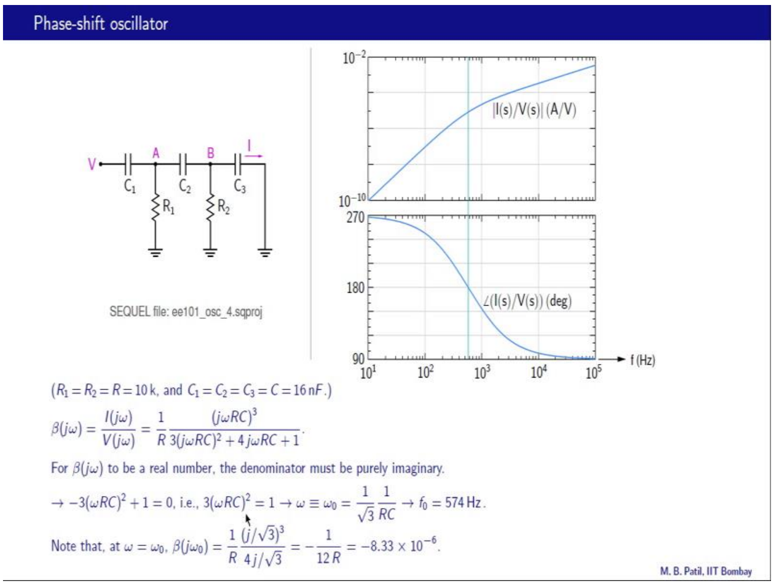

Let us now find our frequency of oscillation by analytical means and we will also find the gain of the amplifier that is required for oscillations to occur. Here is our beta as a function of j omega and that is of course, obtained by substituting s equal to j omega in our previous expression, and this is what we get. Now, we want beta to be a real number. Assuming that our amplifier again is going to be real that is it has no frequency dependence. What does it imply then it implies that this numerator divided by denominator must be a real number. Now, the numerator has j cubed in it that is j times j times j or minus j. So, the numerator is a purely imaginary number. And for beta to be real, what we require is the denominator should also be purely imaginary.

So, let us look at the denominator now, there is a j squared here. So, that is minus 1, this is purely imaginary. So, the real terms here are this and that. So, we have minus 3 times omega R C whole squared that minus 1 is coming from j squared here plus 1, 1 is coming from here and that should be equal to 0 that leaves only this purely imaginary number over here and then beta will be real in that case. So, that gives us omega 3 times omega R C squared equal to 1. So, omega R C would be 1 by square root 3. And therefore, omega would be 1 over square root 3 times 1 by R C. And we can obtain f 0 by dividing omega 0 by 2 pi and that gives us 574 hertz. And if you recall graphically we estimated our frequency of oscillation to be about 500 hertz somewhere here, when the phase was 180 degrees. So, this agrees well with that.

Now, let us find the value of beta at omega equal to omega 0. What happens in the denominator, we have already seen that this term and this term will cancel, and we are left with 4 j omega R C. Now, at omega equal to omega 0, omega times R C is 1 over square root 3. So, we can substitute for omega R C, 1 over square root 3 both in the numerator and in the denominator and that will give us beta like that. So, beta at omega 0 is 1 over R that term as it is times j by square root 3 cubed and this is coming from j omega R C, omega R C is 1 by square root 3, so that is that.

And in the denominator, we have 4 j times omega R C and omega R C again is 1 over square root 3, so that is what we get. Now this j cubed is j times j times j, so that is minus j all together j will cancel with this j and we are left with a minus sign, and this whole thing turns out to be minus 1 by 12 R. And with R equal to 10 k, this turns out to be minus 8.33 10 raise to minus 6, and the units of this are ampere per volt. And if you recall, we had estimated this quantity as well from the plots here that is the magnitude of beta at omega equal to omega 0. And we had estimated that to be about 10 raise to minus 5 in the last slide. So, this does agree well with that estimator.

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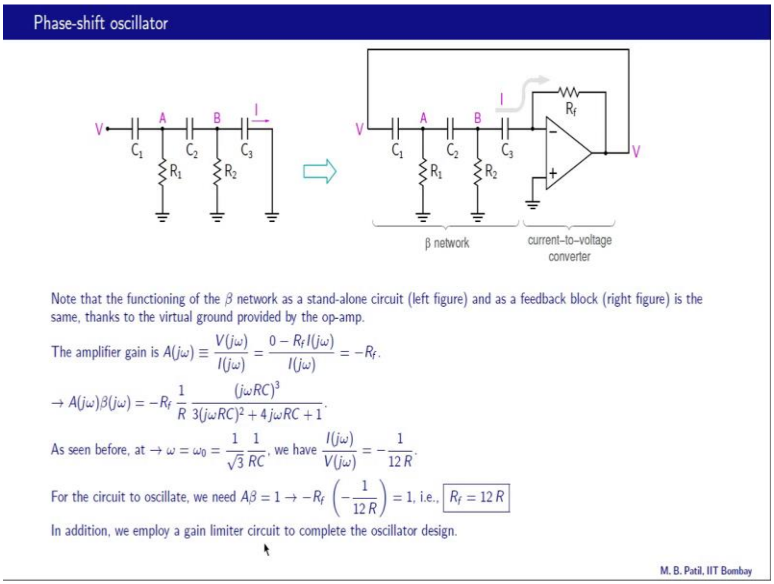

We can now combine our beta network with a suitable amplifier to make up the complete phase shift oscillator circuit and that is shown over here. This is the beta network, and this is the amplifier. Shown on the left here is our beta network as a standalone network, and this is what we considered earlier to derive the relationship between I and V here. Now, this network and this one as it appears inside the oscillator are actually equivalent, because look at this node, this node is at 0, and this node is also at 0, because for the op-amp V minus and V plus are at the same potential. So, this is at virtual ground. So, therefore, whatever we derived for this circuit also applies for the beta network as it appears inside the oscillator circuit.

Notice that our amplifier in this case is not a voltage to voltage amplifier, but it is a current to voltage amplifier. And the reason for that is easy to understand. What is our beta it is I divided by V and that has got units of conductance; and since we want A times beta to be one that is a dimensionless quantity, the units of A must be resistance and that is what a current to voltage converter gives us. Let us now find that gain that is V divided by I. So, the amplifier gain is a equal to V by I, and what is V, it is V minus minus this voltage drop V minus is 0, the same as V plus; and this voltage drop is I times R f. So, we have 0 minus R f times I divided by I simply minus R f, so that is the gain of the amplifier.

We can now write A times beta as minus R f. This is our A times beta this whole expression is our beta, and we want that to be equal to 1 for oscillations. And we have done that calculation already and we found that for this beta to be real we require omega defined as omega 0 to be equal to 1 over square root 3 times 1 over R C. And if we substitute that then I over V is minus 1 by 12 R. So, this quantity which is beta of j omega is minus 1 by 12 R at omega equal to omega 0.

So, therefore, for the circuit to oscillate, we need A beta equal to 1 and that implies that minus R f times this quantity, this is our beta at omega equal to omega 0 must be equal to 1 and that gives us the condition for R f; R f must be 12 times R. So, this feedback resistance here must be 12 times this R, R 1 and R 2 are equal and each one is equal to R; similarly C 1, C 2, C 3 are equal and each one is equal to C. So, that is almost the complete phase shift oscillator except we need to employ a gain limiter circuit to complete the oscillator design. So, let us see how that is done.

To summarize, we have seen how a sinusoidal oscillator is implemented in practice. We have seen two examples namely the Wien bridge oscillator and the phase shift oscillator. In the next class, we will see how amplitude control can be implemented in a sinusoidal oscillator. See you in the next class.