Basic Electronics
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Lecture - 05
Phasors

Welcome back to Basic Electronics. So far we have looked at circuits with dc sources, there are many situations in which there is a sinusoidal voltage source in the circuit and in particular we are interested in the solution in the so called sinusoidal steady state. In this lecture we will first look at the meaning of the term sinusoidal steady state; we will then look at a convenient way to represent voltages and currents in that situation using a new concept called Phasor. We will also look at how phasors can be used to represent R L and C in the sinusoidal steady state.

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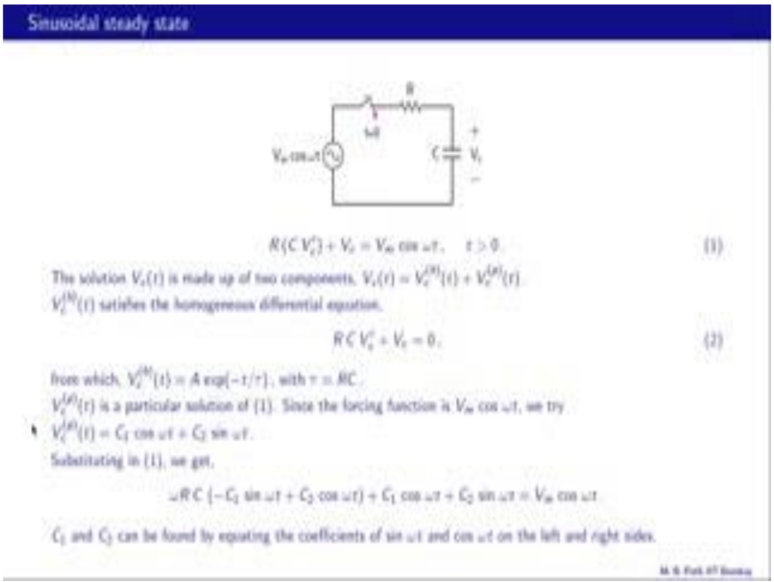

So let us begin; let us try to understand the meaning of this term sinusoidal steady state with help of this example, it is an R C circuit with a sinusoidal input voltage, there is a switch here which closes at t equal to 0 and initially the capacitor is uncharged; that means, V c is 0. Let us begin with the circuit equation this equation 1 here, what does it say? It says that this voltage drop plus this voltage drop must be the same as the source voltage, this voltage drop is R times the current and the current is c b b c d t. So, that is what this says here R times C V c prime, plus V c must be equal to V m cos omega t, for t greater than 0 that is when the switch is closed.

The solution V c of t is made up of 2 components, a homogeneous component indicated with this superscript h here and a particular component indicated with this superscript p. The homogeneous component V c h t satisfies the homogeneous differential equation, R C V c prime plus V c equal to 0. So, we drop the source term and that is all we get this equation and this equation has this solution A e raise to minus t by tau, with tau equal to R times C. Let us now look at the second part the particular solution, since the forcing function is V m cos omega t, we can try V c p of t equal to C 1 cos omega t, plus C 2 sin omega t as a candidate solution and when we substitute this in equation 1, we get this equation here and the constants C 1 and C 2 can be found by equating the coefficients of sin omega t, and cos omega t on the left and right sides.

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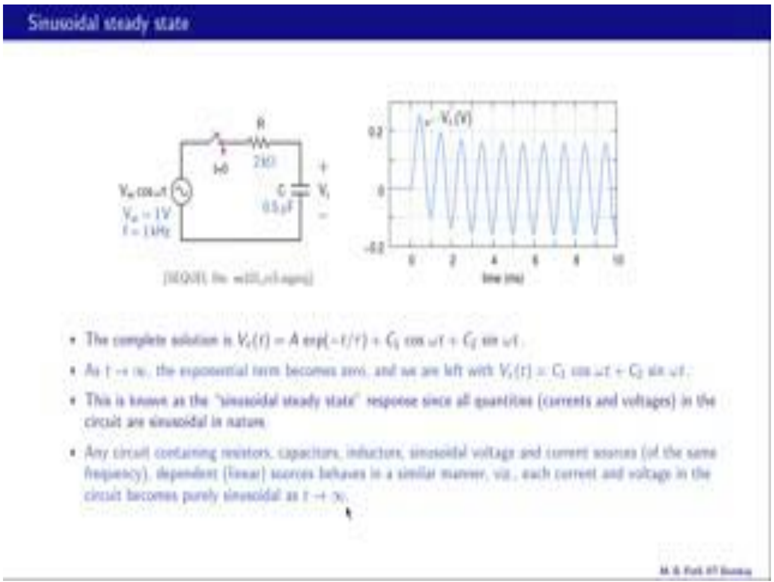

Here is a specific example with V m equal to 1 volts, f is equal to 1 kilo hertz 2 k here; 0.5 micro here for the capacitance and then we get this V c shown in this figure. The complete solution is A e raise to minus t by tau from the previous slide, that is the homogeneous part plus C 1 cos omega t, plus C 2 sin omega t this is the particular part of the solution. Now as t tends to infinity, the exponential term becomes 0 this goes to 0 and we are left with V c of t equal to C 1 cos omega t plus C 2 sin omega t this part here.

And we can observe that in this plot as well, this is our t equal to 0 that is the time when the switch closes, and in the beginning there is some exponential transient and then finally, the transient vanishes and we have the sinusoidal steady state. So, this is known as the sinusoidal steady state response, since all quantities currents and voltages in the circuit are sinusoidal in nature and this turns out to be generally true for any circuit, containing resistors capacitors inductors, sinusoidal voltage and current sources dependent sources such as CCCS, CCVS etcetera. So, any circuit containing these components behaves in a similar manner that is each current and voltage in the circuit becomes purely sinusoidal as t tends to infinity. So, each quantity each current and each voltage, in the circuit would have this form in the sinusoidal steady state ok.

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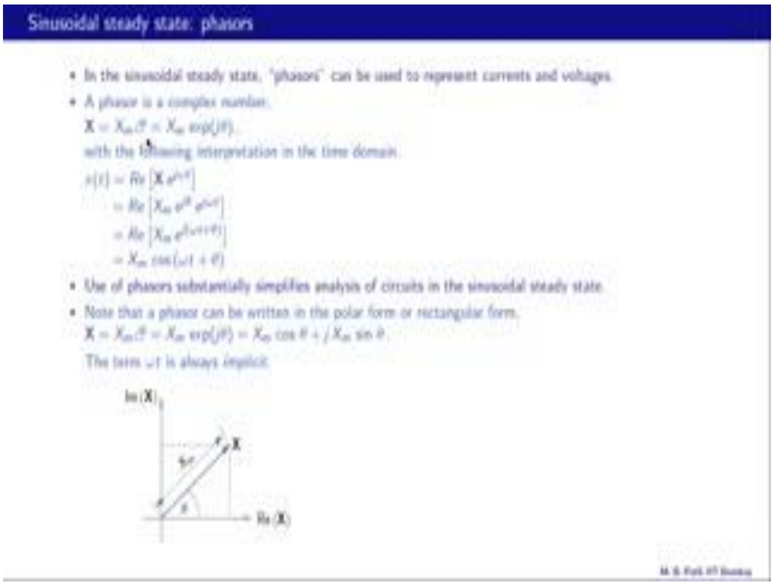

Let us continue and now we want to introduce Phasors. In the sinusoidal steady state phasors can be used to represent currents and voltages. So, let us see what a phasor is a phasor is. A complex number that is why it is written in boldface here, it has got a magnitude of X m and an angle of theta. So, that is the phasor X and we can rewrite this as X m here is to j theta, and it has the following interpretation in the time domain. Corresponding to the phasor X we have a time domain quantity denoted by X of t and the interpretation is like this X of t is D l part of this number that is phasor X multiplied by e raised to j omega t.

So let us see what that turns are to be; our phasor is nothing but X m times e raised to j theta that gets multiplied by e raised to j omega t. Now we can combine these 2 terms to get e raised to j omega t plus theta, what is e raised to j omega t plus theta? It is cos of omega t plus theta, plus j sin of omega t plus theta and we are only taking the real part of that expression, so therefore, we get X m cos omega t plus theta. So, that is what a phasor X corresponds to in the time domain.

Let us make some more comments, use of phasors substantially simplifies analysis of circuits in the sinusoidal steady state and we will look at some examples of that. Note that a phasor can be written in the polar form or rectangular form, this is the polar form X m angle theta also called the magnitude angle form, that is the same as X m e raise to j theta and that can be written as X m cos theta plus j X m sin theta this is called the rectangular form. This figure shows how the polar and rectangular forms of X can be represented, this axis is the real part of X which is the complex number, this axis is the imaginary part of X, this is our phasor it has got a magnitude of X m and angle of theta.

Alternatively we can also write this component which is the real component of X and this component which is the imaginary component of X, and that gives us the rectangular form. Now one final remark very important the term omega t is always implicit. So, when we write a phasor for example X here, we do not write omega t over there, but it is understood that omega t is always implicit and that is how it comes in this time domain expression here.

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Let us now take a few examples to see how something can be represented in the time domain and in the frequency domain. Here is a voltage V 1 of t 3.2 cos omega t plus 30 degrees volts, we can write this in the phasor form; the magnitude is 3.2 and the angle is 30 degrees. So, that is what the phasor looks like 3.2 angle 30 degrees.

We can also write that as 3.2 e raise to j pi by 6, because 30 degrees is the same as pi by 6 radians. Another example I of t is minus 1.5 cos, omega t plus 60 degrees. Now it is not quite in the form that we would like, because we have this minus sin over here. So, let us rewrite this expression as 1.5 cos omega t plus pi by 3 that is 60 degrees minus pi; this minus pi accounts for this minus sin here and that follows form integrametric identities. Now we can combine these, these 2 terms and get omega t minus 2 pi by 3 and now we can write the corresponding phasor like that. Now we took 1.5 and angle minus 2 pi by 3.

Next example V 2 of t equal to minus 0.1 cos omega t; once again we do not want this minus sin here so we rewrite this expression as 0.1 cos omega t plus pi, this plus pi accounts for this minus sin over here and note that we can either write plus pi here or we can write minus pi like we did in the last example here. Now the corresponding phasor is magnitude 0.1 and angle equal to pi; next we have I 2 of t equal to 0.18 sin omega t. Now we require cos over here not sin so therefore, let us rewrite this expression as 0.18 cos, omega t minus pi by 2 and now we can write the corresponding phasor like that, I 2 equal to 0.18 that is the magnitude 0.18 angle minus pi by 2, so that is the angle. Let us take an example of the reverse transformation now, the frequency domain description is given phasor I 3 that is 1 plus j 1 in amperes.

Now we want to convert that into the time domain form, now this is in the rectangular form we first convert that into the polar form like that. So, they are square root 2 angle 45 degrees, and this we can write in the time domain directly like that.

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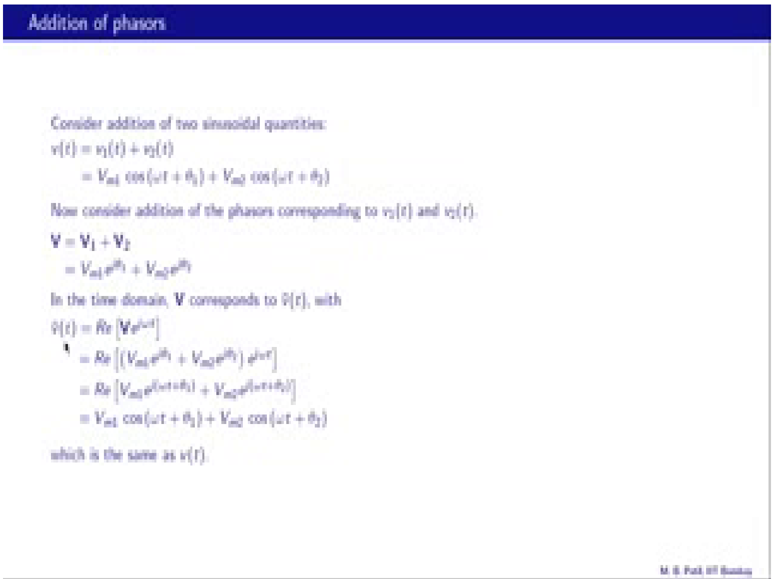

So I 3 of t is square root 2 cos omega t plus 45 degrees, and remember always that this omega t is implicit. So, we do not write that in the phasor, but we understand that it is always there. Let us now talk about addition of Phasors, first let us consider adding 2 sinusoidal time domain quantities like this V of t equal to V 1 of t, plus V 2 of t. V 1 of t is given by V m 1 cos omega t plus theta 1, and V 2 of t is given by V m 2 cos omega t plus theta 2. Now consider the addition of the corresponding Phasors; this phasor corresponds to V 1 of t, this phasor corresponds to V 2 of t. So, V 1 therefore, is V m 1 e raise to j theta 1 and it would V m 1 and angle theta 1. V 2 similarly is V m 2 e raise to j theta 2. In the time domain this phasor V which is V 1 plus V 2 corresponds to V tilde t, with V tilde t equal to real part of phasor V times e raised to j omega t.

As we have seen earlier this is how a phasor can be written in the time domain. So, now, we substitute for V that is V m 1, e raise to j theta 1, plus V m 2 e raise to j theta 2 like that; and now we can combine this e raise to j theta 1 and e raise to j omega t to obtain e raise to j omega t plus theta 1 and similarly e raised to j omega t plus theta 2 here. Now we need to take the real part of this entire expression, and that turns out to be V m 1 cos omega t plus theta 1, plus V m 2 cos omega t plus theta 2. This in fact, is the same as V of t right there. Now what is the conclusion? The conclusion is that if we have 2 sinusoidal varying quantities such as V 1 of t and V 2 of t here, we can obtain their sum by either directly adding them or we can add the corresponding phasors like here and then from the resulting phasor V here, we can obtain the time domain quantity like that.

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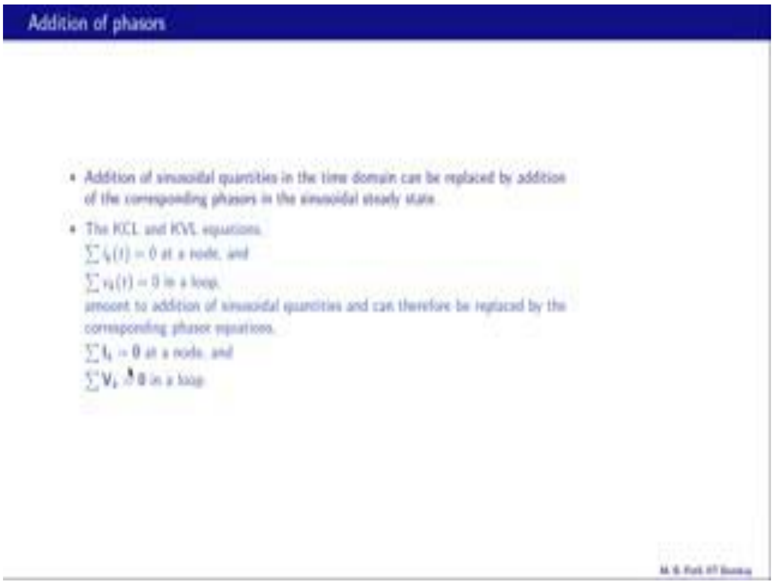

So from the last slide we can say that addition of sinusoidal quantities in the time domain can be replaced by addition of the corresponding phasors in the sinusoidal steady state. Now this is very important for us because of the following point; the KCL and KVL equations, that is sigma I k of t equal to 0 at a node that is all currents at a given node add up to 0 and sigma v k of t equal to 0 in a loop, that is all voltages as we go along a loop add up to 0.

These equations amount to addition of sinusoidal quantities and can therefore, be replaced by the corresponding phasor equations, what are the phasor equations? Sigma of the phasor currents equal to 0 at a node and sigma of the phasor branch voltages is equal to 0 in a loop.

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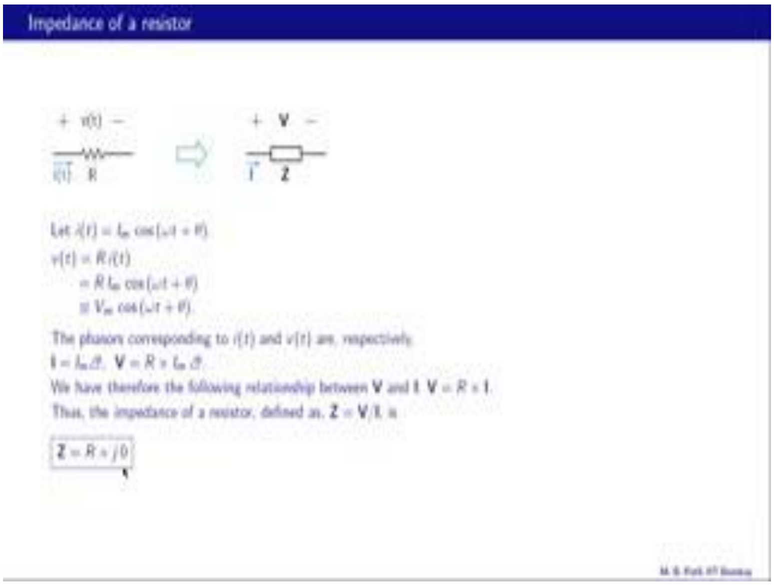

Let us now introduce the concept of impedance starting with impedance of a register first; here is a register in the time domain that is the register current I of t, and V of t is the voltage across the resistor.

Now in the frequency domain we replace this current with phasor I and the voltage with phasor V; and now we can write phasor V equal to phasor I times something called Z, where Z is called the impedance and let us actually show that we can do that. Let I of t V I m cos omega t plus theta, then we have V of t equal to R times I of t that is valid at all times and that is equal to R times I m, cos omega t plus theta after substituting for I of t from there, and we might define that as V m cos omega t plus theta where V m is R times I m. Now the phasors corresponding to i of t and v of t are respectively for I we have I m angle theta, I m is the magnitude and theta is the angle; for V we have R I m angle theta, R I m is the magnitude and theta is the angle.

We have therefore, the following relationship between phasor V and phasor I, that is V is equal to R times I as simple as that and we can see that from here V is R I m angle theta I is I m angle theta. So, therefore, V is simply R times I and therefore, the impedance of a register which is defined as Z equal to V by I is, Z equal to R plus j 0 from here Z is equal to R and R is a real number. So, we write that real number as R plus j 0. So, the impedance of a resistor is the resistance itself.

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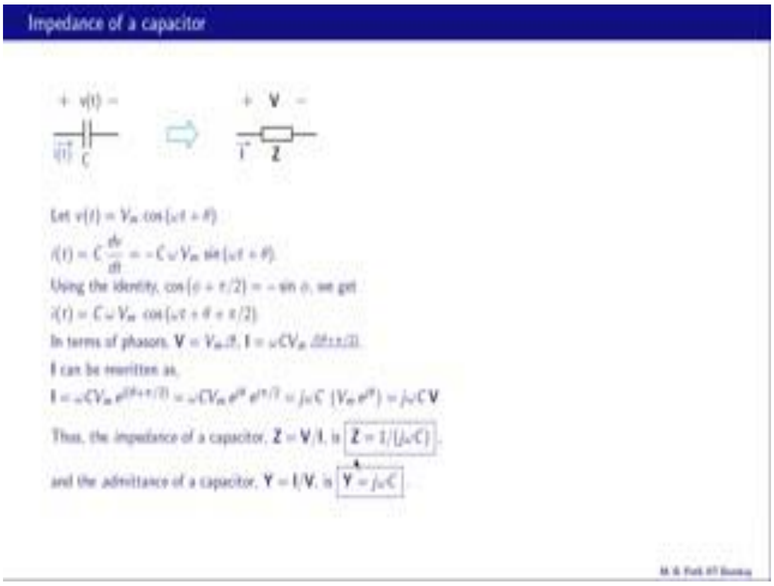

Let us follow the same process now for a capacitor, and find it is impedance marked as Z over here again.

So let this voltage across the capacitor v of t be equal to V m cos omega t plus theta, what is the current? Current in that direction it is C d v d t and when we differentiate this quantity we get minus c omega V m sin omega t plus theta. And now we can use the identity cos phi plus pi by 2 is minus sin pi and we get I of t equal to C omega V m these 3 terms here, times cos of omega t plus theta plus pi by 2 like that. In terms of phasors phasor V is V m angle theta, V m is the magnitude and theta is the angle and phasor I is omega C V m angle theta plus pi by 2. Omega C V m is the magnitude and theta plus pi by 2 is the angle.

Now we can rewrite I as I equal to omega C V m, e raise to j theta plus pi by 2 that is omega C V m, times e raise to j theta times e raised to j pi by 2 and this quantity e raised to j pi by 2 is nothing but j. So, we get then j omega C times, V m e raise to j theta. Now this quantity is nothing but the phasor V and therefore, we can write I as j omega C times phasor V. In short phasor I is equal to j omega C times phasor V, and therefore the impedance of a capacitor Z is equal to V by I is Z is equal to 1 over j omega C and the admittance which is defined as Y equal to I by V is Y equal to j omega C notice that we have omega here now; that means, at low frequencies the impedance of a capacitor is large and at high frequencies the impedance is small.

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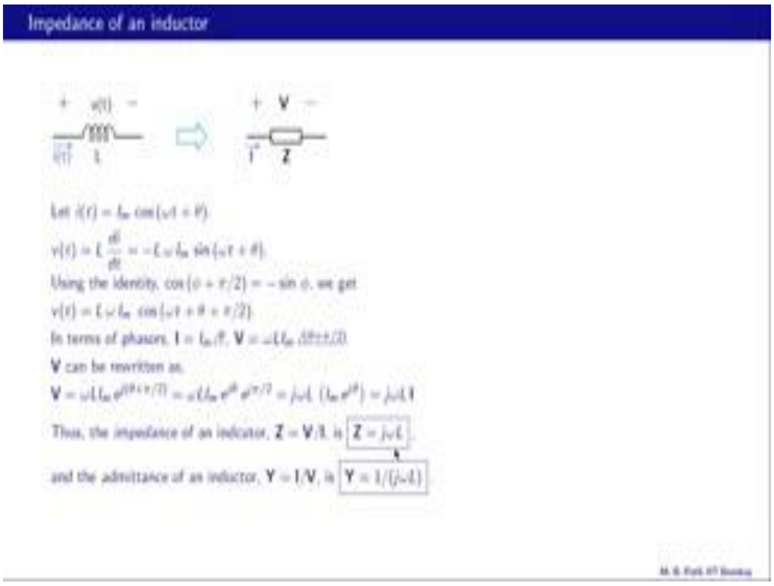

Let us now repeat the process for an inductor let i of t b i m cos omega t plus theta, then we get v of t as l di dt, we can differentiate I of t and get v of t equal to minus l omega I m times sin omega t plus theta.

Now, we can use the identity cos phi plus pi by 2 is equal to minus sin phi and get v of t has L omega I m times cos, omega t plus theta plus pi by 2. So, we have replaced this minus sin omega t plus theta by cos omega t plus theta plus pi by 2. Now in terms of phasors we have I equal to I m angle theta, I m is the magnitude and theta is the angle and V equal to omega l I m; omega l pi m is the magnitude angle theta plus pi by 2 that is the angle and V can be rewritten as V equal to omega l I m times e raised to j theta plus pi by 2, that is omega l I m e raised to j theta times e raised to j pi by 2, this is nothing but j; so that becomes j omega l times I m e raised to j theta and this quantity is nothing but our phasor I.

So finally, we get phasor V equal to j omega l times phasor I and from there the impedance of an inductor Z equal to V by I is Z equal to j omega l, and the admittance of an inductor Y equal to I by V is Y equal to 1 over j omega l. Notice that the inductor impedance also depends on the frequency, but it is directly proportional to the frequency; so at low frequencies the impedance is small and at high frequencies the impedance is large.

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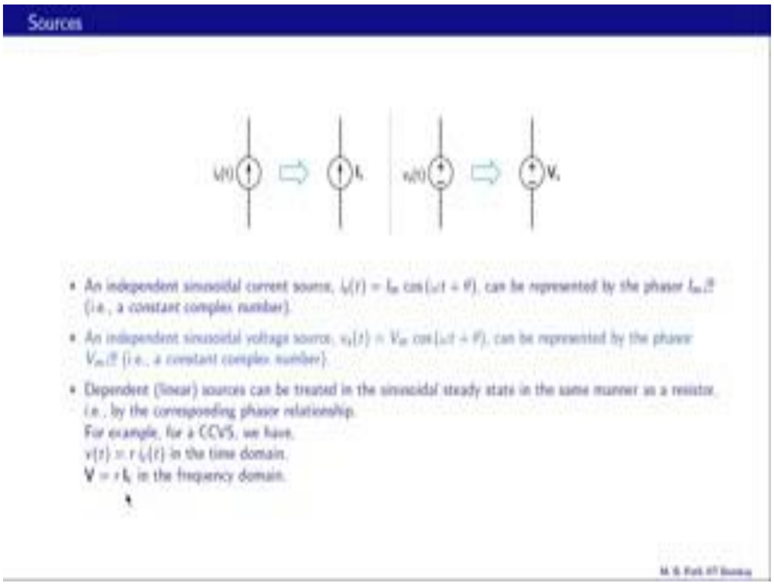

Let us now look at sources sinusoidal current source and sinusoidal voltage source and independent sinusoidal current source I s of t equal to I m cos omega t plus theta can be represented by the phasor, I m angle theta magnitude I m angle theta that is simply a constant complex number. So, we have this source in the time domain and the corresponding phasor in the frequency domain, and that I s is a constant. Similarly an independent sinusoidal voltage source v s of t is equal to V n cos omega t plus theta can be represented by the phasor V m angle theta, that is the constant complex number like that. What about dependent sources? Dependent sources can be treated in the sinusoidal steady state, in the same manner as we treated a register that is by the corresponding phasor relationship.

For example, let us consider a CCVS that is current controlled voltage source; in the time domain we have v of t that is the voltage across the CCVS equal to r, a constant times i c of t where I c of t is the controlling current. Now in the frequency domain this relationship becomes V phasor equal to R times I c phasor as simple as that; and similarly we can write relationships for the other dependent sources such as CCCS or VCVS etcetera.

In summary we have seen what phases are and how they can be used to represent currents and voltages in the sinusoidal steady state. We have also seen how R l and C can be represented in the sinusoidal steady state using Phasors. In the next lecture we will extend these ideas to circuits, until then goodbye.